AD QUADRATUM

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A·STUDY·OF·THE·GEOMETRICAL BASES·OF·CLASSIC & MEDIEVAL RELIGIOUS·ARCHITECTURE

WITH SPECIAL REFERENCE TO THEIR APPLICATION IN THE RESTORATION OF THE CATHEDRAL OF NIDAROS (THRONDHJEM) NORWAY

BY

FREDRIK MACODY LUND

Profusely Illustrated by Plans·Sections·Views and Details of notable Temples, Churches, Cathedrals & other buildings in Greece, Italy, Germany, Denmark, France, England & Norway

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TO

THE MEMORY OF

SNORRE STURLASON

THE GREATEST OF ICELANDIC SAGA WRITERS

WITHOUT WHOSE WORK THE NORWEGIANS

WOULD NOT HAVE EXISTED

AS A NATION

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THE CATHEDRAL OF NOTRE-DAME, PARIS

THE CATHEDRAL OF COLOGNE

ENGLAND

OTHER CHURCHES OF CENTRAL EUROPE

DENMARK

NORWAY

THE SECTIO AUREA

NOTE

Nidaros, Bjorgvin, and Oslo, are the old names of the cities of Throndhjem, Bergen, and Christiania.
The Norwegian symbols of division and subtraction are used in the text : $: = \div$; $\div = -$.
The decimal fractions in the text have unfortunately been printed as $5m5$ instead of $5{\cdot}5m$, and so on.

Owing to omissions, alterations, and wrong renderings due to the translator, the following list of corrections and insertions is unfortunately long.

ERRATA ET CORRIGENDA

Page 1, line 3, *read* design *for* jet.
,, 1, ,, 8, *read* logical *for* consequent.
,, 2, ,, 1, *read* determines *for* determinates.
,, 2, lines 13 and 14, *read* logic *for* consequences.
,, 2, line 29, *read* eo *for* ed.
,, 3, ,, 15, *read* on *for* over.
,, 3, ,, 32, *insert* oblique *before* lines of direction.
,, 3, ,, 35, *read* obviously *for* probably.
,, 3, ,, 36, *read* perhaps *for* even.
,, 4, ,, 13, *read* prædictam *for* prædictum.
,, 5, ,, 15, *read* cella *for* whole construction.
,, 7, ,, 5 from bottom, *read* Norman *for* Romanesque.
,, 7, ,, 5 footnote, *read* editor *for* publisher.
,, 8, ,, 7, *read* theory *for* teaching.
,, 8, ,, 30, *read* laity *for* faithful.
,, 8, ,, 3 from bottom, *read* logical *for* consequent.
,, 17, ,, 4 from bottom, *read* 1096 *for* 1070.
,, 17, ,, 3 from bottom, *insert after* squares : towards east.
,, 23, ,, 6 from bottom, *insert* Max Hasak, l.c. 288.
,, 24, ,, 2, *read* symbolical *for* imaged.
,, 27, ,, 3, *read* $360 \div 5$ *for* $360 + 5$.
,, 33, ,, 8, *read* radius *for* line.
,, 40 footnote, *read* V. *for* vol.
,, 50, line 8 from bottom, *read* feeling *for* judgement.
,, 52, ,, 9, *read* 30 *for* 50, and in note 42, 5 *for* 425.
,, 54, ,, 2 from bottom, *read* (B) *for* B.
,, 55, ,, 16, *read* (B) *for* B.
,, 59, ,, 17 from bottom, *nsert after* House : east to west.
,, 82, ,, 8, *read* before *for* about.
,, 85, ,, 4 from bottom, *delete* a.
,, 88, *insert at end of line 8 from bottom :* The stringency and clarity of its plan is visible at first sight.
,, 93 footnote, *insert after* vault : in Chartres.
,, 95, line 21, *read* consciousness *for* subconsciousness.
,, 96, ,, 5 from bottom, *insert at end of line :* without regard to the principle of the five-aisled church in a three-aisled one.
,, 99, ,, 6, *insert at end :* cp. figs. 97 and 98.
,, 99, ,, 18, *insert after* example : of this un-Gothic construction.
,, 99, ,, 24, *insert after* buttresses : there.
,, 100, ,, 17, *insert after* Germany : pp. 22 and 23.
,, 100, ,, 6 from bottom, *read* single *for* arched.
,, 109, Fig. 126, *read* Cathedral of Roskilde *for* Church of Our Lady of Helsingborg.
,, 113, line 2 from bottom, *read* I *for* L.
,, 115, ,, 5, *read* Bjorgynjar *for* Bjorgyngar.
,, 115, ,, 5, *read* of *for* on.
,, 116, ,, 10 from bottom, *read* royal house *for* throne.
,, 119, footnote, line 3, *read* de jure *for* de facto.
,, 119, ,, ,, 8, *insert after* information : given by.
,, 119, ,, ,, 8, *read* regeneration *for* raising.
,, 122, line 3 from bottom, *read* certainly *for* probably.
,, 124, lines 1 and 2, *read* : this strong demand for unity was the essence of medieval culture whose aims were not those of our railway stations' architecture, involving utilitarian requirements at the expense of the artistic ones.
,, 124, line 14, *insert after* independent : as a philosophic art.
,, 124, ,, 18, *insert after* precisely : by scholasticism.
,, 124, ,, 3 from bottom, *insert after* police : asphalted streets.
,, 127, ,, 21, *insert before* temple : grove of the.
,, 128, *insert* vivit *after* beatus.
,, 130, line 2, *read* Radius for Cirkelens.
,, 130, ,, 6, *read* is moved to *for* continues to.
,, 130, ,, 2 from bottom *read* BıF *for* BıT.

Page 134, line 14 from bottom, *read* cuts *for* sections out.
,, 135, ,, 11, *insert at end of line :* This construction is doubtless a popular misunderstanding of the "Hippocratic" formula of remembrance.
,, 135, ,, 2 from bottom, *read* cuts *for* sections out.
,, 136, ,, 13, *read* q, q *for the* g, *twice*.
,, 139, ,, 10, *read* continuous *for* stetige.
,, 143, ,, 12, *read* Anregung *for* Angerung.
,, 145, ,, 6, *read* cathedral *for* grammar.
,, 146, footnote, *read* computistic *for* computative.
,, 147, line 4 from bottom, *read* church *for* cults.
,, 150, ,, 20, *read* irrational *for* unreckonable.
,, 150, ,, 5 from bottom, *read* silent greatness *for* great silence.
,, 152, ,, 15, *read* evident *for* glaring.
,, 157, ,, 14, *read* consciousness *for* subconsciousness.
,, 157, ,, 1 from bottom, *read* part *for* term.
,, 158, ,, 4 from bottom, *read* proportions *for* time.
,, 163, ,, 16, *read* statement *for* data.
,, 163, ,, 25, *read* complete *for* absolute.
,, 164, ,, 13, *read* intersecting *for* sectioning out.
,, 164, note to be deleted.
,, 166, line 11 from bottom, *read* oNB *for* ONB.
,, 167, ,, 11 from bottom *read* a r, *for* a, r.
,, 177, ,, 17, *read* proved *for* enforced.
,, 178, ,, 13, *read* dispositio *for* disposition.
,, 178, ,, 14, *read* cothurnus *for* cothurnum.
,, 178, ,, 31, *insert* Pl. XVI *after* (fig. 181).
,, 181, lines 4 and 11, *read* irrational *for* unreckonable.
,, 182, line 4, *read* irrational *for* unreckonable.
,, 182, under figure, *after* stars *insert* : around the cella.
,, 183, ,, 6, *read* words *for* figures.
,, 185, ,, 5, *read* cathedral *for* grammar.
,, 185, ,, 20 from bottom, *insert after* above : pp. 25–26.
,, 185, ,, 3 from bottom, *read* on *for* at ; congruent *for* identical.
,, 185, ,, 2 from bottom, *insert* congruent and *before* identical
,, 188, ,, 18, *insert* of *after* groins.
,, 188, ,, 20 from bottom, *read* written capital R for a small r.
,, 188, ,, 14 from bottom, *read* (a written R) *for* (the cursive r).
,, 197, ,, 12, *read* uncritical *for* criticless.
,, 197, footnote, *read* l.c. *for* anf. St.
,, 198, line 11, *read* documentation *for* notes.
,, 198, ,, 11 from bottom, *read* Normans *for* Northmen.
,, 198. Instead of the three lines at the bottom before "In this, etc.," *insert :* In spite of this being recognised, opinion and appreciation regarding the Normans at home, and their—i.e. the Norwegians'—national architecture, have been almost the opposite, and strangely self-contradictory.
,, 199, line 1, *read* an echo *for* a reflection.
,, 199, ,, 1, *read* annalists *for* historians.
,, 199, ,, 2, *read* intelligible *for* understandable.
,, 199, ,, 3, *read* described *for* looked upon.
,, 199, ,, 7, *delete* from except to, etc., *and insert* : by being considered as built by foreigners called in, and therefore late imitations of work already achieved in Europe.
,, 200, lines 2 and 3, *read* Norwegian *for* northerner.
,, 200, ,, 14 and 15, *read* norse Norman *for* Northmen.
,, 200, line 26, *insert* emigrated *before* Norwegian.
,, 201, ,, 20 from bottom, *read* Ollamh *for* Allamh.

Page 201, lines 13 and 12 from bottom, *read* heathen *for* Christian.
,, 203, line 16, *read* isolated *for* spontaneously.
,, 203, ,, 16 from bottom, *read* grjotsmid *for* grjötsmeld.
,, 204, ,, 21, *read* consciously *for* obviously.
,, 204, ,, 6 from bottom, *read* unrestrained eloquence *for* ungoverned fertility.
,, 209, lines 8 and 7 from bottom, *insert after* talent: to transfer and develop the ornamental art and splendour of the heathen time.
,, 212, ,, 17–15 from bottom, *delete from* honour *to* stone, *and insert:* were well equipped to erect an edifice which could fully measure itself with the sanctuaries they had admired in their kinsmen's settlements in Northern France.
,, 213, line 13, *read* which *for* whom.
,, 213, ,, 19, *read* cathedral *for* cathedrals.
,, 213, ,, 9 from bottom, *read* Kirkjuvaag *for* Kirkejuvaag.
,, 214, ,, 5, *read* into a society *for* into the society's religion.
,, 214, ,, 10 from bottom, *read* The Kings of the Lodbrok family *for* The Kings of the Lodbrok clans.
,, 214, ,, 6 from bottom, *read* the Lodbrok Kings *for* King of the Lodbrok.
,, 215, ,, 13, *read* seat *for* sit.
,, 215, ,, 15, *read* worthy church over *for* dignified church on.
,, 216, footnote, *read* "tyrannic" *for* "hard judgement."
,, 217, line 4, *read* Officium *for* Office.
,, 217, ,, 24, *read* flock *for* stream.
,, 217, ,, 1 from bottom, *read* heathen cult *instead of* cult of the "blot."
,, 219, ,, 6 from bottom, *read* meetings *for* councils.
,, 219, ,, 4 from bottom, *insert before* minister: ecclesiastical and educational.
,, 219, ,, 3 from bottom, *read* head *for* came first.
,, 220, ,, 19 from bottom, *read* any *for* a.
,, 220, ,, 3 from bottom, *read* chapters *for* assemblies.
,, 221, ,, 18 from bottom, *read* compared with *for* obtained from.
,, 222, ,, 6, *read* choir *for* Cross (the canons in Norway being called the brothers of the choir).
,, 222, lines 6 and 5 from bottom, *delete from* Presbyterii *to* that, *and insert* it had to be takĕn into consideration.
,, 223, ,, 18 and 21, *read* baldachin *for* dais.
,, 225, line 14, *read* twenty-four *for* twenty-two.
,, 225, ,, 13 from bottom, *read* Eyktartid *for* Eykartid.
,, 225, ,, 6 from bottom, *insert* comma between high *and* one.
,, 226, Fig. 207, *insert after* chancel: the choir, presbytery, and high choir (retro-choir).
,, 227, line 4 from bottom, *read* made *for* used.
,, 229, ,, 5, *read* viz. *for* vitz.
,, 229, ,, 16, *read* the brink of the river *for* Mælen.
,, 232, Fig. 210, *insert*: The circle drawn by Macody Lund. to illustrate the words of Gervasius, quoted p. 17.
,, 232, line 10 from bottom, *read* south of *for* before.
,, 234, ,, 7, *read* p. 229 *for* p. 228.
,, 234, ,, 10, *read* south of *for* before.
,, 234, footnote, *read* Samlede Afhandlinger *for* collected accounts.
,, 238, Fig. 215, *read*: Remains of the arch over the procession-path bonded in the north wall of the choir-screen.
,, 239, line 3, *read* uncritical *for* criticless.
,, 239, ,, 6 from bottom, *read* scholars *for* scientists.
,, 240, lines 4 and 5, *read* the sill under the jambs of the archway to the Presbyterium *for* voussoir belonging to the retro-choir.
,, 240, line 1 from bottom, *read* edifice *for* work.
,, 240, footnote, *read* l.c. *for* c.l.
,, 241, footnote, line 7 from bottom, *read* know *for* recognise.
,, 244, line 20 from bottom, *read* of the Orkneyingasaga *for* the saga of Orkneyinga.
,, 245, ,, 5, *read* politics *for* politic.
,, 245, ,, 22, *after* chiefs *insert*: of the greatest seafaring nation of the time.
,, 245, ,, 12 from bottom, *read* poverty of the traditions *for* lack of folk-lore.
,, 246, ,, 15 from bottom, *after* St. Carileph, *insert* in.
,, 250, Fig. 246, *delete* of interior *and insert* south *before* transept.

Page 252, line 15, *read* screen *for* pulpitum.
,, 254, ,, 7, *insert* wall *before* surface.
,, 254, ,, 5 from bottom, *insert* "d" *before* under *and delete comma after* under.
,, 256, ,, 7, *read* proved *for* witnessed.
,, 256, ,, 11, *insert after* known: from documents.
,, 258, ,, 15, *read* repositories *for* aumbries.
,, 258, ,, 18 from bottom, *read* to the advantage of.
,, 259, ,, 2 from bottom, *read* endowments *for* legacies.
,, 260, ,, 21 from bottom, *read* 1299 *for* 1229.
,, 262, *for the 6th King read* Hakon Sigurdsson.
,, 262, between lines 5 and 6 from bottom, *insert* translation there stands high in the head church the Shrine of Olaf over the Altar.
,, 265, footnote, *read* series *for* column.
,, 265, Fig. 254, *insert after* transept: with Olaf Kyrre's unchanged triforium from before the restoration.
,, 267, Fig. 257, *read*: Porch of north transept before restoration *for* door, etc.
,, 268, Fig. 258*b*, *insert after* from: the nave of.
,, 270, Fig. 268, *insert after* transept: after H. E. Schirmer's measured design.
,, 270, line 3 from bottom, *delete* "Forhall."
,, 271, ,, 23, *read* forms *for* shapes.
,, 273, ,, 9 from bottom, *read* annulet-like *for* amulet-like.
,, 274, ,, 22 from bottom, *read* r1 *for* R1.
,, 274, ,, 17 from bottom, *read* capital S *for* s.
,, 276, ,, 14, *read* pallium *for* palladium.
,, 276, ,, 17, *read* archiepiscopacy *for* time office.
,, 277, ,, 7 from bottom, *read* express statement *for* direct saying.
,, 277, ,, 8 from bottom, *insert after* after: the establishment of the archbishopric and the chapter, 1152.
,, 278, ,, 23, *read* fig. 253 *for* 263.
Pages 279–282, line 1 from bottom, *insert after* arcades: (see fig. 277) were found in the raw supporting walls, which were built in the sixteenth century, *and delete text*.
Page 282, Fig. 276, *insert after* remains: of the opening for the procession-path.
,, 282, line 10 from bottom, *insert after* 247: and fig. 278.
,, 283, Fig. 278, *insert after* nave: wall of the northern aisle with springers of the vault.
,, 321, line 2, *read* upwards *for* in height.
,, 322, footnote, *insert* superior wisdom of the committee of supervision *and delete text*.
,, 323, line 7, *read* tangents *for* tangent.
,, 325, ,, 22 from bottom, *read* belong to *for* take part in.
,, 326, ,, 15, *read* story *for* floor.
,, 326, ,, 16, *read* portals *for* porches.
,, 326, ,, 3 from bottom, *read* proved *for* witnessed.
,, 331, Fig. 311. *Insert*: The third row of figures reconstructed by architect Solberg, by means of found stones from the screen of the west front. Analysis inscribed.
,, 333, Fig. 312. *Insert after* Nidaros: after Macody Lund's reconstruction.
,, 333, line 5, *read* proofs *for* evidences.
,, 333, ,, 12, *read* stones *for* stone.
,, 334, ,, 3 from bottom, *read* documentation *for* data.
,, 336, ,, 11, *read* plus one *for* and one.
,, 336, ,, 17, *read* parts *for* part.
,, 337, ,, 10, *read* archi-episcopal *for* episcopal.
,, 342, lines 14 and 13 from bottom, *insert*: This trace of roof shows that the parapet had been pulled down then *and delete text*.
,, 343, line 8 from bottom, *read* facing *for* surface.
,, 344, ,, 4, *insert after* traces: of roof on the west side of the central tower.
,, 344, ,, 9 from bottom, *insert before* architects: instruction for the.
,, 344, ,, 4 from bottom, *insert after* previously: p. 296.
,, 344, ,, 3 from bottom, *insert before* a proof: my being entrusted with this task for our present work.
,, 345, Fig. 324, *insert after* 1916: his 11th design.
,, 352, line 14 from bottom, *read* deserved *for* self-brought.

INTRODUCTION

THE great Norman-Gothic Cathedral of Nidaros, Throndhjem, the national sanctuary of Norway and one of the most beautiful buildings of Europe, has been under reconstruction since 1869.

The admitted principle for its restoration has been during all this time that—as far as possible—the church should be given back its original appearance. This task was entrusted to Mr. Christian Christie, a learned and scientific architect, who, for thirty-four years, carried out the work with a reverent adherence to the lines and character of the original fabric. In 1900, however, when confronted with the problem of restoring the west front, of which but the ground story remained, he experienced great difficulty in evolving a suitable design. The scheme submitted by him in 1903 was very severely criticised, and, during the public discussion which ensued, we pointed out that no design was likely to prove satisfactory unless it was conceived in harmony with the principle of geometrical proportion which pervaded the original fabric—a principle which Mr. Christie had not fully understood.

In 1906, to the intense regret of his many admirers, Mr. Christie died, and the great work of restoration was then placed in the hands of the young architect, Mr. Olaf Nordhagen, who was impressed with our recommendations, which he adopted in certain of his designs.

In December 1914, however, Mr. Nordhagen officially declared that he lacked sufficient archæological material to be able to continue the work according to the principle demanded, and he submitted an " original " or " personal " design, which was directly opposed to the terms of his official appointment. We then felt it not only our duty to protest, but to take up afresh and extend our researches in order to demonstrate the truth of the principles annunciated and their applicability in connection with the restoration of the fabric as a whole. We therefore gave a résumé of our studies at a meeting of the Christiania Videnskabsselskab, the Royal Society of Science, on May 28th, 1915, subsequently repeated before the Norwegian Parliament, with the result that the latter ordered the publication of a full and detailed report of our researches so that our scheme could be further considered.

It is this official commission that we have been carrying out since then.

While going through our investigations, we found full confirmation not only of the truth of our theories, as regards the cathedral of Nidaros, but we have been able to trace the idea through the Middle Ages right back to classic times.

It appears from this, that medieval church architecture is a direct continuation of the art of building classic temples, which in its turn expresses the perception of Greek philosophy concerning the harmony of the universe. The temple was supposed to be the material image of the mystery of existence. Its proportioning was therefore established according to an irrational measure, in an ascending, harmonic, geometrical progression, from the unit to the totality,

as this progression appears in the pentagram, which for the Pythagoreans was the symbol of the harmonious system in the Cosmos, the masterpiece of the universe.

Our treatment of these architectonic questions is therefore a new chapter in the history of Greek philosophy. It seems that these rules of religious architecture were a secret science in the Middle Ages, just as they were in classic times, with the object that the temple should create the same respectful astonishment as did life itself.

Through this secrecy, the rules became forgotten, and as a reflection of the ancient science there only existed in the following centuries the superstitious use of the pentagram, as a charm and protection against evil powers. It is this forgotten science which we have discovered, and which we develop fully in this work, giving argumentative and conclusive evidence of its use by the temple-builders of antiquity.

This science is not only of help in obtaining harmony in the building, but it gives also full guidance for the stability of the overwhelming and daring cathedral architecture of the Middle Ages.

* * * * *

This science came to Norway with the introduction of Christianity. In the cathedral of Nidaros, the Norman and Gothic parts of which belong to the oldest of its kind in Europe, it is used with a full and independent knowledge, as it can only be done by a complete surrender to an acknowledged science.

The cathedral stands out as a splendid memorial of the spiritual development of Norway, just as Norse literature and customs have long been famous in Europe.

But, moreover, this cathedral, situated far north, shows what an intimate and living connection existed all over Europe in the Middle Ages, thanks to the universality of the Catholic Church.

Every nation lived quite apart, it is true, but thanks to the uniformity in their perception of life, due to the teaching of the Christian Church, there ripened in each a civilisation, to a certain extent local and independent, but fully universal, just as the local vegetation of various countries is brought forth by the light of the sun rising and setting each day on them all.

Life in the Middle Ages was in God's honour. And the thought of mankind was also in His honour. His cathedrals, lying in the path of the sun from east to west, raised their high spires towards heaven, as a sign of the gratitude and of the praise of man for the wonder of life.

The building of cathedrals was a religious lyric in stone, the visible symbol of the miracle of existence.

But Nidaros cathedral was more than that. It was the memorial of the king who suffered martyrdom for the divine teaching and its world-wide dissemination. It was also the point of contact of all Norsemen in memory of him who had united Norway and the Norse tribes in obedience to humane Christian laws.

Consequently we have not been able to treat the geometrical system of the cathedral of Nidaros without explaining also more fully the conditions of history which are so foreign to the way of thinking of our days.

* * * * *

We are convinced that this discovery of the past rules of religious architecture will have a fertilising influence upon the science of building as a whole. It creates a new life in ancient Greek estheticism, to whom beauty was not an indescribable matter of feelings, but on

the contrary the result of a conscious intellectual perception of harmonising laws existing in nature itself.

But for the present, what is important is that the discovery of these harmonising rules has brought fixed laws for the completion of the great work of rebuilding the cathedral of Nidaros.

* * * * *

The extremely limited time which has been allowed us to fulfil our commission, and the continual pressure with which we have had to conduct our work under the most unfavourable conditions, have prevented us from including in our task questions of great interest and importance belonging to it.

* * * * *

It would be unjust to finish this introduction without expressing unreservedly our gratitude to the two architects who have assisted us, each in his way. The first, Mr. Chr. Lange, is director of a department for the town architect of Christiania; the other is Mr. Holger Sinding-Larsen, architect.

Mr. Lange has made complete scientific studies of Gothic architecture at the Technical High School of Hanover. When his studies were finished, he assisted his teacher during several years, the well-known professor, C. W. Hase; he was then for a time with the first director of *Das Germanische Nationalmuseum* in Nuremberg, Professor Essenwein; and, finally, he stayed for about ten years with Professor Emerich Steindl in Budapest. It was on the recommendation of the latter that he was made "Dombauleiter" by the Royal Hungarian Ministry of Education. In that capacity he worked at the restoration of the cathedral of St. Elizabeth, Kaschau, and of the church of St. Egidius in Bartfeld, the last of which he completed.

When the Norwegian Parliament gave us our distinguished and responsible commission, Mr. Lange put at our disposal all his knowledge of Gothic art and his artistic talents—like the learned gentleman he is. The beautiful and correct carrying out in details of the drawing of the west front is his own work according to our conception.

The other architect, Mr. Sinding-Larsen, on his side, has helped us with idealistic unselfishness, in secretarial and designing work, and he has also endeavoured with pamphlets and writings in periodicals and newspapers, as well as with public lectures and at the sacrifice of his valuable time, to create an understanding of our work.

We have also to thank the Royal Archivary, Mr. Chr. Brinchmann, and the architect, Mr. Harald Sund. Mr. Brinchmann has helped us in secretarial work and in various ways, among others by setting up the index. Mr. Sund has put much interest in the execution of the excellent perspective drawings.

Furthermore, we acknowledge gratefully the readiness with which the First Burgomaster of Christiania, Cabinet Minister Mr. Arctander, has put at our disposal the necessary rooms and drawing-offices. We thank also, and not least, the chief librarian of the University, Mr. A. C. Drolsum, for the rare goodwill he has shown towards our work, as not only did he allow us for a long time a separate work-room during the proof reading, but he has eased many difficulties by the extremely liberal and lengthy loan of books of plates and similar literary means of help.

Finally, as to the English translation, we beg to express our thanks to our friend, Dr. Jon Stefansson, Lecturer in Old Norwegian at the London University, for his valuable help

in revising the English text, although most of the matter had already been printed when he commenced the work.

* * *

We beg to call the attention of the reader to the important notice on p. xviii, and claim his indulgence for the imperfections of the translation.

VILLMARK, IN SVARTSKOG,
NEAR CHRISTIANIA
September 14, 1919
and
LONDON, *May* 18, 1921

F. Macody Lund.

AD QUADRATUM

CHAPTER I

"ARS SINE SCIENTIA NIHIL"

THE study of the cathedral of Nidaros which we have carried on for half a generation has convinced us more and more that this building must have been planned in one jet. In order to reconstruct the missing portion of the west front, it was necessary to find the system lying at the base of the whole conception. As our demonstration will prove, we have succeeded in showing the geometrical system according to which not only the cathedral of Nidaros was planned, but all the large medieval cathedrals as well as the most ancient sacred buildings of antiquity. This geometrical system for sacred architecture is derived from the consequent and yet richly varied use of a certain proportion between the width, the length, and the height of the building, the construction being planned upon an ever-varying play of the elementary regular polygons and their angles.

These laws of geometrical proportions had sunk so deeply in the minds of the ancient builders that they were able to create not only beautiful architectural types, but, as we shall see further, they were also able to vary, without ever infringing, the regularity of these laws; thus it was that, keeping strictly to the principles of the construction, the unity of the whole was attained and always kept, even in the variations.

The romantic period with its craving for the past having created a certain understanding of the greatness of medieval building construction, the study that naturally followed brought with it a desire to understand its undisputable principle which alone could have created the wonderful, not to say mysterious, harmony which radiates from these magnificent and at the same time infinitely complicated buildings. In France, England, Germany, and other countries, a number of theories were brought forward for the solution of this riddle, each of them laying down some kind of geometrical system for medieval architecture. All these theories have, however, one thing in common: they are more or less artificial. This is due to the already mentioned assurance of the builders in the art of varying, which forced the discoverers of systems to grope through all sorts of doubtful makeshifts in order to bring the selected one to fit the obvious variations which presented themselves. A splendid exception stands out in the crowd—*Viollet-le-Duc*.

With his analytical method, his intimate and thorough study of details, this brilliant thinker acquired the yet unsurpassed understanding of the art of building in the Middle Ages. The building rose in his mind as the result of geometrical constructions; its axis, like a skeleton, contains the idea of the building from which the lines of the triangles must spring according to a mathematical necessity. In accordance with the proposition, " It is the harmonic system

used for the interior which determinates the visible proportions of the exterior," * he begins his examination of the proportioning of a building at the beginning of the construction above the plinth which is the finish of the foundation. One would have thought that Viollet-le-Duc, once having admitted this, would have found the key of the riddle already in the plan of the building, and need only have first looked for it in the proportion of the angles of the various main divisions; from this he afterwards made his famous theory of the three triangles, using the three at the same time. Even Viollet-le-Duc's sound analysis did not lead to the solution of the question concerning the geometrical system of medieval architecture.

Professor Dehio of Strasburg, in his attempt to correct Viollet-le-Duc, whom he dares to accuse of operating "mit gewohnter Leichtlebigkeit" and "gewohnten Mangel an Kritik und Konsequenz" ("with the usual carelessness" and "usual lack of critique and consequences"), and with "hervorgesprudelten Kombinationen" ("rushed combinations"), explains that Viollet-le-Duc was led astray because he combines his triangle lines with the axes in the walls where they are invisible. Dehio himself places his lines in the openings, although these are the outcome of the function of the auxiliary lines, and therefore at best can only be used for trying the right placing of the constructive lines: in this way he shows the usual incompetence of the man who, having derived all his knowledge from books, is incapable of grasping and treating realities. Dehio is also taken thoroughly to task by that intelligent expert Regierungs und Baurat *Max Hasak* of Berlin, in his book *Der Kirchenbau des Mittelalters*, second edition (1913), pp. 279 ff.: here he explains the importance of the auxiliary lines to obtain harmony and unity in a building.

That, in the Middle Ages, the geometrical system of a building should have been considered as the very point of its planning is shown in the famous extract of the *Discussion of the Building Council of the Cathedral of Milan in* 1398. The Italian masters at a critical point during the work had summoned Master *Jean Mignot* from Paris. In their answer to his criticisms of the construction, they said, rather unfortunately, "Quod scientia geometriae non debet in iis locum habere ed quia scientia est unum et ars est aliud" ("that the Science of Geometry was of no importance here, because Science is one thing, Art another"), to which the Frenchman answered irrefutably, "Ars sine scientia nihil est" ("Art without Science is nothing"). After having also given an extract of a building-discussion during the construction of the cathedral of Gerona in Spain in 1417, Hasak remarks (l.c., 292): "Hätten die Baumeister nicht eine *feste Theorie* besessen, so hätten sie solche bestimte Antworten nicht abgeben können" ("If the constructors had not possessed a fixed theory, they could not have given such an answer").

The words "Art without Science is nothing" apply especially to the art of building churches. *The conception of the House of God* as a dwelling-place, having a definitely fixed ritual for the glorification of the Highest Being, demands definitely fixed rules for the division of the space horizontally as well as vertically. It is obvious that such a fabric, both in plan and elevation, must reflect the thought that makes of it a unity. This would be especially the case with God's House in the Middle Ages, the age which more than any other was under the domination of a real uniting idea, because of the unity of religion then existing in the whole of Europe.

We are now going to explain the fundamental idea underlying Christian architecture in the Middle Ages as the result of an intelligent consciousness and an ancient ideal conception of the mystery of the world; therefore we need not make the cathedral of Nidaros our starting-point. It was, however, during our repeated attempts to find a geometrical system

* "C'est le système harmonique admis pour l'intérieur qui commande des proportions visibles à l'extérieur" (Viollet-le-Duc, *Dictionnaire de l'Architecture*, vol. vii, p. 538).

for the rebuilding of the destroyed upper parts of the west elevation of this cathedral that, when examining the proportions in the remaining parts of this elevation, we recognised everywhere *an angle* of 63° 26′, or more exactly 63° 25·982′, which, according to school-days' recollections, was connected with the construction of the *sectio aurea* and of the pentagon. This angle equals the angle between the hypotenuse and the base of a rectangular triangle, when the length of the base is half the height, viz. when this same hypotenuse forms the diagonal in a rectangle composed of two squares and where the proportion between base and height is as 1 : 2—geometrical constructions which from the time of Euclid have existed in the school curriculum. The pentagon and the pentagram are frequent symbols of witchcraft in popular superstition—a proof that something of an intellectual value from a bygone age has been preserved. Various experiments on constructions convince us of the meaning of the Pythagorean teaching concerning the pentagon as a symbol of the harmony of the spheres. The geometrical treatment of the pentagon leads to an infinity of combinations, constantly leading back to the square. The presence of this angle with such marvellous functions threw a light over the difficulties that until now had baffled our attempts to rediscover the *principle of unity* in the conception of the cathedral; it even gave us the *key* to one riddle after another in medieval cathedral building art.

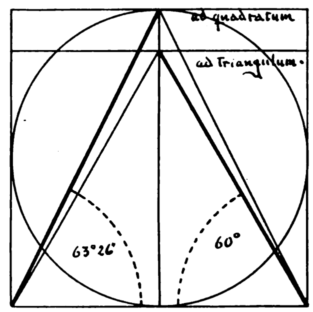


Fig. 1.—*Ad quadratum* and *ad triangulum*.

In the same way we also found the meaning of the significant sentence in "Annali della fabbrica del duomo di Milano": "Utrum ecclesia ipsa non computando in mensura tiburium fiendum debeat *ascendere ad quadratum an ad triangulum?* Declaraverunt quod ipsa posset ascendere usque ad triangulum sive usque ad figuram triangularem et non ultra" (Hasak, l.c., 286). *A Discussion of the Council of Milan Cathedral in* 1392 is referred to, where an "expertus in arte geometriae," *Gabriele Stornalocho,* had been summoned from Piacenza, and whose drawing gives equilateral triangles as a guidance for the raising of the church. The expert Hasak, otherwise so sharp, has here misunderstood the connection and maintains that the mention of a system of triangles only refers to the oblique auxiliary lines which appear in the drawing of Stornalocho—"Richtungsschrägen" (lines of direction)—which by themselves form triangles. "Der Ausdruck 'Dreieck' ist ersichtlich eine abgekürzte Bezeichnung—ein terminus technicus, den die Italiener seinem Wesen nach vielleicht gar nicht einmal verstehen"* ("the expression 'triangle' is probably a short name, a technical term—of which the Italians did not even understand the meaning"). The protocol of the Building Council is translated word for word as follows: "Ought the church to be erected according to the square, or according to the triangle, not including the measurement of the central tower which is being built? It was declared (by the designers) that it ought to be erected according to a triangle or a triangular figure and no higher." The question was therefore the possibility of building the church according to a constructive auxiliary line forming with the base an angle of 63° 26′ (*ad quadratum*) or of 60° (= *ad triangulum*: that is to say, the equilateral triangle). The answer was that it would be impossible to build the church higher than the exact height of the equilateral triangle, that is to say, according to the auxiliary lines system of 60° and not, as originally intended, as high as according to the auxiliary lines system of 63° 26′. The Building Committee had, therefore, to choose between two different principles of building, *ad quadratum* or *ad triangulum,* as the attached drawing (fig. 1) will show.

That the cathedral of Milan was originally intended to be built *ad quadratum* is proved

* Max Hasak, ch. i, p. 288.

by its fundamental plan (fig. 2), which is designed over two squares; as we shall see, it is a typically medieval church plan. With its immense width, 58 metres interior measurements, this building was bound to have, according to its plan *ad quadratum*, 58 metres in height up to the vault. This made the Building Committee doubtful regarding its stability, and they resolved to be satisfied with the principle *ad triangulum*, according to which the height to the vault would be only 48 instead of 58 metres.

Can a more striking evidence be demanded of the strictly mathematical way of reasoning of the medieval architects than the reference to the above-mentioned discussion of the Building Council of the cathedral of Milan six years later, when Master Jean Mignot and the responsible constructors were discussing the stability of the church? By his geometrical arguments the

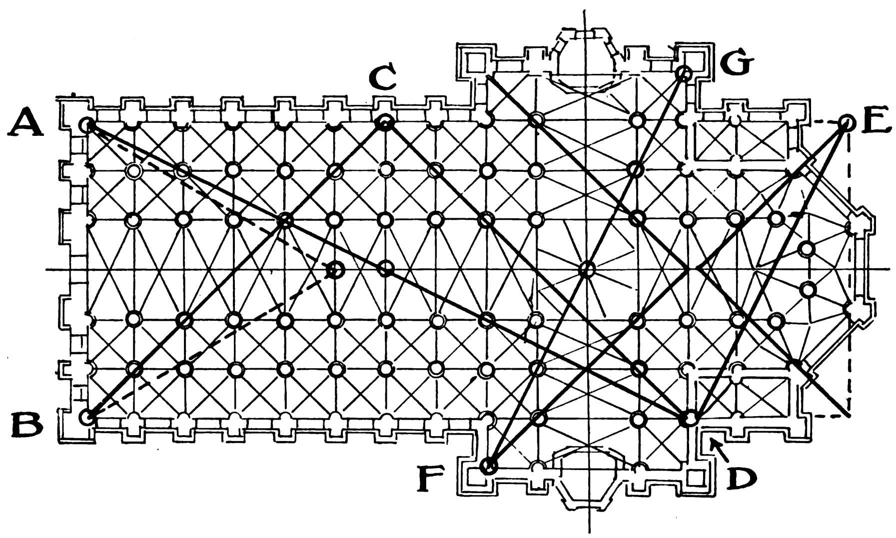


Fig. 2.—Plan of the cathedral of Milan, analysis.

foreign theoretician compels the architects to defend their plans by the same scientific method. They point out, as the first of the reasons for building the towers as they wish to have them, that it is "*pro retificando* * praedictum ecclesiam et croxeriam quod respondent *ad quadrangulum secundum ordinem geometriae*" ("to square out like a net the already mentioned church and its vault, in order that they should correspond with the square according to the laws of geometry"). We can see clearly in our mind the Milanese architects and the Parisian theoretician sitting round the drawing-boards debating the original plans, all squared up with geometrical lines *ad quadratum*, that is to say, with the angle of 63° 26′.

We will now examine the church buildings themselves, and it will be made evident that all typical medieval churches are built *ad quadratum*, according to the auxiliary lines system, with the angle of 63° 26′ in the horizontal as well as in the vertical plan.

* *Hasak*. This quotation, which we have taken from the only obtainable source, stands in the Latin text as "*reti*ficando," but he translates it "*recti*ficando"; either there has been a misprint in his Latin quotation, or he has been drowsy while translating.

CHAPTER II

"AD QUADRATUM"

GENERAL SUMMARY OF CONTINENTAL AND ENGLISH CATHEDRALS

As a starting-point in our attempt to gain a general view of the constructive main proportions of the various medieval churches, we will conveniently select the cathedral of Milan, of which we have just spoken.

The plan of this church (fig. 2) is, as we have already noticed, designed *ad quadratum*. The diagonal A D marks the two squares B C and C D. The transverse diagonal F G shows equally that the transept is designed over two squares. Finally, the diagonal of the chancel D E shows also two squares.

It could be objected that all this is only a necessary outcome of the system of vaulting. But we will show how unvaulted churches were also built *ad quadratum*, and that this principle is a usual inheritance from Antiquity.

In his rules for the building of a basilica, *Vitruvius* teaches that the proportions are as 1 : 2 or as 1 : 3 (*De Architectura*, V, I, 4,4), which is proved on examining the plans of the existing basilicas.

We take at random the *basilica of Pompei* (fig. 3). Its width is, as we see, in proportion to its length as 1 : 3, while the whole construction itself shows a proportion of 1 : 2. The plan of the building is therefore composed respectively of three and two squares. The hall of Trajan's immense basilica in Rome has the same proportion of 1 : 2.

The *basilica of Constantine* in the same place has, however, a different horizontal plan; but, on the other hand, it has a vertical plan (fig. 4) with a fully developed system of squares. The total height to the top of the ridge is in proportion to its length as 1 : 2; the height of the walls of the nave is as 1 : 3. The whole building to the top of the ridge consists also of two large squares and the walls of the nave of three small squares, while the height from the top of these walls to the ridge consists of six smaller squares.

St. Paul's basilica in Rome. The proportion of the width to the total length, apse included, is as 1 : 2, that is, two squares. The total height to the ridge in proportion to the length is as 1 : 4 (fig. 5), that is, four squares. The height of the walls of the nave to the length is as 1 : 3, that is, three squares. The height of the arcade to the length is as 1 : 7, that is, seven squares, to which can be added three further squares, for the transept, chancel, and apse; altogether ten squares.

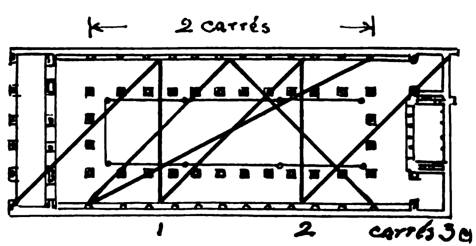


Fig. 3.—Plan of the basilica of Pompei, analysis.

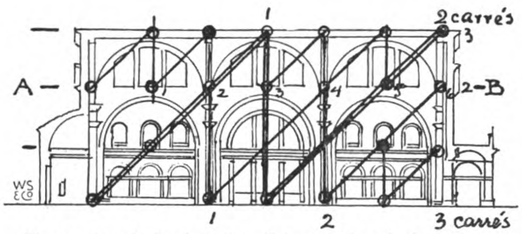


Fig. 4.—Longitudinal section of the basilica of Constantine, Rome, analysis.

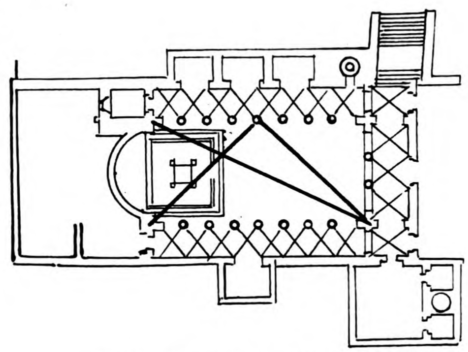


Fig. 6.—Plan of St. Agnese, Rome.

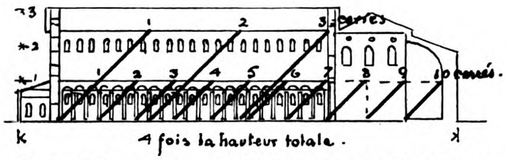


Fig. 5.—Longitudinal section of the basilica of St. Paul, Rome, analysis.

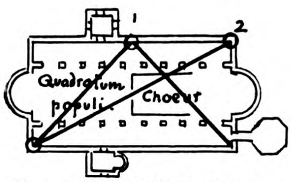


Fig. 7.—Plan of the Anglo-Saxon cathedral of Canterbury, analysis.

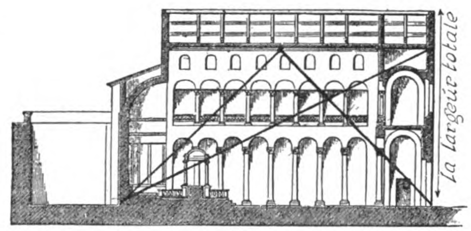


Fig. 8.—Longitudinal section of St. Agnese, Rome, analysis.

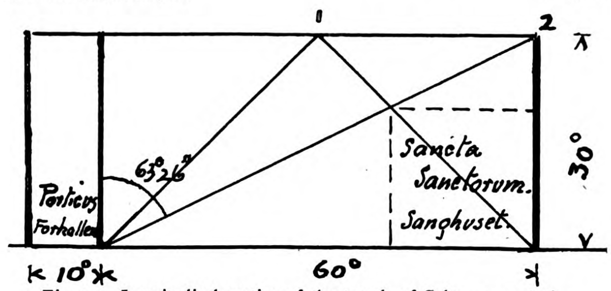


Fig. 9*a*.—Longitudinal section of the temple of Solomon, analysis.

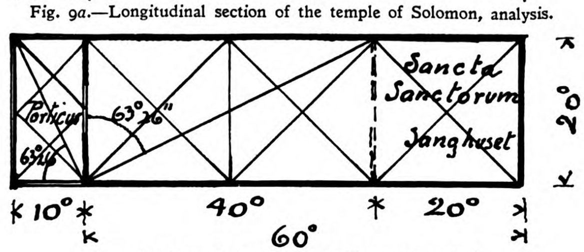


Fig. 9*b*.—Plan of the temple of Solomon, analysis.

Fig. 6 is the plan of *St. Agnese* in Rome. The proportion of this is as 1 : 2 with one square for the chancel and one for the church.

Fig. 8 is the same church in longitudinal section, where the same proportion is carried through without exception. The total height, that is, to the ridge, is equal to the width; the front is designed also *ad quadratum*.

Fig. 7 is the plan of the oldest Anglo-Saxon cathedral, *the cathedral of Canterbury*, from 597 to 959; the proportion is the same.

These examples from pagan times as well as from the first centuries of Christianity confirm therefore the teachings of Vitruvius. But his rule of proportions for the building of a temple is still more definite: "*The width of the temple must be equal to half its length*" (*De Architectura*, V, I, 4). The proportion is therefore as 1 : 2. We shall see farther on that this is a very ancient proportion in sacred architecture, and becomes a determining factor in the art of building churches in the Middle Ages.

There is a point of considerable interest which should arrest our attention: it is owing to the mother-house of the Benedictines of Monte Casino near Naples, founded about 520, that the work on architecture by Vitruvius is preserved. The zealous church builders of this order * must have had in his book a support for the preservation of the rules affecting the main proportions of the antique building art, the correctness of which must have received a sanction on being found again in the Bible as applied to the Temple of Solomon: "And the house which King Solomon built for the Lord, the length thereof was threescore cubits, and the breadth thereof twenty cubits, and the height thereof thirty cubits. And the porch before the temple of the house, twenty cubits was the length thereof, according to the breadth of the house; and ten cubits was the breadth thereof before the house. . . . And he built twenty cubits on the sides of the house, both the floor and the walls with boards of cedar: he built them for it within, for the oracle, even for the most holy place.

And the house, that is, the temple before it was forty cubits long.

And the cedar of the house within was carved with knops and open flowers: all was cedar; there was no stone seen.

And the oracle he prepared in the house within, to set there the ark of the covenant of the Lord.

And the oracle in the forepart was twenty cubits in length, and twenty cubits in breadth, and twenty cubits in the height thereof: and he overlaid it with pure gold; and so covered the altar which was of cedar" (1 Kings vi, 2, 3, 16–20). Fig. 9*a* and *b* illustrates these figures given in the Bible.

It will be seen by these examples given from among the oldest churches that everywhere the square is used as a basis for church building, not only in the horizontal but also in the vertical plan, in length and in width, as noticed in connection with St. Agnese. This is according to classic antecedents.

The Benedictines were, as we know, the foremost promoters of the medieval art of building, and all the greatest and best churches were raised by monks belonging to the order of St. Benedict. This applies to the cathedral of Nidaros, to its Romanesque as well as to its later rebuilt Gothic portion.

In addition to this, or, most probably, the chief cause for this use of the square, was a thought buried deep down in man's consciousness of the world, in ancient times.

This subconscious thought saw in *number* an expression of the spirit of things, a condition

* The oldest existing MS. by Vitruvius (*Codex Harleianus*, in the British Museum) is supposed to have been written, according to the publisher Valentin Rose, in a German monastery in the time of Charlemagne, and belonged to the first Abbot of the monastery of St. Michael in Hildesheim (1022–30). Valentin Rose affirms that Charlemagne's biographer Einhart possessed a Vitruvius, as did also the monks of Fulda. It is also shown here that the Christian Middle Ages took over directly the scientific inheritance from the Classics, as we shall prove later.

of all existence and of all law in the various forms and expression of life—in other words, the very principle of things.

In this thought the *uneven* number was therefore more perfect than the even, because, being undivisible, it remains unchangeable, equally so when the even number is added, which makes the even alone become uneven. In geometry the comparison between the even and the uneven can be represented in the same way by the square and the rectangle, the first without parts, the other made up of parts; while in the teaching of harmony the uneven is represented by the square in opposition to the perfect circle, and in the treatment of bodies the cube is put in opposition to the sphere, which possesses in itself its own equilibrium.

In this symbolical philosophy it is therefore natural that the square and its materialised form, the cube, should be perfect representations of the surface and the solid respectively, while the circle and its materialised form, the globe, become the representation of the sphere and of the universe respectively, that which is self-supporting; thus also the pentagram with its infinite power of combination represents the harmonious combinations within the spheres and the universe.

This pictorial, or we may call it esthetic, philosophy was the very core of the theology and symbolism of the Middle Ages. The French archæologist *Didron* notices this fact already when he mentions the square nimbus that was given to the pictures of those recognised as saints during their lifetime. He explains it in this way: " The square, as it has been said, is inferior to the circle with the Neoplatonists. According to their thoughts, the square is the symbolical expression given to the earth in geometry; the circle is the symbol of heaven. The circle is, according to heraldic teaching, a perfected square, and the square is a circle broken or diminished" ("Le carré ainsi qu'on l'a dit est inférieur au rond chez les néo-platoniciens. Suivant ces idées, le carré est l'expression symbolique donnée par la géométrie à la terre; le rond est le symbole du ciel. Le rond est un carré perfectionné; le carré est un rond brisé ou diminué, suivant l'expression heraldique" (M. Didron, *Iconographie chrétienne. Histoire de Dieu*, Paris, 1843, p. 79). He might have added that the square is also a symbol of the House of God; similarly the ancient expression " quadratum populi," limits the symbol to indicate that part of the church allocated to the faithful, that is, the nave as shown in the plan of St. Agnese in Rome (fig. 6) and of Canterbury (fig. 7).

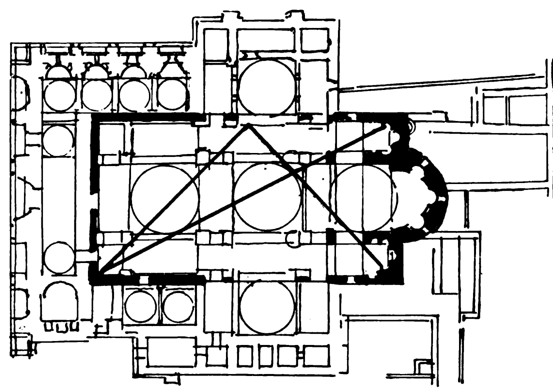
* * *

A general examination of the churches of all countries through the course of time shows that the principle of construction *ad quadratum* runs through them absolutely without exception. The material that can be collected to prove this rule is so abundant that our demonstration can be only of a cursory character. After having first admitted the existence of the principle, we shall make a special analysis of several typical cathedrals in a later chapter. After having followed the development of the idea in respect of space and time, we shall be obliged to admit that it leads to wonderful results by a consequent use of it in medieval architecture, the beauty of which until now we have only admired without being able to explain it. From Italy, we continue through France to England and Germany.

ITALY

Fig. 10 gives us the original plan, although later extended, of the cathedral of *St. Mark in Venice*, built after 830. As it can be seen, the plan is a simple rectangle composed of squares according to the angle of 63° 26′.

The next example is the *cathedral of Pisa* of the eleventh century (figs. 11, 12, 13*a*, and 13*b*), where we find the architect playing with the squares like a Pythagorean. The rectangle of the nave and of the chancel (see fig. 11) is formed of three squares, the transept of four smaller squares with sides half as long as those of the squares of the nave. The extension of the transept towards north and south is determined as follows: the eastern corners by diagonals of 63° 26′ from the main porch of the nave—that is to say, by dividing the larger square (having the east wall as a base) by means of diagonal lines into two equal rectangles; the west corners are fixed by diagonals of 45° from the west gable corner of the nave. Lastly, the west wall of the transept forms the hypotenuse in a rectangular triangle, the summit of which corresponds to the position occupied by the high altar; in other words, this triangle forms one half of an oblique square. The lengthened sides of the other half meet the corners of the west front. It is a complete play with the square.



Hasak s. 196. (1913)

Fig. 10.—Plan of St. Mark, Venice, analysis.

Fig. 12 shows how the transverse section is constructed *ad quadratum*; this is also the case with the west front and the cupola (fig. 13).

In the section (fig. 12) we have written NB against the diagonal which starts from the base of the column between the two aisles and gives the height of the cap on the inside of the outer wall. We have placed this same line in the elevation (figs. 13*a* and 13*b*), marked here also NB, where the line formed by the greater width of the elevation while cutting the outer limits at a higher point marks the height of the plinth of the arcade of the lower story. This NB in the elevation forms again a new starting-point *ad quadratum* because the diagonal (63° 26′) deflected from here meets the wall of the nave and determines the base of the cupola, which then finds itself on a level with the summit of the triangle of the gable in the elevation at point C. We notice that this is designed within a large square having the width of the front as side, in the same way that the height of the wall of the aisle and the height of the cupola correspond to half the height of a square of the same size.

If this result is now compared with the plan (fig. 11), it will be found that the height of the church to the top of the cupola corresponds to the distance from the west front to the west wall of the transept interior measurement. It will be noticed that there is a complete play between the horizontal and the vertical plan.

This really very simple Pythagorean play with the square, such as it appears in the Romanesque time, continues further through the Gothic period.

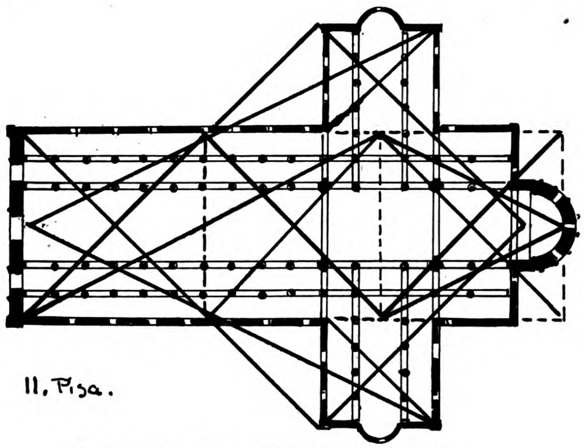


Fig. 11.—Plan of the cathedral of Pisa, analysis.

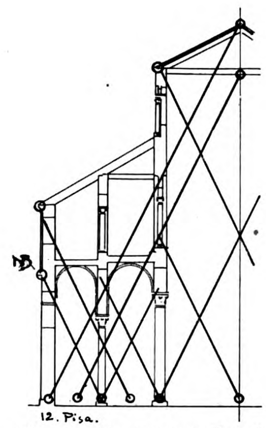


Fig. 12.—Transverse section of the cathedral of Pisa, analysis.

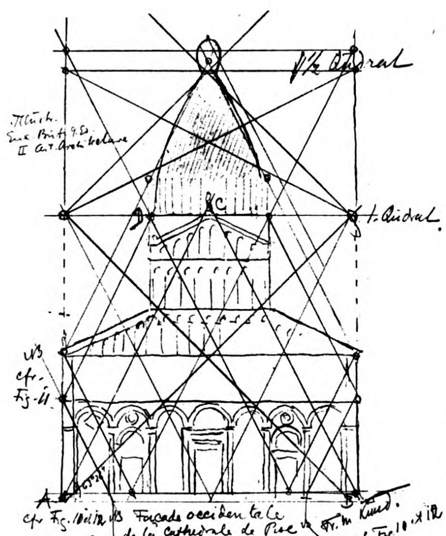


Fig. 13*a*. —West front of the cathedral of Pisa, analysis.

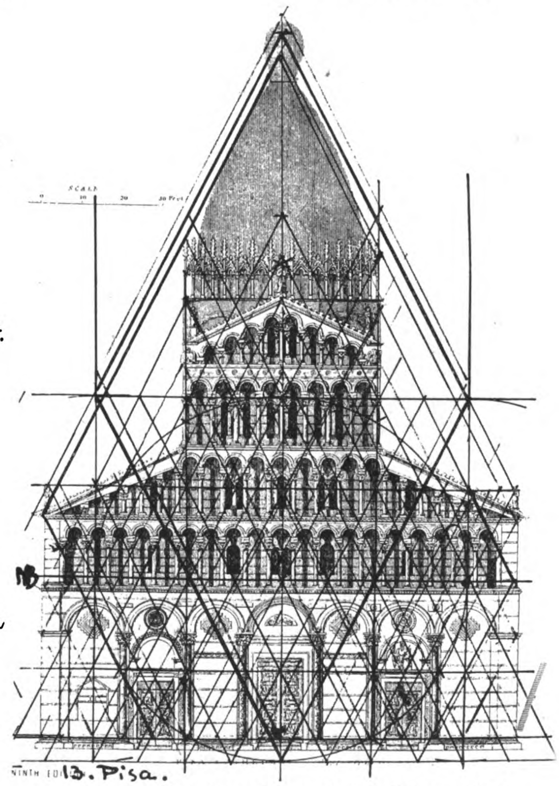


Fig. 13*b*.—West front of the cathedral of Pisa, analysis.

We have now seen this principle *ad quadratum* used in Italy in the ninth, tenth, eleventh, and twelfth centuries, and how it developed according to ancient rules which infused in the art of construction the conception of primitive philosophy. In our later analysis of classic architecture we shall develop this statement.

FRANCE

In France also we take our examples at random.

In figs. 14 and 15 we see the plan and transverse section of the church of *Vignory* from the tenth century (Viollet-le-Duc, *Dict. de l'Architecture*, vol. i, p. 169). Its plan shows a length of three squares; its section is *Ascensio ad quadratum*.

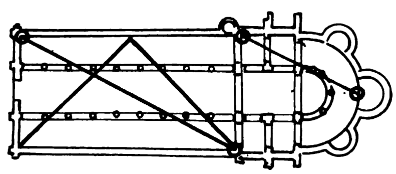


Fig. 14.—Plan of the church of Vignory, analysis.

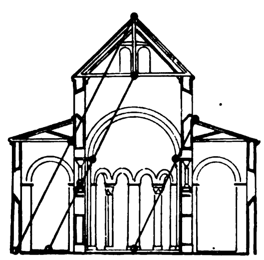


Fig. 15.—Transverse section of the church of Vignory, analysis.

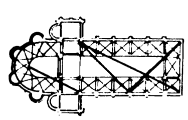


Fig. 16*a*.—Plan of the church of Clermont-Ferrand, analysis.

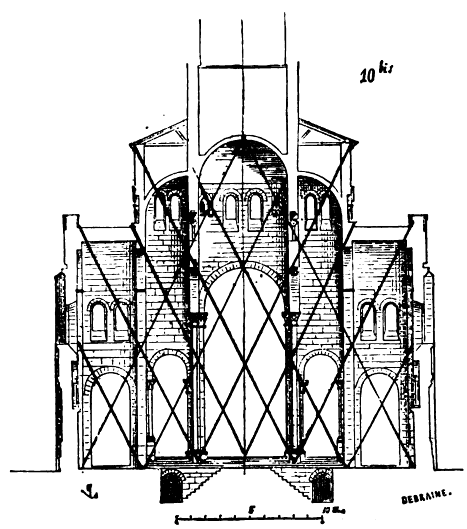


Fig. 16*b*.—Transverse section of the church of Clermont-Ferrand, analysis.

Figs. 16*a* and 16*b* is the church of *Clermont-Ferrand* from the eleventh century, which shows the same proportion in plan and section. Fig. 16*b* gives the longitudinal section of the transept.

The Benedictine *abbey of Cluny* (fig. 17), from 1089 to 1131, is built *ad quadratum*, in exactly four squares, according to the plan of Viollet-le-Duc in *Dict. de l'Arch.*, vol. i, p. 258. The plan reproduced in fig. 17 is from Francis Bond's *Gothic Architecture in England*, p. 150. We find the idea of the square used consequently throughout. It will be noticed that the plan is developed inside two large squares (marked I and II), or a rectangle in the halving line of which is placed the west wall of the transept, in opposition to the cathedral of Pisa, where the same line coincides with the east wall.

We have now come to the use of two squares, which characterise the Benedictine churches of France in the eleventh century. The nave of the church is formed of two squares, and

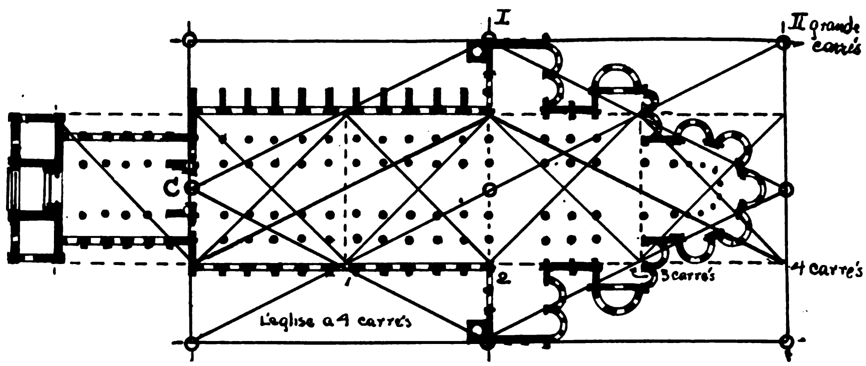


Fig. 17.—Plan of the abbey church of Cluny, analysis.

so is the chancel. It will be remarked that the church falls constantly within a rectangle having the angle of 63° 26′.

In fig. 18 we find the *cathedral of Strasburg*, from the thirteenth century, which is built simply according to the fundamental rules. The transept and the chancel stand from the Romanesque period. It must be noticed that here we find realised the idea of the abbey of Cluny, the older Romanesque west walls of the transept being used as the base of the triangle in the fundamental square, in the planning of the new Gothic nave.

The *cathedral of Metz* (fig. 19), thirteenth century, is included as an especially interesting example. It consists, as we know, of two churches built together, the older small Liebfrauenkirche and the later cathedral; the latter being simply designed over three squares.

In fig. 20 we have a longitudinal section of the first two bays of the chancel in the church of *Vezelay*, which shows how the square is used, not only for determining the main proportions, but also as the means of developing the architectural divisions. After what has been said, the drawing speaks for itself.

The church of *St. Sernin in Toulouse* (fig. 21) is about contemporary with the abbey of Cluny, and ought therefore chronologically to have been classed with it. It is interesting, however, to put it together with the cathedral of Bourges, whereby the increased use of the square in the eleventh, the twelfth, and the thirteenth centuries will be shown strikingly. These examples have the additional architectural and historical interest of having been used by Viollet-le-Duc to prove the correctness of his triangle theory based on the rectangular, isosceles, and equilateral triangles. It can be seen how near but at the same time how far the great master was from the truth.

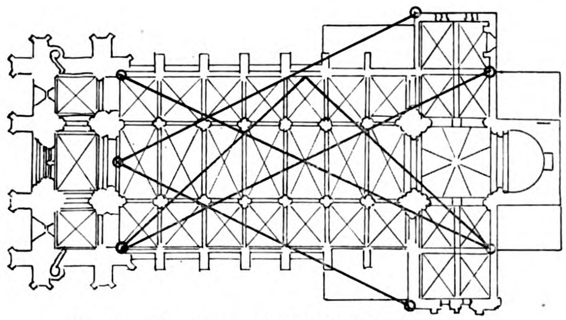


Fig. 18.—Plan of the cathedral of Strasburg, analysis.

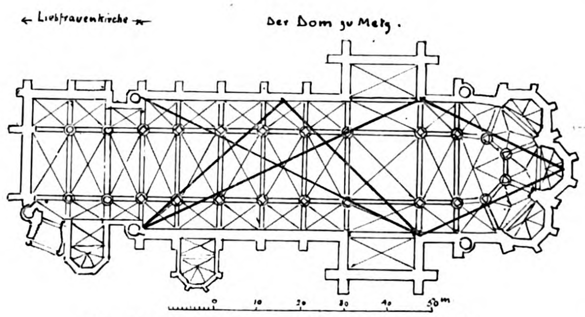


Fig. 19.—Plan of the cathedral of Metz, analysis.

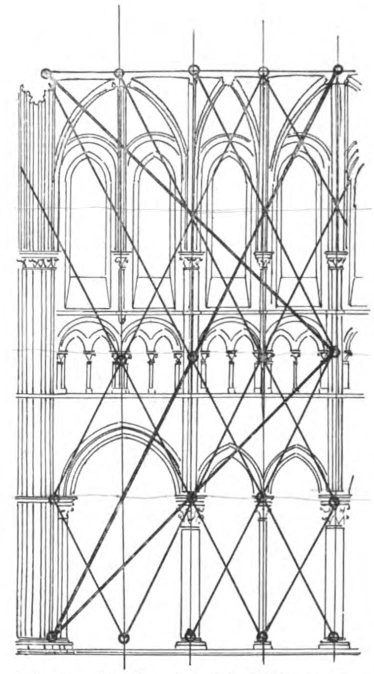


Fig. 20.—Interior side wall of the church of Vezelay, analysis.

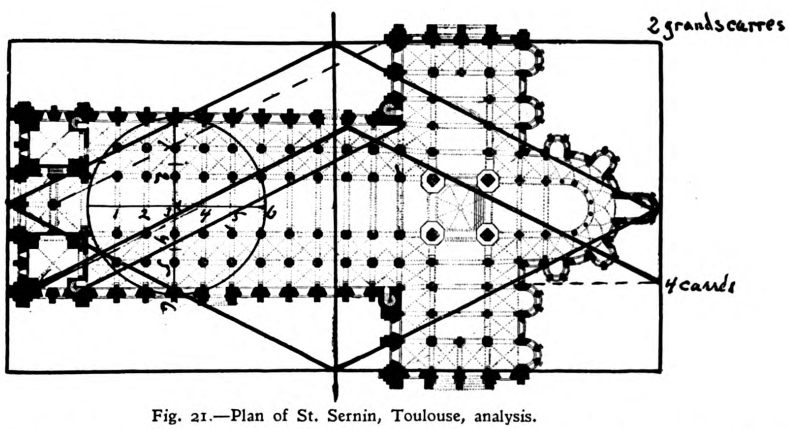


Fig. 21.—Plan of St. Sernin, Toulouse, analysis.

St. Sernin (fig. 21) is on the same principle as Cluny, as its plan is formed within two large squares, within which again are formed two squares on the west part and two squares in the chancel. The central point of the church (which in Cluny lies on a line with the west wall of the transept) lies in St. Sernin two bays to the west, by which the summit of what we

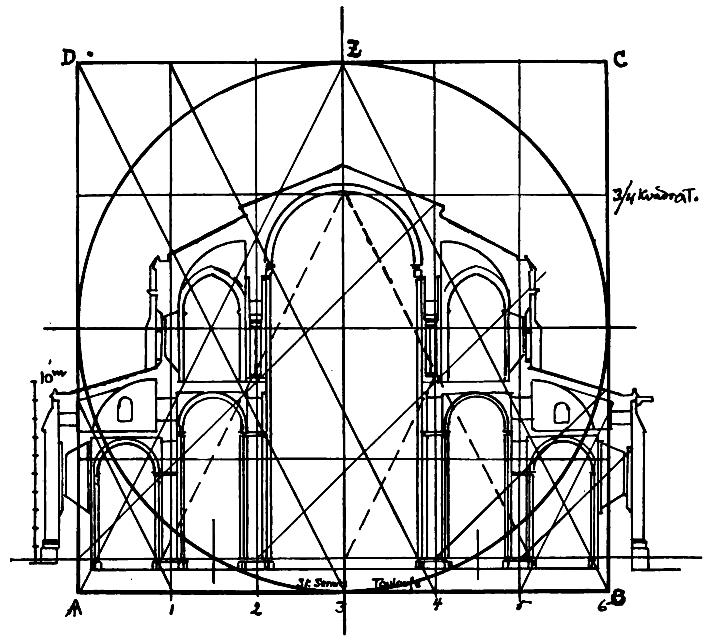


Fig. 22.—Transverse section of St. Sernin, Toulouse, analysis.

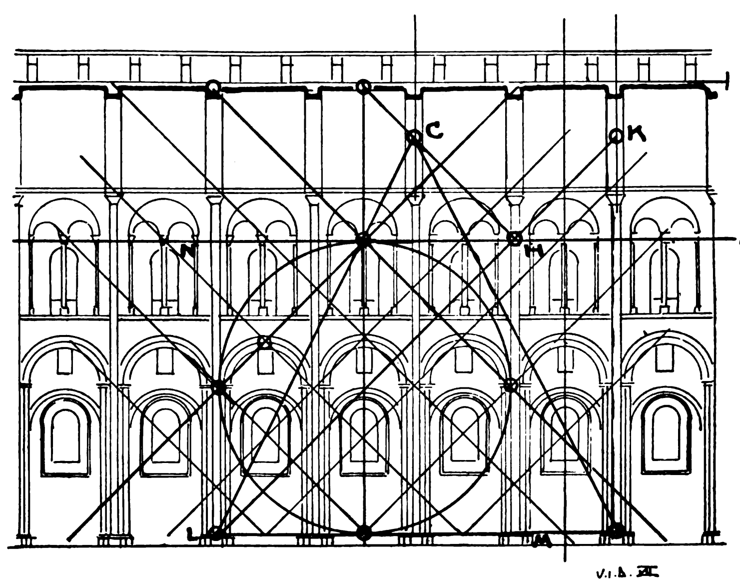


Fig. 23.—Longitudinal section of St. Sernin, Toulouse, analysis.

call the triangle in the square (which in Cluny lies in the west front) is found in St. Sernin through the central axes of the west towers. As shown by the circle inscribed in the plan, the latter is designed within a square which is divided into six, both transversally and longitudinally. We shall also treat this subject in the next chapter, and prove gradually that this way of dividing is a fixed rule for the regular five-aisled cathedral of Western Europe.

Fig. 22 shows this division of the square as it concerns the transept. The transverse section is interesting because it shows how the church is proportioned *ad quadratum*, according to the angles of 45° or of 63° 26′, but nevertheless only *ascendit ad quadratum* to three-quarters of the height of the square. The reason for this is probably that in the use of the semi-circular vault—whether this is a barrel vault as in St. Sernin or a cross vault, with or without ribs—one has not been able, or dared, to let the walls with the vaulting rise *ad quadratum* on the great width of the church; the top of the vault would in this way have been at the zenith of the circle, such as we find it to be the rule at the time of the pointed arch.

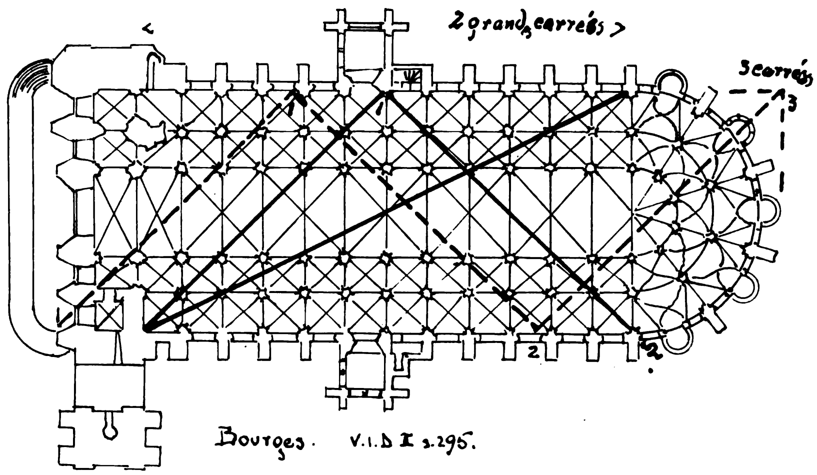


Fig. 24*a*.—Plan of Bourges cathedral, analysis.

This case is clearly illustrated in fig. 23, which represents the longitudinal section of the church. The points C and K represent accurately the height of the walls of the nave. From the diagonal L—K according to the angle of 45° we see that four bays go into the square, the side of which is equal to the height of the wall, while the walls should have had a width of six bays, if the church had been, as to height, as a five-aisled church *ad quadratum*.

As we remarked already when mentioning the abbey church of Vezelay, four bays to the square are characteristic of a regularly constructed three-aisled church. We find, therefore, that the cathedral of St. Sernin in Toulouse, although its plan is traced as a regular five-aisled church, in its raising is carried out according to the principle of the three-aisled one. In spite of this deviation, the reason of which has been alluded to, it appears nevertheless that the walls are built perfectly after the system.

As regards the designing of the plan within two large squares, as we noticed in Cluny, we shall be convinced by what follows, that this method becomes gradually a rule for the cathedrals built by Benedictines in the west of Europe.

Most of the Norman cathedrals were built after this rule, whether they have central towers or not. Among churches having central towers we can name in France the cathedral of Laon, in England the cathedral of Durham, and in Norway the cathedral of Nidaros, this one being the oldest of the three and erected somewhere about 1070 to 1093. In these three cathedrals the central tower is placed in the middle of the plan. The transverse axis of the latter through the middle of the tower forms the common base for the two adjoining squares, or the dividing line of the large rectangle. Inside the two large squares we shall see the development of the nave and transept.

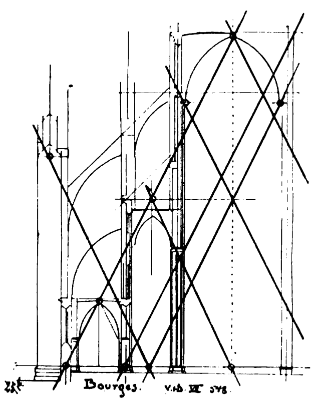


Fig. 24*b*.—Transverse section of Bourges cathedral, analysis.

But before doing so we must show how in the twelfth century the French used these two large squares in their undivided original simplicity, and in this way created the large festive church hall on the great unbroken lines; for example, Notre-Dame of Paris and the cathedral of Bourges, and in Germany the cathedral of Cologne, and many others.

The *cathedral of Bourges* is reproduced in plan (fig. 24) and transverse section (fig. 24*b*). Here we can see the church as a single large and clear rectangle. With apse and tower we have the traditional three squares; the length of the church is equal to three times its internal width.

In the cathedral of Bourges we have a typical example from Gothic times. We notice the specific French plan where the antique temple form is realised in its purity *ad quadratum*. In its transverse section (fig. 24*b*) we notice how the pointed arch allows the possibility of realising

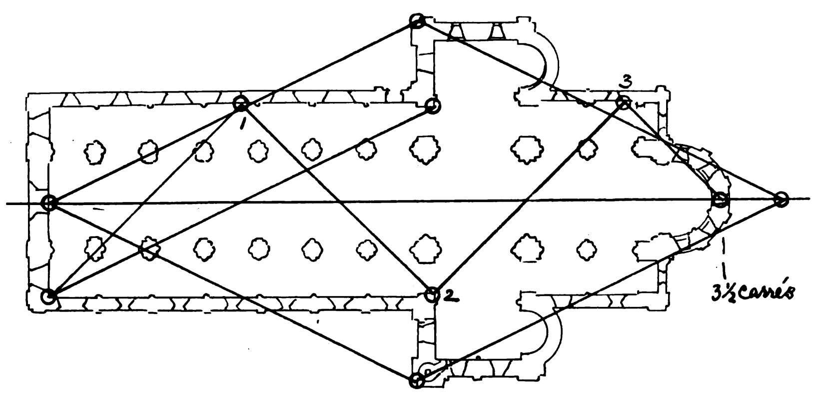


Fig. 25.—Plan of the abbey church of Lessay, analysis.

the square vertically also; everything is built according to the angle of 63° 26′, or *ascendere usque ad quadratum*. Viollet-le-Duc is therefore undoubtedly in the wrong when, misled by his theories, he concludes that the cathedral of Bourges had originally a lower vault (*Dict. de l'Arch.*, vol. vii, p. 547).

With fig. 25, *the abbey church of Lessay,* we go back to a church dating from the end

of the eleventh century; this will allow us to reach across Normandy to England. In Lessay, according to the rules, we have the west wall of the transept as base of one of the large squares and the nave forming the necessary two squares, while a corresponding square is situated east of the west wall of the transept. The church contains, therefore, like the previous one, the three small squares, to which is added the apse, with half a square.

After this very cursory examination of French examples, we turn now to—

ENGLAND

In the *cathedral of Southwell* (1110–15) (fig. 26) we find a plan following the rule in its main points, as in the French monastic church of Lessay, but with this difference, that towers are added towards west, and in their transverse axis the summit of the triangle in the square is situated, with its base on the west wall of the transept. Between the towers in the west and the transept are situated the two smaller squares or rectangle A—B, and to the east one and a half squares. Externally the length of the church has, however, four squares. If the church had originally an apse as we suppose, it was perfectly according to the rule: two squares to the west and two to the east of the west wall of the transept. The Norman cathedral of Southwell was therefore built according to the continental rule, and this ought to be noticed, because in the Norman period the English frequently broke or, we might more correctly say, modified these rules, by imitating the great length of the French cathedrals without at the same time following them in their width, that is to say, without building their long churches with five aisles.

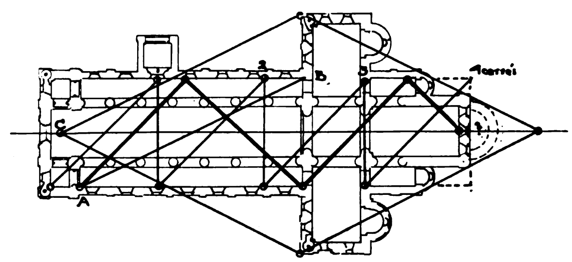


Fig. 26.—Plan of the cathedral of Southwell, analysis.

If two aisles were added to the large three-aisled English-Norman churches, they would become quite according to the continental rule.

We shall first consider *the cathedral of Canterbury* (fig. 27). Similar to Southwell, it has towers placed west. Towards the time this church was built (1070–77) the custom began to mark the division between the two large squares, or the centre of the church, by the central tower; thus the *monk Gervasius* relates that *the tower is erected on large pillars and placed in the middle of the church like the centre in the middle of the circle.** We shall prove later that these words of Gervasius, far from being only a figure of speech, express in reality a rule. The interior of the cathedral of Canterbury, before 1070, has two squares between the west towers and the west wall of the transept, and east of this, 1 square; to this is added the apse; while the total length of the church externally from the west elevation to the apse, also externally, has four squares, that is to say, the old rule unaltered.

By Lanfranc's extension in 1070, according to the new rule with the central tower in the middle, the church was lengthened of two squares; in this way the total length becomes six squares, according to the internal width of the building. It did not, however, get any extension in width to retain the proportion; like most of the Norman cathedrals in England, it

* Bell's Cathedral Series: *Canterbury* (London), p. 4

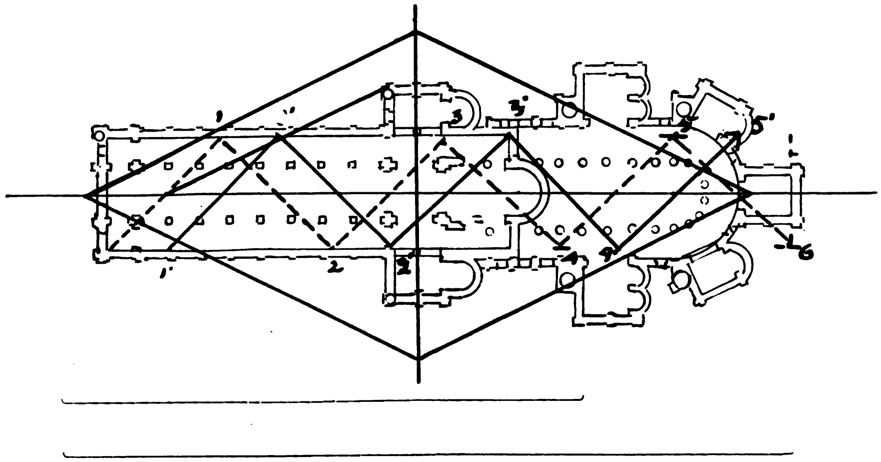


Fig. 27.—Plan of the cathedral of Canterbury, ~~analysis.~~ 1096–1130.

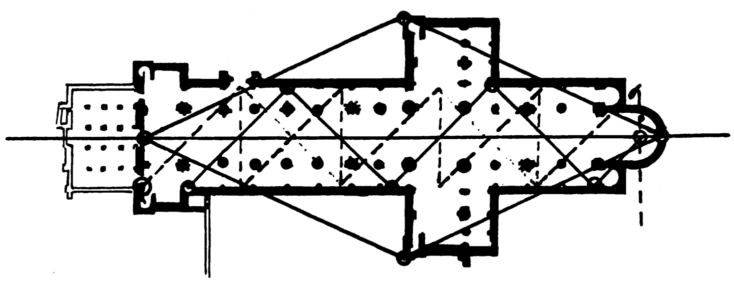


Fig. 28.—Plan of the cathedral of Durham, analysis.

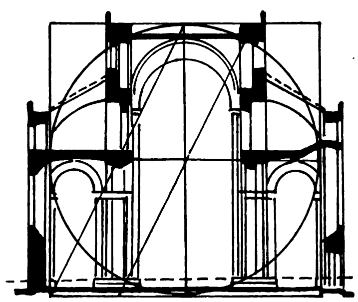


Fig. 29.—Transverse section of the cathedral of Durham, analysis.

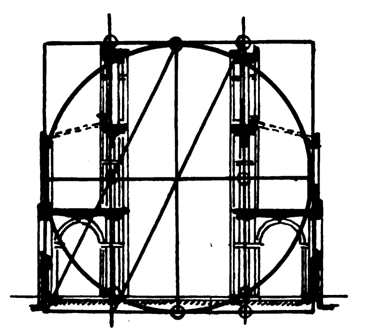


Fig. 30.—Transverse section of cathedral of Ely, analysis.

obtained a disproportionate length. This malformation was continued by the later extension towards east (not shown in the plan) after the fire of 1174.

In *Durham*, where, unlike Canterbury, the builders were not tied to an older building, but where they pulled down the existing and probably insignificant Saxon church, the rules for the raising of the cathedral get more freedom. The cathedral of Durham was, as we know, founded in 1093. From its plan (fig. 28) we see a harmonious design conceived within two large squares, one to the west and the other to the east of the west wall of the transept; in the same way the nave with its two smaller squares is situated between this and the east side of the west towers.

The total length of the church, apse not included, is internally five squares instead of four, which were the rule as we saw it in the cathedrals of the Continent.

Moreover this long church, dating from about 1100, is not five-aisled like the French, neither has it obviously the height of these. It has been able, however, in spite of its Norman style of three aisles, to be built *ascendere ad quadratum*, as seen in section, fig. 29.

At the side of it we have given the section of the *cathedral of Ely* (fig. 30), a building scarcely less old, but having only a secondary interest because it is not vaulted.

In *Beverley Minster*, built in 1225 (fig. 31), we have an example of English churches dating

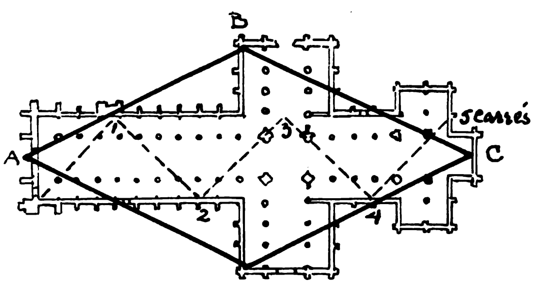


Fig. 31.—Plan of Beverley Minster, analysis.

from Gothic times. The transverse section, which has not been reproduced here, is also designed *ad quadratum*; the height to the vault is almost equal to the width. But, as it appears from the dotted diagonals of 45° in the plan, the nave of the church is too narrow in proportion to the length, there being more than two squares on the west of the transept.

The above-mentioned disproportion between length and width in Norman times, and which began in England with the cathedral of Canterbury, is shown to have been fatal, as it continued in English Gothic. The English masters do not seem to have understood the inner meaning of the rule of the square, which will be proved in our later analysis.

In the *cathedral of Wells* (fig. 33) we find the *central tower placed in the centre of the church* and the *west front* placed at the conventional distance from the west wall of the transept, *therefore within the large square*. Between the east side of the west towers and the west side of the central tower we find the rectangle D—E of the nave, and from the east side of the central tower begins the rectangle of the chancel F—G, to which is attached the Lady Chapel according to English custom.

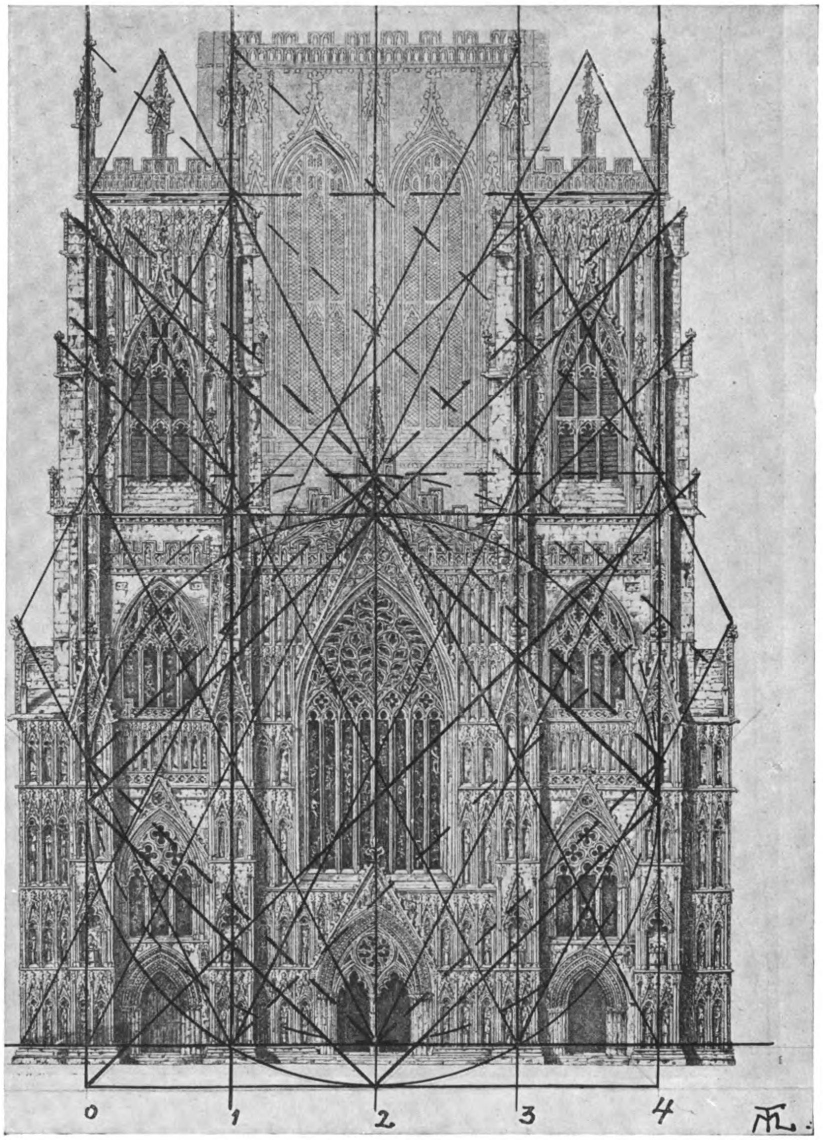


Fig. 32.—The cathedral of York, west front, analysis.

If we disregard these additions as well as the fact that the large transept is too broad in proportion to the whole plan according to the conventional building rules, then the cathedral of Wells is a typical English church such as the continental rule made them through the unfortunate example of Canterbury.

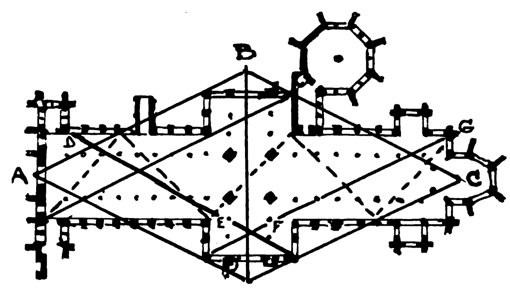


Fig. 33.—Plan of the cathedral of Wells, analysis.

The *cathedral of York* (fig. 34) is conceived *ad quadratum*, not only in its horizontal but also in its vertical plan. The west front (fig. 32) is built logically and correctly, even to the distribution of the details.

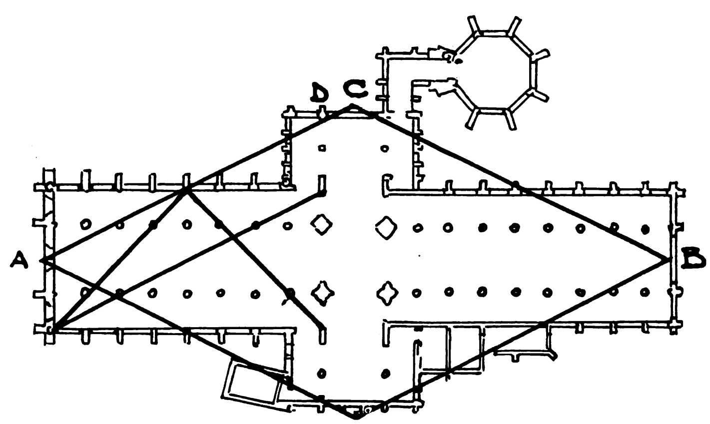


Fig. 34.—Plan of the cathedral of York, analysis.

It is built like a French church, quite regular according to the two large squares A, C, B, and right opposite C the central tower is placed in the centre. From point D in the northern transept the diagonal falls just in the central line of the west front.

The *cathedral of Exeter* (fig. 35) has also, as we see, a regular design. The drawing speaks for itself.

We have now stated the use of the universal and ancient rule for building churches in England.

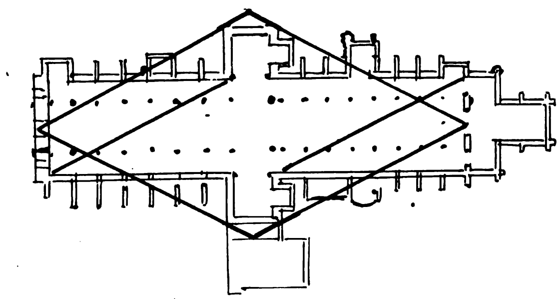


Fig. 35.—Plan of the cathedral of Exeter, analysis.

GERMANY

It is not surprising to find this rule also in Germany; we have already given examples of its building art in the cathedrals of *Strasburg* and of *Metz*. As they are strongly related to the French on account of their character, they have been classed with the latter. Here we shall consider only the Romanesque churches, as later on, we shall examine especially the cathedral of Cologne as a Gothic type.

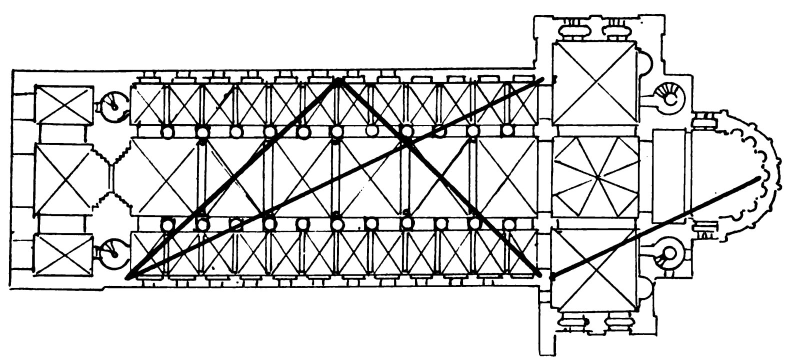


Fig. 36.—Plan of the cathedral of Spire, analysis.

Through the constant intercourse between the Roman Empire and Italy, German Romanesque churches seem to have strongly preserved the original antique form. We find an example of this in the *cathedral of Spire* (fig. 36). We see the two squares to the west of the

transept and one square to the east. This applies also to the cathedrals of *Mainz*, *Worms*, and *Bamberg* (see *Handbuch der Arch.*, II, IV, 3, *Der Kirchenbau des Mittelalters*, Leipzig, 1913, pp. 22, 27, and 30)—in short to most of the *Romanesque* churches in Germany which can with good reason be called *Romanesque* in opposition to the *Norman* churches of France, England, and Norway.

We see a variation of the theme in the two old churches of *St. Michael* and *St. Godehard of Hildesheim* (figs. 37 and 38).

In connection with this, it is interesting to point out that in Germany there exist written proofs which fully support and comment the Milanese document already quoted.

One of these proofs is written by Lorentz Lacher, "der Pfalzbaumeister und Pixenmeister" in the year 1516, for the education of his son Moritz, "Zu Underweissungen und Lerungen sein Handwerk desto besz und khünstlicher zu volpringen" ("to teach him to improve his work and to finish it in a more artistic way") (A. Reichensperger, *Vermischte Schriften über christliche Kunst*, Leipzig, 1856, pp. 153–5). The second is a manuscript which is supposed to date from 1650, and which contains an extract of the rules for building constructions in the medieval Building Guilds (C. L. Streglitz, *Von altdeutscher Baukunst*, Leipzig, 1820, p. 240, etc., quoted by Dedekam, *Gotik og Geometriske Systemer*, pp. 10 and 13).

These rules being perfectly identical, they can be referred to under one quotation, according to Dedekam, such as follows:

A chancel which is 20 feet wide ought to be made one and a half to twice as high. The author calls this last height very appropriately the *real* height. It is said further that the height of the nave should be twice the width of the chancel, and also—characteristic to note—reckoned from the plinth.

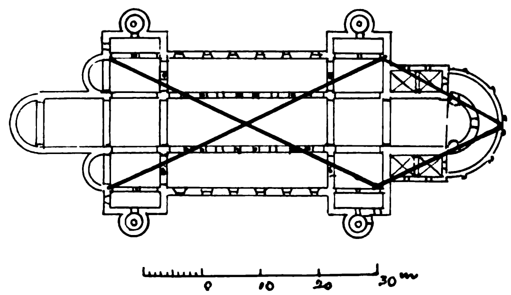


Fig. 37.—Plan of St. Michael, Hildesheim, analysis.

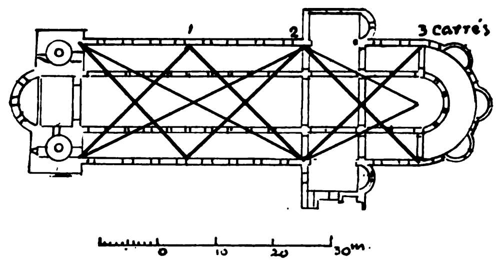


Fig. 38.—Plan of St. Godehard, Hildesheim, analysis.

This is fully in accordance with the rule *ad quadratum*, such as we have explained it above.

Max Hasak maintains that the expressions *ad triangulum* and likewise *ad quadratum* in the Milanese document only signify the use of parallel oblique lines, by means of which the principal points of a building can be determined. He means therefore that "the expression 'triangle' is a short name, a technical term of which the Italians did not understand the meaning."

Our general survey of quite arbitrarily chosen examples from old sacred architecture shows already that this opinion of Hasak—as well as the denial of auxiliary systems in architecture—are without foundation.

We can already state that the expressions *ad triangulum* and *ad quadratum* of the Milanese document, so far from being convenient artisan's terms, without any deeper meaning, are,

on the contrary, in their fullest sense a "terminus technicus," which in a brief expression contains the ancient building rules, fixed by the imaged language of classic philosophy, for its deep consciousness, for the unity in existence, and for this unity as a condition of harmony.

These rules have been realised in the pagan temples, as we will show more clearly in the later chapters, and they have been continued in the churches of the Middle Ages.

We know, furthermore, that the rules of antique art—such as they have been understood by Vitruvius—have been spread by copies from the mother-house of the Benedictine order of Monte-Casino through their branches in the eighth, ninth, tenth, and eleventh centuries all over Europe, and thus formed a firm and concise guide for the Christian temple-builders.

This teaching found its divine sanction in the description of the temple of Solomon in the Bible.

We have also seen how this antique method *ad quadratum* is in a state of continual development, without a break, when used in the medieval churches.

From the original rectangle of two squares the plan of the church is developed within a larger rectangle of four squares, while at the same time it develops transversally by the addition of a transept north and south, in such a manner that the whole again comes within a rectangle of two larger squares—one to the east and one to the west of the axis of the church.

Later on, when we are treating of the elevation of the church, we shall discover that it is designed *ad quadratum* both longitudinally and transversally, even to the very summit of the towers, and that the whole building can finally be included in a large square.

Unity is carried through from beginning to end—it is the harmony of perfection.

The cathedrals of Paris, of Cologne, and of Nidaros will furnish us with typical examples.

But we shall first consider the connection between the scientific geometry of antiquity and the artisan's practical geometry in the Middle Ages.

CHAPTER III

THE SQUARE AND PRACTICAL GEOMETRY IN MEDIEVAL HANDICRAFT

BEFORE leaving this general examination we must define more exactly the rule *ad quadratum*, by means of which and within which the Middle Age built its churches. We do this in diagrams, as seen in figs. 39 and 40. The first is the square for a three-aisled and the second for a five-aisled church. An explanation of how we found this rule *ad quadratum* will better explain the development of the construction.

When we saw the angle of 63° 26′ constantly repeated in all the principal and secondary features of the cathedral of Nidaros, and when we remembered that this angle was the diagonal of the rectangle with proportion of 1 : 2, and moreover that this rectangle represented half the square, it occurred to us to investigate how such a rectangle could be found to fit into the building. With the purely intuitive feeling that we had to start at the plinth above which a building really begins, we resolved to carry our investigations from that point.

When we found later that Viollet-le-Duc had done the same, we felt on safe ground. As a preliminary to this we drew in the transverse section of the nave of the cathedral a line A—B at the top of the plinth and drew up the axes in the walls of the nave (refer to C and D in fig. 39). We chose then as radius the distance between these, and found the centre of a circle by starting from the intersecting points of the axes with the line on the plinth, E and F. By dividing the circle in four parts and introducing them in a square, the vertical sides of this square coincide with the axes in the walls of the aisles, and the whole transverse section of the building falls within it and with all the geometrical combinations that the square allows.

In accordance with this, we have constructed in fig. 39 a diagram for a three-aisled church.

On the line A—B we fix first the arbitrary length E—F, which we use as the base line of an equilateral triangle, at the summit of which we place the centre of a circle having the side E—F of the triangle as radius. Having divided a circle in four parts and introduced them in the square, we divide the horizontal diameter in four equal parts, and through the halving points of the radii we draw the vertical lines C—G and D—H.

The square which in this way has been divided into four contains, then, four equally large rectangles, of which the rectangle on the left of the vertical dividing line C—G makes one-fourth, the rectangle between the dividing lines C—G and D—H makes two-fourths, and the rectangle to the right of D—H one-fourth of the square.

The diagonals of 63° 26′, I—K, K—L, C—H, and D—G, and also C—M and D—N, are introduced. In the intersections of the diagonals with the plinth line A—B we draw vertical lines by means of which the wall of the nave and the inner side of the walls of the aisles are

determined. In the intersecting points between the axes of the walls of the nave C—G and D—H and the upper portion of the circle at O and P, where the line between these two points cuts the diagonals, we find that the height of the vault on the walls of the nave is determined.

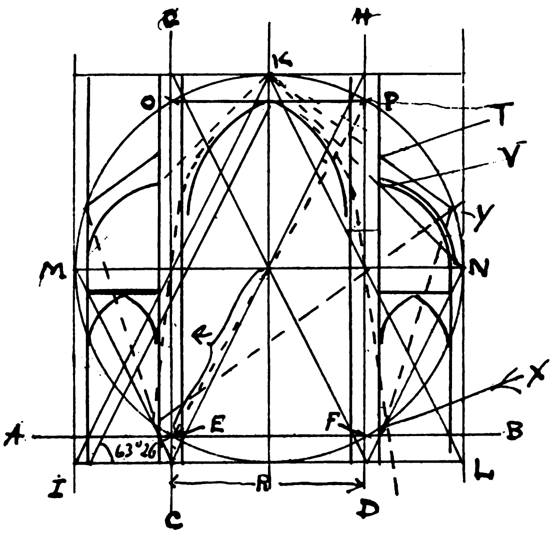


Fig. 39.—Diagram for the construction of a three-aisled cathedral, analysis.

Similarly, the height of the vault over the aisles is found by means of the intersection points of the diagonal lines C—M and D—N with the wall lines of the aisles.

A church is here produced, with a nave and two aisles, all according to the proportion of 1 : 2 and all constructed *ad quadratum*, that is, according to the square, within the square, and by means of the square. Through our investigation of the cathedral of Nidaros and the general study based upon it of medieval churches of Italy, France, England, and Germany, we have in reality found the explanation of the clear and simple meaning of the Milanese document, which document, it must be particularly noted, was entirely unknown to us during our work until it came to our notice later when reading Hasak.

The diagram of the five-aisled church (fig. 40) can be made, after the previous demonstration, as for the three-aisled church, but with this difference, that the horizontal line is here divided in six instead of four parts. In the three-aisled church the nave gets two-fourths and each of the aisles one-fourth of the square, each of the three aisles have their rectangle with the diagonal of 63° 26′, or occupy a space according to the proportion of 1 : 2, all reckoned from the line A—B on the plinth.

In the five-aisled church, on the other hand (fig. 40), the nave gets two-sixths and each of the four aisles one-sixth of the square. From this are produced five rectangles, with a proportion between width and height as 1 : 3, with a diagonal forming an angle of 71·564° with the base, instead of the diagonal of 63° 26′, according to which the proportion of space is determined in the three-aisled church.

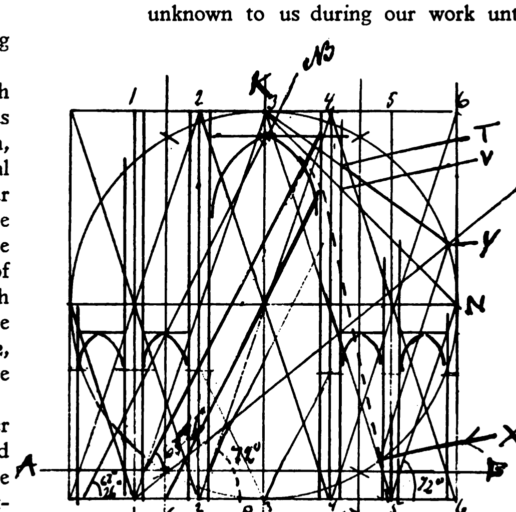


Fig. 40.—Diagram for the construction of a five-aisled cathedral, analysis.

We obtain thus in the five-aisled church a new diagonal which comes very near to the diagonal of the pentagon, as with the side of this used as base it forms an angle of 72° or $360° \div 5 = 72°$.

Therefore it might be said that the space in the five-aisled church is practically proportioned *ad pentagonum*, to use the expression of the Milanese document.

This angle of about 72° is, however, as might have been noticed, only the practical result of dividing the square into six. As the general examination has shown us already, and as the special analysis will prove it later, the construction *ad quadratum* with the diagonal according to 63° 26′ of the half square is the leading principle for proportioning the entire building and the architectural parts—equally so as regards the five-aisled church.

It can be pointed out at the same time that a diagonal according to the angle of 72° has been used for proportioning the space of some churches, both in Romanesque and in Gothic times; the pentagon and its diagonals with their angles have been used in various ways in medieval church building.

At present we shall only consider the part which the pentagon in connection with the square has obviously played as static guide in the construction of the system of abutment to receive the thrust of the vault.

It is true that we have no written proof for this *regula ad pentagonum*—as we might call it technically—such as for *regula ad quadratum*; but in the buildings themselves we will find quite plausible evidence of it.

In accordance with our observations, the pentagon is therefore introduced in both diagrams according to *regula ad quadratum* (figs. 39 and 40). It is not drawn in full, but, as can be seen, it is represented by two of its five sides. These sides are marked K—Y and Y—X. At the same time a square placed diagonally is introduced into the circle and described by the line K—N.

The points where the sides of the pentagon and of the diagonally placed square intersect the outer surface of the wall of the nave are in both drawings marked by the letters T and V.

It is most important to notice that the part between these two points T and V is just that part of the wall where the thrust of the vault is greatest.

The object of the flying-buttress is, as we know, to counteract this thrust, and wherever these arches have been correctly built, they butt against this part of the wall, between T and V.

At the same time we shall point to another observation of theoretical interest.

The thrust of the vault is neutralised by the buttress and forced in a line corresponding to the axis of the pillar of the nave; while the line of thrust without abutment runs obliquely out from this pillar—such as we have shown by the dot line from the top of the vault, in the drawings after Viollet-le-Duc.

As it will be seen, the line goes down to point X. It is, therefore, interesting to notice that point X is an angle of the base of the pentagon K—Y—X inscribed in the circle.

As far as we have been able to judge from existing scientific writings, the Middle Ages did not possess the theory of mechanics of our days.

It was probably limited to some empirical notions, summed up in simple geometrical formulæ. Some of those used in the sixteenth century are still preserved: see Viollet-le-Duc, *Dictionnaire de l'Architecture*, vol. iv, 63. The pentagon and the pentagram have undoubtedly served as static forms. But history has forgotten what it knew formerly. The cathedral of Ely presents a most interesting indication of the correctness of our view. In fig. 41 the transverse section of the Gothic chancel is given. It dates from 1235 to 1252.

In our general analysis we saw that the Norman nave of this cathedral is built *ad quadratum* (fig. 30). Fig. 41 shows that the chancel is also built according to the same method.

To the right of the drawing we see how the system of abutment was carried out originally.

The higher arch thrusts against the upper part of the wall, the lower had to oppose the thrust of the vault, but, as in most English cathedrals, it was placed too low on the wall, so that in the fourteenth century the vault was on the point of thrusting out the wall. It was necessary, therefore, to lift the abutting-point of the arch higher, as illustrated on the left of the drawing and described by Francis Bond in his *Introduction to British Church Architecture*, p. 411. It can be seen that when repairing in the fourteenth century, the flying-buttress was lifted just to that part of the wall which is included between the mentioned lines: the side of the pentagon and the side of the square placed diagonally.

The objection has been brought against this theory that in the Middle Ages the science of geometry was on too low a level to know the pentagram. The learned German mathematician Cantor is quoted, and it is claimed that "the mathematical figure consisting of a

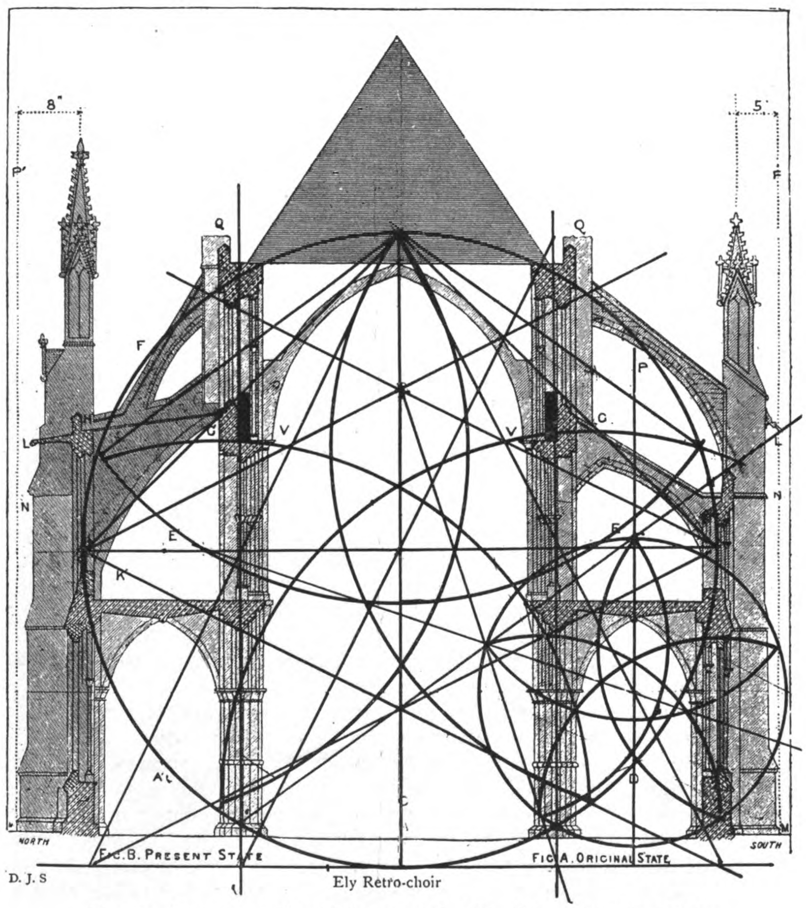


Fig. 41.—Transverse section of the cathedral of Ely, flying buttresses, analysis.

combination of the pentagon and the pentagram cannot be traced in mathematical works until it appears in an anonymous French *Traité de Géométrie*, dating from about 1275" (see Dedekam, *Gotik og Geometriske Systemer*, pp. 21, 42, etc.).

This quotation taken from Cantor is quite unwarranted. Cantor did not at all deny that the pentagram was known in the Middle Ages, but, in his historical treatise of preserved writings, he only pointed out where the pentagram is mentioned. This appears clearly when he quotes the English mathematician Bradwardinus (1290–1349), and his works on star-polygons (*figuris angulorum egredientibus*), where he says:

"The pentagram in the history of mathematics has been frequently noticed. We have seen its shape appear not only in the darkness of ages, but also in the mentioned French book on geometry of 1275, we have seen also the sum of its angles in the book on Euclid by Campanus, in the second half of the thirteenth century" (*Geschichte der Mathematik*, 2nd edition, 1900, vol. ii, p. 114).

It is evident, therefore, that Cantor had not the foolishness to believe that a thing is first known when a written proof of it is produced and preserved accidentally. The learned Historian of Mathematics would not have observed, in that case either, that there exist cinquefoils used as ornaments on churches of the twelfth century, as in Notre-Dame of Paris; the pentagram being also used as a stonemason's mark on buildings of the eleventh century, among others in the cathedral of Nidaros.

But Cantor is not without mistakes, however; when, mentioning the contents of *Geometria Deutsch* written about 1487, he speaks of various geometrical problems and carries back their ancient origin to Euclid, Heron, and others, and he says that the instructions for constructing a pentagon in the above-named work had not been seen in any book before it appears in *Geometria Deutsch*.

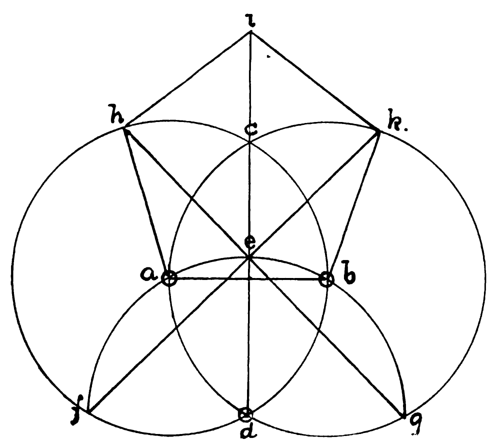


Fig. 42.—Construction of a pentagon after *Geometria Deutsch*, analysis.

This construction is shown here in fig. 42.

It seems that this writer, otherwise so sharp, was not aware that this figure dates from the time of the Greek philosophers, being identical with another figure which he gave himself earlier in his work while writing on the Greek geometrician Hippocrates (about 400 B.C.). Cantor says on this figure (fig. 43):

"Still it is not without interest that a pentagon with inverted angles does appear, but is only called 'the figure with straight lines consisting of three triangles, ZBH, ZBK, ZKE.' We leave the question open: it can be concluded from this, that this figure was not considered as a pentagon; the combination of the three triangles into one figure is, however, clearly shown, and in this way it becomes the first-known occurrence of a polygon with inverted angles in a geometrical treatise" (*Gesch. d. Math.*, 2nd edition, vol. i, p. 195).

It is not clear what theoretical interest can be attached to this figure because it is turned "inside-out." But what can be of interest to us is that it appears in Hippocrates, and it seems to be simply a formula of remembrance for the construction of the pentagon, showing at the same time that the Pythagoreans, as it might be expected, had studied the properties of the angles developed by the diagonals of the pentagon, something Cantor has entirely overlooked.

In connection with this, it will be remembered that the merchant Hippocrates of Chios, after being ruined, went to Athens, where he joined the Pythagoreans, from whom he learned geometry, being, however, excluded from their midst later, because he taught their secret science for money.

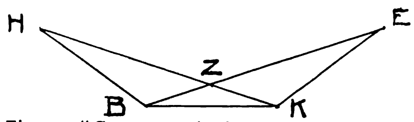


Fig. 43.—" Commemorative formula " of Hippocrates for the construction of a pentagon.

The pentagon with " inverted angles " (fig. 43), which owes its origin to Hippocrates, seems to be, as mentioned, a quite simple fundamental formula for the construction of the pentagon, the sign of the Pythagoreans (see fig. 44). Starting from this figure, and taking Z as centre, a circle is drawn through points H and E, and from the intersecting point F of the vertical diameter with the circle, the chords through B and K are drawn; they cut the circle in points C and D.

It will be noticed that we have already drawn the diagonals of the regular pentagon C, D, E, F, H, or, in other words, the pentagram.

If we try now to see in what relation this formula stands to the method given for the construction of a similar figure in *Geometria Deutsch* (fig. 42), we continue in fig. 45 by taking the distance B K as radius, and having B and K as centres we draw two dotted circles; these will go through the points of the formula H and E, and also through the corners C and D of the large pentagon thus produced. Then if we draw a circle with the same radius having point *d* as centre through B and K, and draw from the intersecting points of this circle with the previous dotted circles at points *f* and *g*, lines through Z, we get by the intersecting points of these lines with the two circles *h* and *k*, the small pentagon *i, k,* K, B, *h,* completed, which, in parenthesis, is perfectly in the proportion of the " sectio aurea " to the large pentagon. This small pentagon developed on the formula of Hippocrates differs from the pentagon *a, h, i, k, b* in *Geometria Deutsch* (fig. 42) in this, that the first is a perfect pentagon, whereas the last is somewhat too high, and therefore only approximately a regular pentagon (see fig. 45).

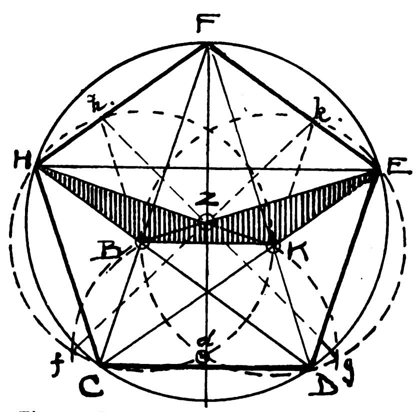


Fig. 44.—Pentagon constructed by means of the formula in fig. 43.

It is outside our subject to explain on which geometrical functions this small discrepancy depends in *Geometria Deutsch.*

What is of importance as regards this construction also, is to have proved the connection between antique science and the practical geometry of the medieval artisan and the general tradition in history and art.

This tradition, which appears in liturgic and heraldic symbols, seems to have been preserved by the formula of Hippocrates and in connection with it, such as it reappears in *Geometria Deutsch.*

A study of the following fig. 46 will further prove this. Taking the construction from *Geometria Deutsch,* we get here this obvious combination: a circle is drawn round point *m,* and by means of this we get the spherical three-pointed star. The three circles in *Geometria Deutsch* form from this the figure in a shield, which in heraldry is named " litera Pythagorica," and which symbolises the pallium, being the sign of the dignity of an archbishop, and whose likeness with what is known as the " Pythagorean Cross " is striking: fig. 47.

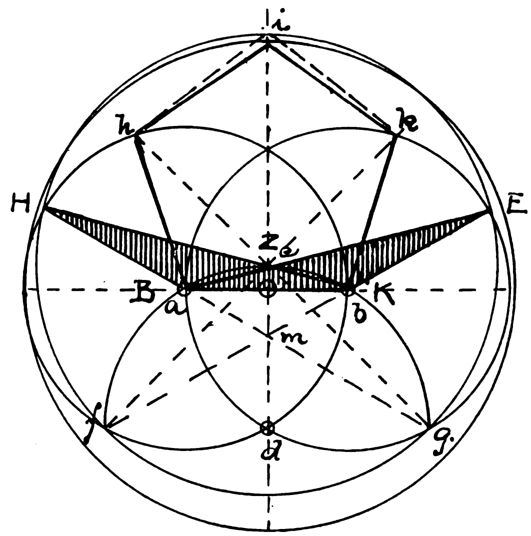


Fig. 45.—Comparison of pentagon constructed by means of the construction in figs. 42 and 43.

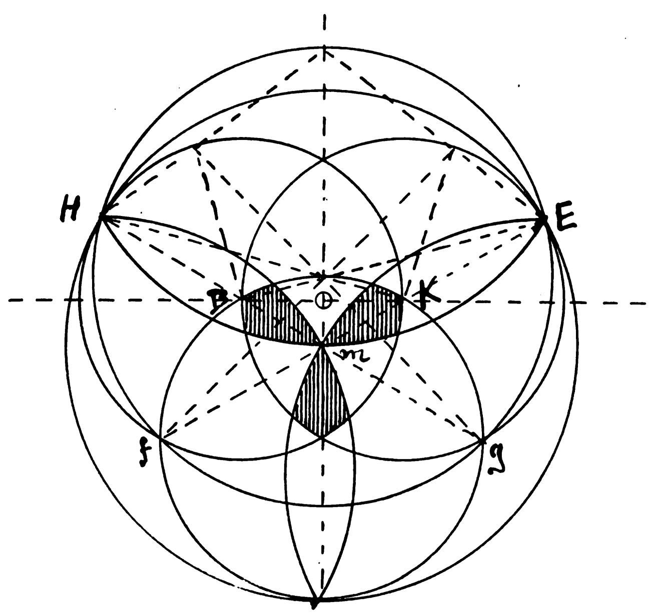


Fig. 46.—The curved triangular star and "Litera Pythagorica" developed in the construction of *Geometria Deutsch.*

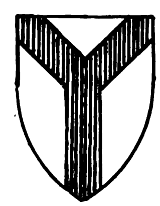


Fig. 47.—Heraldic Pythagorean cross.

But quite apart from this formula of remembrance of Hippocrates, even quite apart from the whole of antique geometry, we venture to maintain that to every intelligent builder who is obliged to use compasses and ruler in his work, a number of obvious combinations of the square and of the circle inscribed into it will present themselves, all leading to a practically perfect and satisfactory construction of the pentagon and of the pentagram—if he does not try his problem directly on the circle, as architects do.

As an example of such combinations we have already mentioned that by dividing the square into six, the angle of 72° of the pentagram is produced approximately. By halving this angle, as we know, the pentagon is produced at once in the circle which is inscribed in the square.

Other practical methods which present themselves will be pointed out in the construction (fig. 48):

(*a*) By means of a triangle *e—f—g* drawn in the circle; if the radius is cut in half by

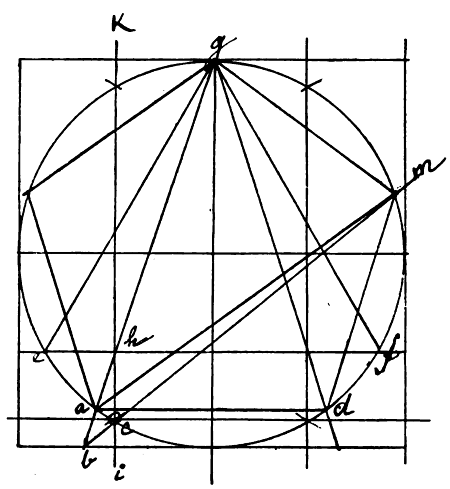


Fig. 48.—Various constructions of pentagons.

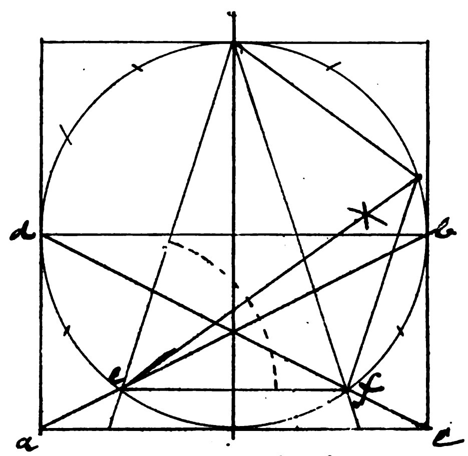


Fig. 49.—Various constructions of pentagons.

the line *i—k*, this line will cut the base *e—f* of the triangle at point *h*. If a line *g—a* is drawn through the summit *g* of the triangle and the point *h*, and if in the same way we draw a corresponding line *g—d*, the chord *a—d* forms the side of a regular pentagon; as a variation of this construction:

(*b*) By means of a square and a hexagon. It will be seen from the drawing that when the line *g—a* is lengthened to point *b* at the base of the square, and if from this point through the corner of the hexagon at *c* we draw a line, this will cut the circle at point *m*, when the line *m—g* will form the side of the pentagon.

In fig. 49 we see a construction with the diagonals *a—b* and *c—d* in the halved square, therefore according to the angle 63° 26′. These cut the inscribed circle at points *e* and *f*. The line between these points will again give the side of an approximately regular pentagon. The figure speaks, however, for itself.

In fig. 50 we see as a last example the well-known *achtort* from the medieval building tradition employed as foundation for the construction of the pentagon in connection with the

diagonals of the rectangle, as in the previous figure. The achtort consists, as we know, of two equally large squares, of which one is placed horizontally and the other diagonally round a common centre, whereby an octagon is formed. From the summit of the inscribed circle a line is drawn to corner *b* of the square placed diagonally, by which the chord *a—c* becomes the side of the pentagon. For the sake of comparison we introduce the side *e—d* of the construction of the pentagon, by means of the diagonals of the half square. Both methods give very nearly the ideal pentagon, which, as everyone knows, is very difficult to construct absolutely correctly, even according to the construction of Euclid or to the division of a line in extreme and mean ratio.

We have pointed out here, as an introduction, the connection which exists between antique scientific geometry and the practical geometry of the artisan in the Middle Ages. While doing so, we take the opportunity to point out geometrical combinations which intelligent medieval artisans would naturally arrive at when using their compass and ruler.

We intend to show later the connection between the classic times and the Middle Ages as regards scientific geometry.

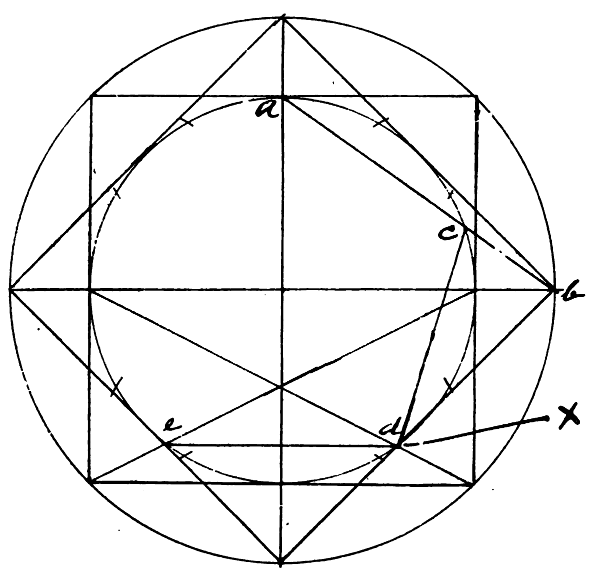


Fig. 50.—Construction of a pentagon by means of the "achtort."

CHAPTER IV

THE CATHEDRAL OF NOTRE-DAME, IN PARIS

IN the foregoing general survey we saw how the principle *ad quadratum* had been gradually realised in the cathedral of Bourges, and similarly in the height of the vault in a five-aisled church. The cathedral of Bourges was planned in 1172, but it was not commenced before the thirteenth century. Part of the chancel was finished in 1220.

We now turn to the cathedral of Paris. It was founded in 1160, and represents therefore an earlier stage of development. But for that very reason it is interesting to notice how the principle *ad quadratum* is there carried out whenever it is possible, while one has not been able to let the great vaulting *ascendere ad quadratum*, or dared to do so. Fig. 51 represents the original plan of this church in 1160, without the chapels added in the thirteenth century between the buttresses. Here we find the large, open, and clear-lined sacred hall, similar to that of the cathedral of Bourges built twelve years later, where we see the same disposition *ad quadratum*.

The total width of the interior is 36 metres, the inner length, reckoning from the east wall of the west towers to the continuation of the outer aisles round the chancel, is 108 metres; the plan therefore contains three squares (*a*—*b*—*c*).

The diagonal of 45° H—I—K shows also that between the east side of the front wall and the chancel there are two squares, while the diagonal K—L, according to the same angle, gives a square for the chancel—apse included.

The width of the front is 42 metres, the length from the west side of the towers to the ambulatory is 126 metres; therefore we have again three squares, which are shown by the diagonals D—E—F—G according to 45°.

If we start from the distance between the axes of the towers which is marked on the plan as "le rayon du cercle," that is the radius of the circle, we obtain, when using the diameter of the semicircle described by means of this radius, exactly the distance between the north and the south gable wall of the transept, externally—such as this became through the extension north-south of the transept in the thirteenth century. As regards the northern gable, this is indicated by the point marked NB. If, from the extreme point A of the diameter, through NB, and from the other extreme point through the corresponding but unmarked points, we draw lines towards the east, and if, for instance, we place a diagonal from point A to B, according to an angle of 63° 26′, and from thence a diagonal of 45° to C, we obtain a rectangle composed of two and a half squares. The length of this rectangle is also the total exterior length of the church.

As shown by demonstration above the plan, the west wall of the transept is situated on the middle line of this rectangle. Moreover the dotted diagonals R—NB—S show that on each

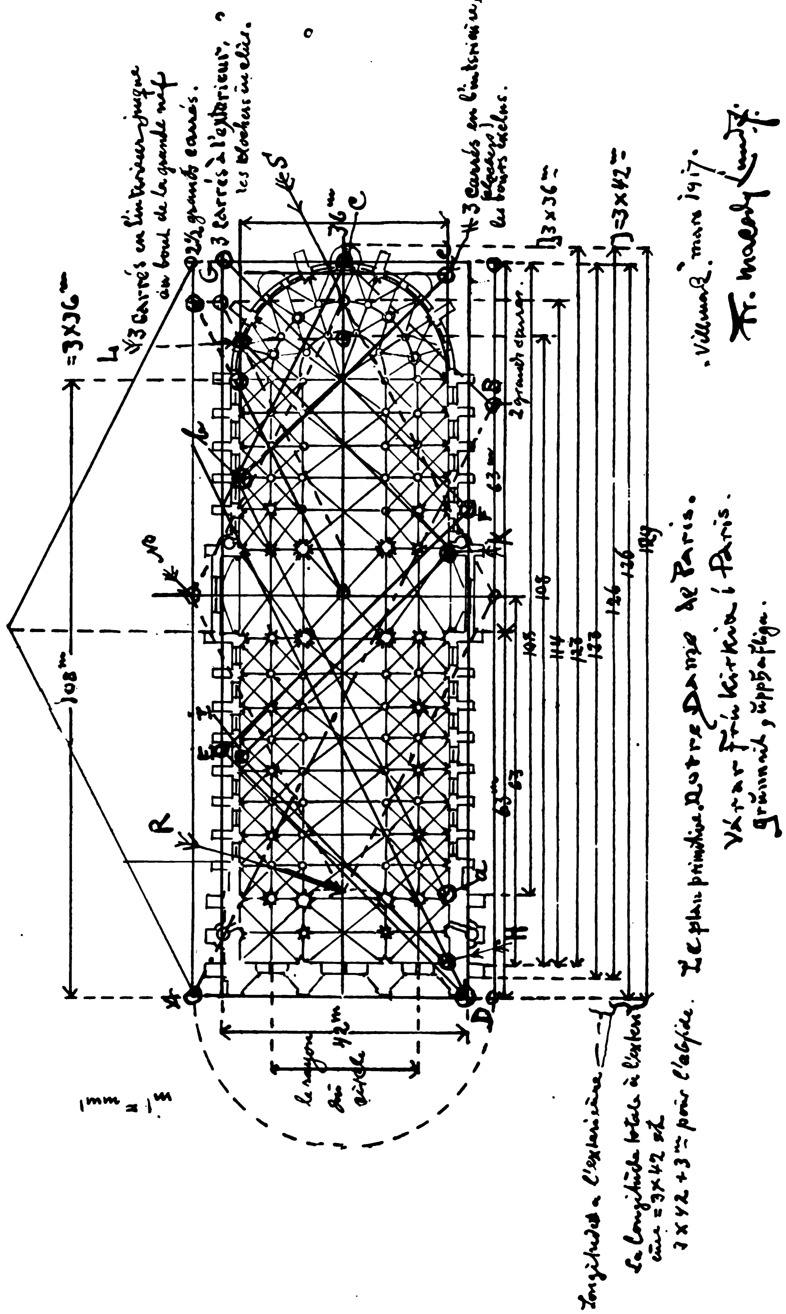


Fig. 51.—Plan before the reconstruction in the thirteenth century, analysis.

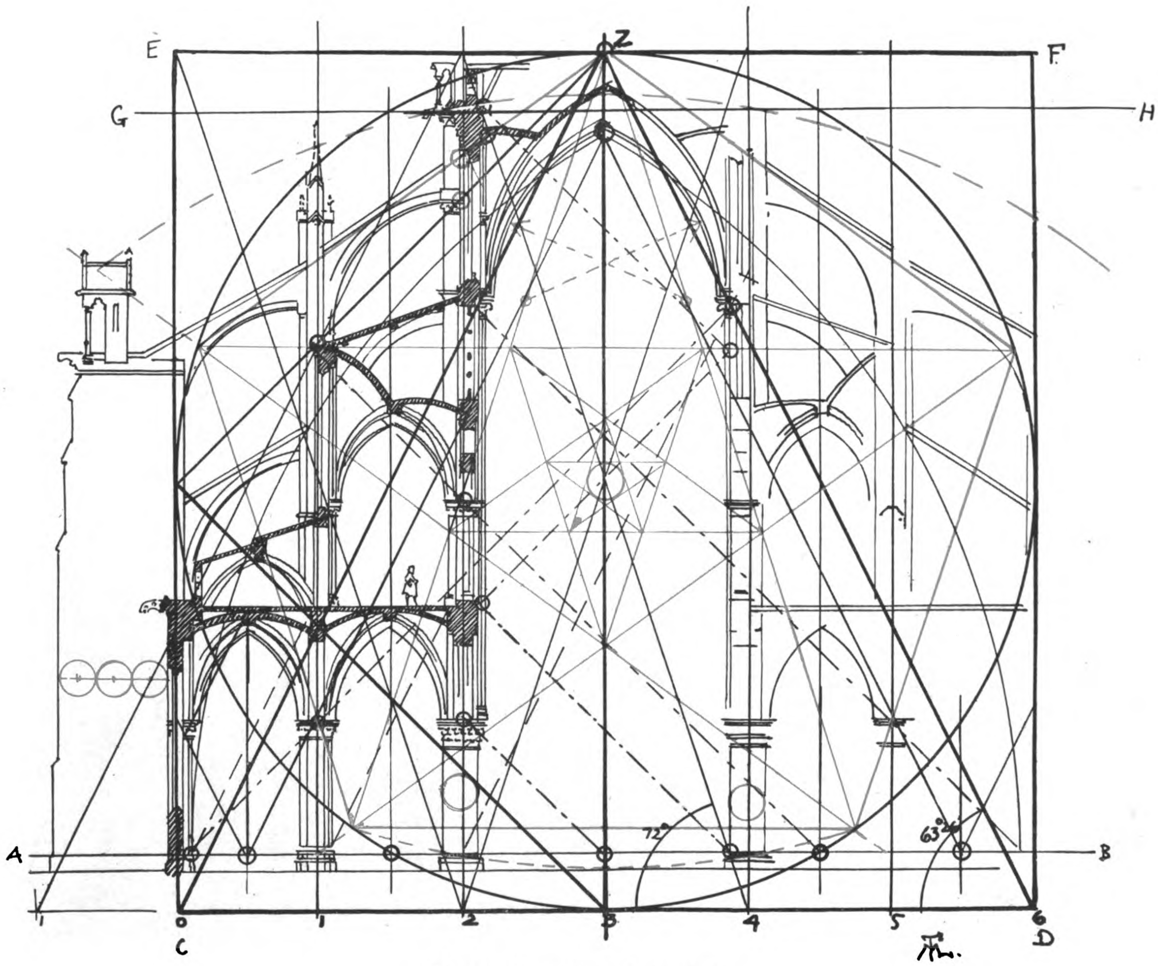


Fig. 52.—Transverse section, analysis.

side of the centre line of the transept there is situated a square, one extending to the east wall of the towers and one reaching to and including the circling of the first aisle.

A further study of the drawing of the plan and of the measurements introduced will, however, convince the reader that the principle *ad quadratum* has in every respect determined the proportioning.

We can see that the *transverse section* (fig. 52) has also been constructed *ad quadratum*. After the previous explanations the careful reader will be convinced when he observes that all the main divisions are determined by the angle of 63° 26′, as, for instance, the height of the aisles from the top of the plinth to the beginning of the vault. The church is planned

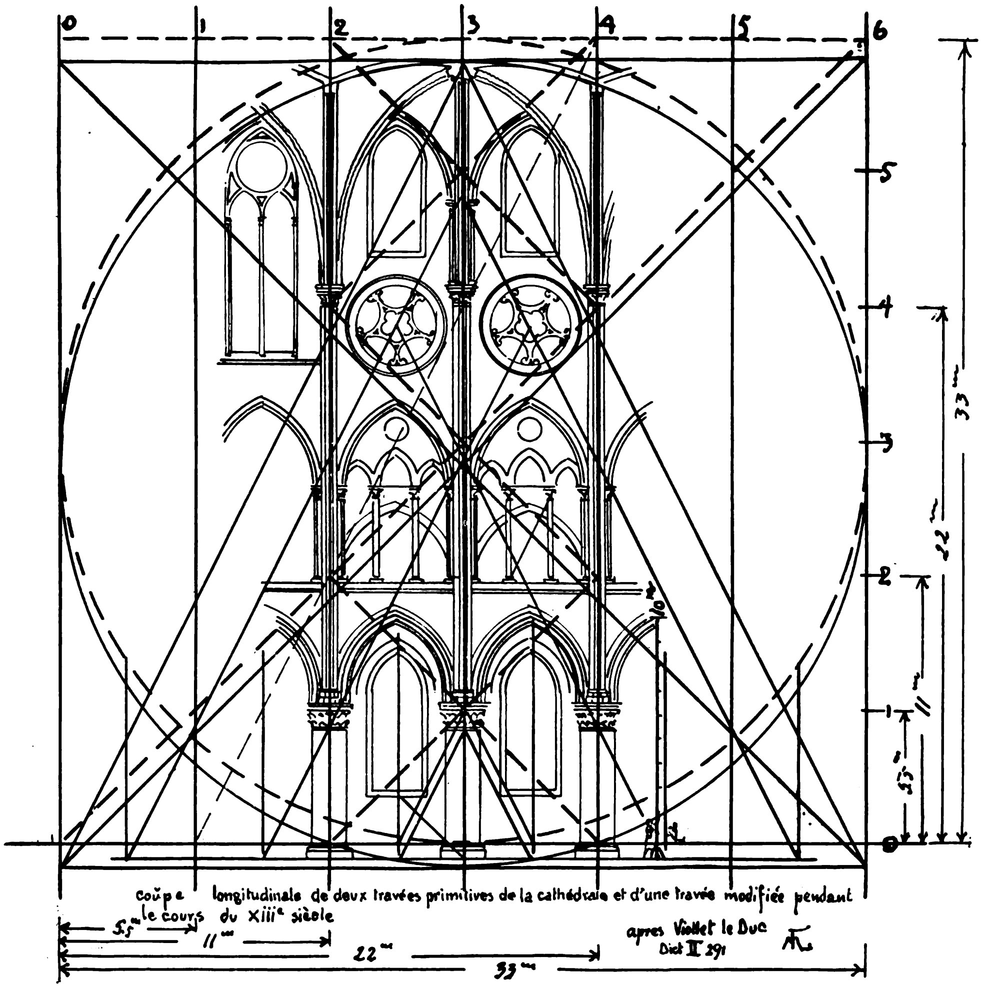


Fig. 53.—Part of the longitudinal section, analysis.

as a five-aisled church *ad quadratum*, but the consequences of dividing the square into six with its resulting diagonal angle of 72° have not yet been completely arrived at. The aisles are proportioned by the angle of 63° 26′, while the nave is proportioned by an angle of nearly 72°.

According to the principle of dividing into six, the summit of the vault of the nave ought to have been situated at point Z. It is, however, situated considerably lower, because in proportion to the total width it is built vertically *ad triangulum*, according to the angle of 60°, as shown by the intersection of the circles. As yet one has not dared or wished, to rise

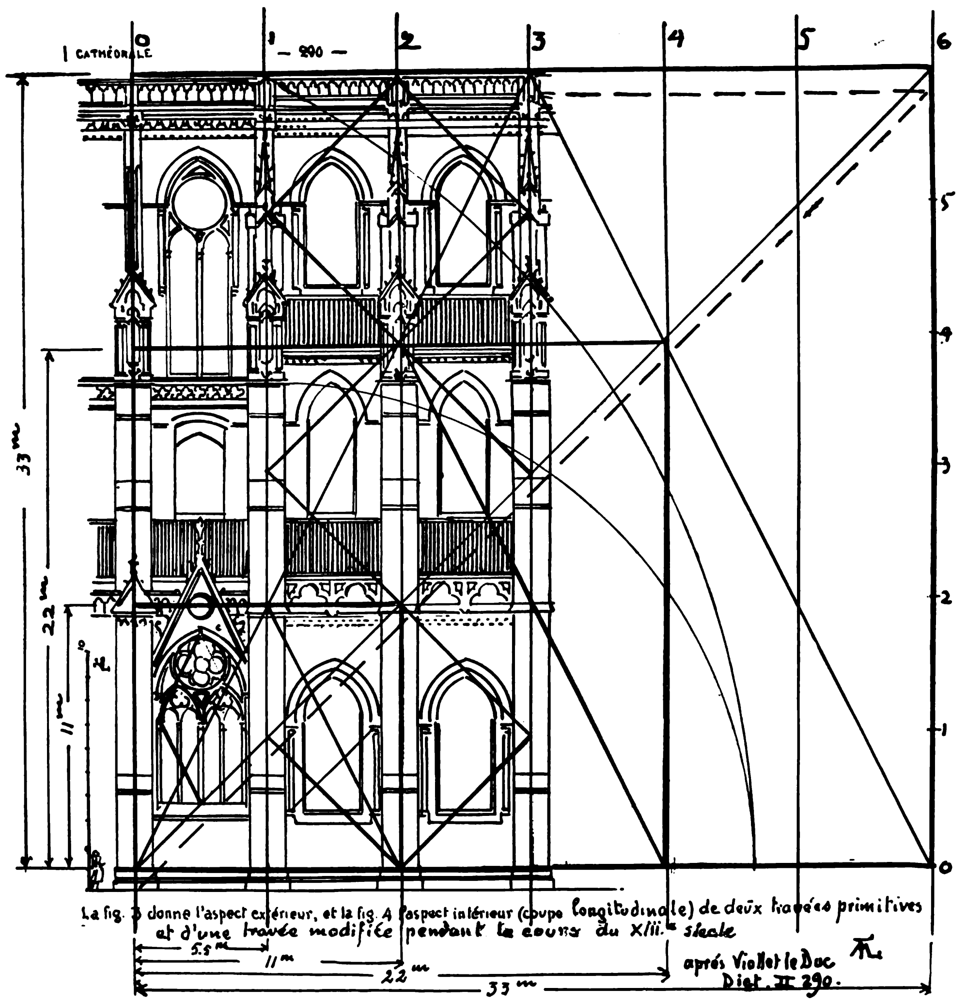


Fig. 54.—Part of the side elevation, analysis.

higher in a church so broad. It will be remembered that the vault of St. Sernin does not reach higher than three-fourths of the square, that is to say, it lacks one-fourth of the height it should have correctly, according to the principle. After the year 1160 the architect has dared to rise considerably higher in Paris, so that here scarcely more than one-tenth of the height of the square is lacking. The proportion between width and height in the nave is, in St. Sernin as 1 : 2½, while in Notre-Dame, as seen by the diagonals of 45°, it approaches very nearly the proportion of 1 : 3—as in a fully developed five-aisled church *ad quadratum*.

Finally, we must not forget to point out how the flying-buttress thrusts against the wall

just between the two red circles, which mark where the side of the pentagon and the diagonally placed square coincide with the surface of the wall. It will also be seen that the radius of the transverse arch has its centre in the diagonal line of the pentagon. Later on, however, we will explain the significance of the pentagon.

In the *longitudinal section* we find the principle *ad quadratum* logically carried out, both internally and externally. Fig. 53, which gives part of the longitudinal section, shows this. The distance between the axis of each bay is 5.m.5, each *double* bay is therefore 11 metres wide. The height to the summit of the vault is 33 metres, therefore according to the angle of 72° of the square divided in six, with the proportion of 1 : 3 belonging to the five-aisled church. The dotted diagonals of 45° show three squares in a double bay, or six squares in every single bay.

With the distance of 5.m5 between the pillars, we obtain, in a longitudinal direction, six bays or three double bays, altogether 33 metres, therefore equal to the height. Here we have six bays in a square, therefore a square divided into six, which we found to be one of the conditions of a five-aisled church.

In fig. 54, which gives part of the side elevation, the same condition is prevalent—six bays of 33 metres altogether. From the bottom of the plinth to the top of the wall and from the top of the plinth to the top of the balustrade, the height is 33 metres, and therefore a square.

Within this square we find not only the height, but also the architectural divisions determined by the angle 63° 26′, similar to what we saw earlier in the monastic church of Lessay. Our later analysis will show that this division of the square is carried out in the whole length of the building.

As a basis for our analysis of the *front*, we have used the drawing of Viollet-le-Duc with his added design of the spires (fig. 55, Plate I). It is necessary at first to find the theoretical frame of the design, or the rectangle within which the front is constructed. We have the choice of three different ways: (1) As explained earlier, to start from the distance between the axes of the walls of the nave; (2) to take half the width of the front, 21 metres, as radius (compare the stroke-lines of the system); and finally, as we should prefer to do here, (3) to use the distance between the axes of the towers as radius of the circle inscribed in the large square of the whole design. (Compare the fully drawn lines of the system.) All these proceedings lead to the same result, which will convince us of the strictly geometrical conception of the building.

After having drawn the base of the analysis, the line A—B on the plinth, we introduce the axes of the towers, lines D—E and F—G. An equilateral triangle on the distance H—I on the plinth gives us the centre C of the circle which determines the square K—L—M—N of the whole design. The vertical sides of this square are extended upwards. From the intersecting point between the axis of the church with the base K L of the square, diagonals are introduced on both sides, according to the angle 63° 26′, to points N and M, from whence they are deflected upwards. From the extreme points K and L of the base, there are introduced corresponding diagonals which are deflected in a similar manner from the extended vertical sides of the square, at points O and P. The first- and the last-mentioned diagonals intersect at points E and G on the axes of the towers. The horizontal line through these two points gives the limit of the extended sides of the square. In this way is found the rectangle K—L—Q—R within which the front is designed.

The above-mentioned diagonals K—P and L—O, also N—G and M—E, intersect the axes of the walls of the towers exactly at the height of the cornice (see the small circles introduced) just as, at the intersecting points E and G, they fix exactly the summit of the spires, where these should have been according to medieval cathedral rules (see *Dictionnaire de l'Archi-*

tecture, by Viollet-le-Duc, under "Flèche," p. 426). The shape of the pyramid of the spires is therefore determined. How firmly this shape is fixed to the design is proved by this: the inner lines of the pyramids, when extended downwards, intersect one another exactly at the central point on the base of the rectangle of the whole design, while the outer lines meet exactly the extreme points K and L of the same base. We shall see later that the same condition, in a varied form, exists in the original medieval drawing of the front of Cologne cathedral.

It will be noticed that the rectangle of the design we are studying contains two and a half squares. If we return to the plan (fig. 51), we find there a diagonal of 63° 26′ drawn from point A of the front; through B it is deflected to C on the longitudinal axis, corresponding to the end of the apse towards east, giving exactly a rectangle of two and a half squares. The total height of the church becomes therefore exactly equal to the total length.

Furthermore, we shall find, drawn from point D in the front line of the plan (fig. 51), a diagonal of 45° going across the total width of the plan and deflected, first at point E, then at point F to point G, by which the total length of the church is reached on the east to the end of the apse. A rectangle with the total external width of the front, 42 metres, as base, and moreover limited by lines through the above-mentioned points D, E, F, and G, is thus composed of three squares. If we now return to the drawing of the front (fig. 55) we shall, of course, find exactly the same condition for the raising of the front vertically, when we use half the width of the front as radius (the above-mentioned method No. 2), such as shown by the stroke lines of the auxiliary system. The drawing speaks for itself; here it will only be pointed out how the front is designed *ad quadratum* on the given base, the top of the square corresponding with the top of the parapet; similarly the parapet of the towers is situated at a height of one and a half square, the top of the spires at a height of three squares, above the plinth—in other words, the total height of the towers is divided into two equally large parts, composed each of one and a half square, the tower and the spire. By using our method *ad quadratum* we have thus proved the truth of the observation of Viollet-le-Duc, when he says that the spires in the first half of the thirteenth century had often the same height as the towers on which they were placed.* We shall see later on that this proportion is quite the rule when building medieval cathedrals.

It is equally interesting to notice how the two methods *ad quadratum* confirm each other. The extended sides of the diagonally placed square of the dotted auxiliary system intersect the diagonals N—G and M—E of the auxiliary system with fully drawn lines, exactly at the points marked by small circles mentioned above, on the cornice of the towers.

It will be noticed that the placing of the large rose window of the front seems chosen arbitrarily; its centre corresponds neither with the centre of the square of the front nor with the circle inscribed in the square of the whole design. How definitely its placing is determined by the system will be seen, however, in the next drawing (fig. 56, Plate I). It will be noticed that the medieval architect has lifted the square of his design, K—L—M—N, the red lines in fig. 56, to the top of the plinth line A—B (the black lines), and determined the placing of the rose window according to the centre produced in this way. By this raising, the top line R—Q of the rectangle of the design is lifted correspondingly; through this, the height of the ornamentation of the spires, with cross, floret, or vane, is fixed. This height above the plinth corresponds exactly to the total length of the church, the small apsidal chapel included, 129 metres in all.

We have introduced besides, in this drawing, the lines of the system with the angle of 63° 26′, in order to show that the architectural and decorative distributions are carried out

* "Elles (les flèches) finissent par devenir très aiguës, a prendre une hauteur égale souvent aux tours qui leur servent de supports" (*Dictionnaire de l'Architecture*, vol., art. "Flèche," p. 426).

ad quadratum as strictly as are the main features of the church; the distribution of the stories, the proportioning of the porches and their arches, the placing of the rose window, the position of the capitals, in short, everything is strictly determined according to the system. It will thus be seen, for instance, that the diagonals coming from the axes of the side porches and of the towers intersect one another in the centre of the rose window, and then intersect the axes of the most northern and southern buttress at the level of the top of the parapet of the towers.

In the following drawing (fig. 57, Plate II) we have put together our observations of the front, and we recommend that it should be carefully examined. The drawing speaks for itself. It will be seen that nothing is accidental or arbitrary or jolly, to use the present-day art slang.

After having in this way completely proved the construction *ad quadratum* of the cathedral of Paris, in all the plans, we shall be able to unite our observations by simply producing diagrammatically the front and the side elevation of this cathedral from its plan.

From our analysis of the longitudinal section and of the side elevation (figs. 53 and 54), it will be remembered that the distance of the axes in each bay is 5.m.5 therefore each double bay is 11 metres. In the side elevation we found the height of the wall of the outer aisle to be 11 metres and the wall of the inner aisle 22 metres; similarly we found the height of the wall of the nave from the plinth to the top of the parapet to be 33 metres.

It will also be seen that in the drawing of the front (fig. 57) we have given the measure of the height to the apex of the gable as being 44 metres.

We have thus found that there exists a certain proportion, 1×11, 2×11, and 3×11 for the side-walls and 4×11 for the gable.

We have utilised this observation on the plan (fig. 58, Plate III). In the lowest line but one, marked 1—2—3, underneath the plan, we have placed together three double bays, or 33 metres in groups. As will be seen, there are three such groups of 33 metres from the east side of the most western buttress to the semicircle of the chancel.

If this manner of dividing is projected from the plan to the elevation, the side wall of the nave must necessarily, with its given height *ad quadratum* of 33 metres, be composed of three squares.

The inner or first aisle is 22 metres high. As shown by the elevation, there are two squares of 22 metres between the tower and the transept, and two squares to the east of this when the ambulatory is included.

Between the tower and the transept there are four squares with sides equal to the height of 11 metres of the outer or second aisle and two and a half squares from the east wall of the transept to where the ambulatory begins; and in the whole length of the church there are twelve squares.

Between the east wall of the tower and the west wall of the transept there is one square, with its side equal to the height of the gable (44 metres), and one square between the east wall of the transept to the ambulatory included, and there are three squares in the whole length.

As a guidance we have marked with a thicker line and larger circles how the second aisle, the first aisle, the nave, and the ridge of the roof are raised on the square of one, two, three and four double bays respectively.

It will also be seen that the height of the spire above the crossing is determined *ad quadratum*, as it is the case with the height of the spires of the west towers.

What has been stated must be sufficient to prove that the same law which governs the side elevation governs also the front and the plan. Similar to the front, the elevation consists of a number of squares which combine in larger squares and finally in one single large square,

as we have alluded to before. In other words, the church in its entirety is designed into a large square. It begins with a square and ends with a square—the symbol of the temporal church. Unity is preserved.

But added to this, there is still another point which helps to give life to this unity—that is, the proportioning according to the sectio aurea. We shall point this out later when dealing specially with this proportion.

* * *

We have made the cathedrals of Paris and Cologne the main object of our documentation. As regards the latter, we have had quite large drawings at our disposal; but for the former, we had no others than the small ones which are found in Viollet-le-Duc's *Dictionnaire de l'Architecture.*

After having read the proofs of the previous four chapters, we succeeded in finding in the University Library the large book of plates by Lassus and Viollet-le-Duc on the cathedral of Paris.

From this work we give five plates in an appendix: the side elevation, the longitudinal section of the nave, the longitudinal section of the transept, the transverse section of the nave with the elevation of the west wall of the south transept, and, finally, the apse with the entire east elevation of the transept, all reproduced by means of photography on a somewhat reduced scale.

Fig. 59 (Plate IV) gives the whole north elevation as it is now, after the alterations in the thirteenth century, when the lean-to roofs of the aisles were removed and the height of the walls of the first aisle was lowered.

We have here introduced the analysis of the side walls, which was made earlier, and through it we have verified the truth of our observation. It is not different from what we expected; but what is of special interest and a decisive proof of the correctness of the theory which we maintain, is that the analysis of the authentic side elevation shows it to correspond with the theoretical side elevation which we gave quite diagrammatically, and is based on the several analyses of the small plan and of the comparatively small drawings of detached parts.

We find that the heights of the second and first aisles and of the walls of the nave determined by the square have the length of one, two, and three double bays respectively. We notice further that the height of the ridge is determined by the elevation of the side walls, as it proves that the level of this ridge is determined by the square having a length of four double bays or eight single bays, as we demonstrated in the diagram and observed it in the analysis of the west gable. Moreover, it will be seen that six double bays go into the height of the towers, to the top of the parapet.

There is one square for four double bays plus half a similar square, or in other words, there is one and a half square in this height. We find thus full correspondence with the west elevation; the same conformity to law which from here permeates the side elevation.

We find this elevation designed within a rectangle of two squares, having as side the height of the towers = half the length of the church (as remarked during the analysis of the front).

In the front we found the height to the ridge to be *ad quadratum* on the width of the front. We found also three squares on the width of the front to compose the length of the church. It is understood that we find again the same three squares in the side elevation of the church which is shown by the diagonal of the rectangle of the squares within which the side elevation is designed. This diagonal starts at the point marked by three small rings from the corner below on the right and upwards towards the left, to the intersecting point with the ridge, also marked by three small rings intersecting the diagonal of 45°, passing from here down to the left corner at the base of the rectangle.

We find, everywhere, therefore, in the horizontal plan, in the vertical longitudinal plan, and in the vertical transverse plan that the same geometrical unit of measure is used.

This unit is the square. In the drawing, we have introduced the auxiliary unit collectively, having placed on the sides of each of the two squares forming the rectangle of the whole design the geometrical value of the length of two bays and marked these by numbers from one to six.

The square is therefore divided into six, here as well! We find altogether twelve double bays for the whole external length of the church.

At level 1, *ad quadratum* of one double bay, we get the height of the second or outer aisle; at level 2, the top of the parapet of the Gallery of Kings in the front and the original height of the wall of the first aisle; at level 3, the height of the wall of the nave; at level 4, the projection of the main cornice of the front and the line of the ridge; and finally at level six, *ad quadratum* of six double bays or half the length of the church, the top of the parapet of the towers.

Within this large square or its squares of units, the side elevation is designed according to the same strict geometrical law which we found in the west gable wall. We have drawn some diagonals in order to prove this fact.

It will be seen from the fully drawn lines of the system in the gable wall of the transept that there is not a single accidental occurrence from the base of the building to the summit of the floret on the gable and the finials of the corners; the division of the stories, the porch, and the lights are proportioned *ad quadratum*, even the centres of the rose windows are determined in the same way. Thus they are connected—thanks to the spirit of unity within which the distribution of the totality is organised—not only with the elevation of the gable wall, but also with the totality as shown by the diagonals going through the centres. From the parapet of the towers down to the plinth and its extreme points, from east, west, and round the west front, continuity is obtained by means of this system *ad quadratum*, and a command over the large masses of this enormous building and its various developments.

It will be noticed that an error has crept into our theoretical drawing of the side elevation. This is not, however, of a theoretical, but only of a practical kind. The tower and its spire have become too low; owing to carelessness the elevation of the west front has been drawn into the plan, an error which anybody with the minimum of experience in drawing will be able to control and correct.

The elevation is otherwise exact to all intents and purposes, and it shows fully how the raising of the building is contained in the plan and developed from it by the logical use of the square in all the plans. The diagram corresponds entirely with the authentic measured drawings.

Fig. 60 (Plate V) is also a photographically reduced reproduction of the longitudinal section of the cathedral of Paris.

It is obvious that the same laws exist here. According to what has been said before, the drawing is explicit. We find the vault built perfectly *ad quadratum*, according to the principle of the division into six parts; we find that this is not only followed within the square having the length of six bays, but within the whole space of the church, because the height of the vault, in its totality, stands in systematic proportion to the whole interior of the church, as a consequence of the continuous method of the principle *ad quadratum*. This is shown by the less inclined diagonals, according to 63° 26′, starting from the top of the arch of the transept and ending respectively at the threshold of the main porch on the west, and at the plinth of the farthest wall of the church towards east.

By means of these diagonals four squares are marked under the vault, two on either side

of the central axis. In this way the central axis of the transept becomes also the central axis of the entire church.

It is equally worth noticing that the diagonal of the half square, according to 63° 26′ from the east side of the towers, after having passed the centre of the rose, corresponds at the top of the vault with the diagonal of 45° from the apse of the central aisle. Thus we obtain three squares.

We find further that the diagonal of the half-square, from the plinth under the column of the ambulatory, corresponds at the centre of the rose with the diagonal, according to the same angle coming from the east side of the towers.

Furthermore, the centre of the rose in the west front, which we noticed was determined

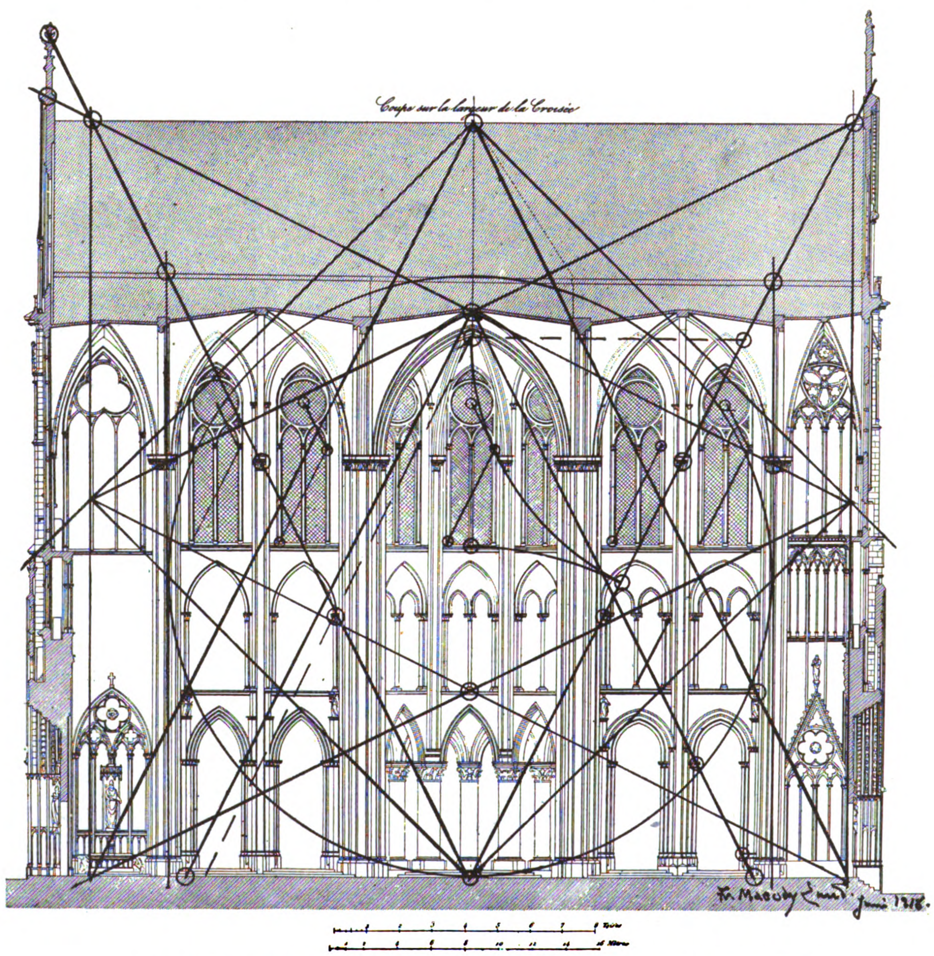


Fig. 61.—Longitudinal section of transept, after Viollet-le-Duc and Lassus, analysis.

by the exterior conditions of the front, stands in a certain connection to the principal features of the interior and to the totality, inside and outside, as shown by the diagonals fully drawn. But this relationship which we have thus found existing between all the principal features, between the separate parts of the building and the totality, is the necessary outcome of a law-bound concordance between the interior and the exterior. The esthetic requirements demanding that a building should give externally a genuine expression of the interior is shown to be realised to perfection in the cathedral of Paris.

Finally, we ask the reader to notice the measurements given on the left underneath the nave. This is the length of the church according to Viollet-le-Duc's drawing of the original plan in his *Dictionnaire*. One hundred and twenty metres give the extension of the second aisle towards the east, while one hundred and twenty metres mean the extension for the small apsidal chapel. It will be seen that the diagonal of 63° 26′ from this point obviously intersects the end of the ridge of the roof, but otherwise stands in no relation with other principal points inside and outside the church itself, such as is the case with the extreme west points and also as concerns the present extreme east points of the church. The diagonal of 63° 26′ from here passed the cornice at the original height of the wall of the first aisle, just as it still passes the cornice of the wall of the nave. It will be seen that all the diagonals from the present east side correspond with the extreme west points of the church as well as with the towers. This fact happening in this regular church is a decisive proof that it had originally the same extension towards east as to-day.

The apses indicated by Viollet-le-Duc are much too small to have been of any use. With the extension given to them in our analysis they are rational and practical.

In fig. 61 we see a longitudinal section of the transept—also photographically reduced, from the same book of plates. According to the two plans, the original and the present one, in Viollet-le-Duc's *Dictionnaire* under the word "Cathédrale," the transept has been somewhat extended in the direction north-south during the thirteenth century. There are many indications of this, not only in the different styles, but in the constructive peculiarities which at once strike the eye. That such an alteration has really taken place is clearly seen by the lines introduced in the section. On account of the raising of the floor of the church towards east the line over the plinth of the nave reaches here, in fig. 61, to the level of the floor; it is therefore clear that, as regards the totality, the base of our geometrical analysis must coincide with the floor line. From the circle inscribed it will be seen that the height of the wall of the nave is determined *ad quadratum* on the distance between the axes of the original outer walls of the second aisle—or, if preferred, of the two outer aisles. According to the entire systematic raising of the

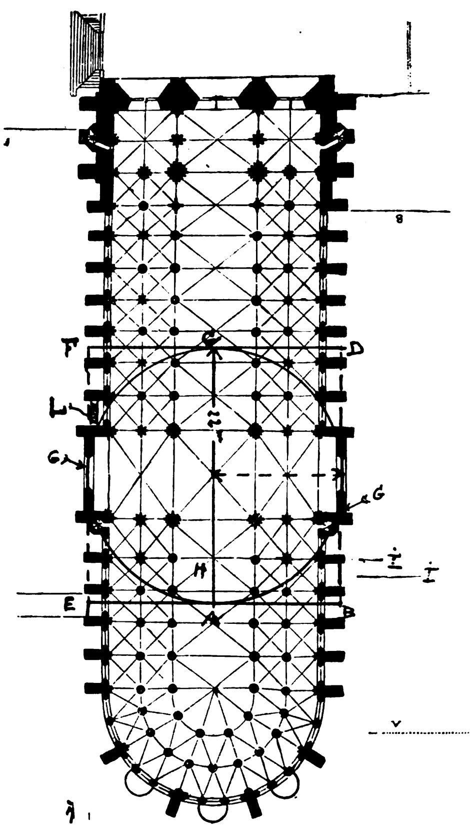


Fig. 62.—Plan of the cathedral, analysis.

church, the length of the transept has been determined *ad quadratum* on the height to the ridge or vice versa; the transept obtains the length of four double bays in the direction north-south and the ridge is determined accordingly. The sides of the square of four double bays must therefore have been situated on the axes of the original gable walls. It will be seen how the introduction of the diagonals of the half-square within the square at the height of the ridge gives just the height of the vault, the division of the stories, the position of the capitals and other principal points of division, as it will be seen that the diagonals of the larger square at the height of the ridge correspond with the diagonals of the smaller square on the distance between the axes of the walls of the outer aisles.

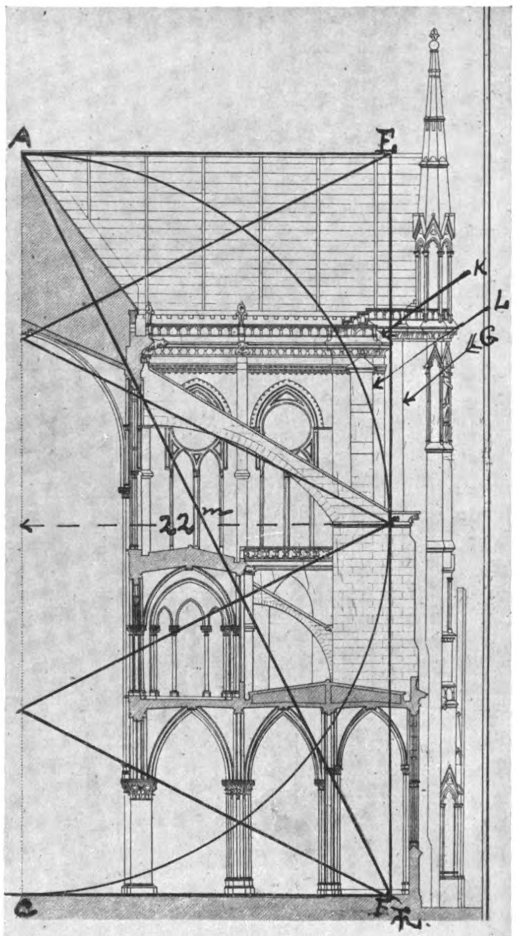


Fig. 63.—Section of the nave with elevation of the west front of the transept, after Viollet-le-Duc and Lassus, analysis.

We had no time to read the text in the book of plates by Lassus and Viollet-le-Duc, and therefore we do not know the explanation which it contains concerning the said extension. It is, however, a satisfaction and a confirmation of our theory to be able to note that the distance between the axes of the gable walls of the transept, in Viollet-de-Duc's ground-plan of the original cathedral, is 44 metres, or exactly as here, according to the system, 4×11 metres $= 4 \times 2$ double bays.

We have demonstrated this fact in fig. 62, having drawn in the ground-plan a circle with a radius of 22 metres or the length of two double bays. It will be seen that the sides E—F and B—D of the square outside the circle coincide with the axes of the gable walls. As already remarked, there still exists externally on the transept, marks of its original length. We see in the plan that the sides of the square E—F and B—D both pass somewhat outside the small towers with winding stairs on the east wall of the transept. Besides being a means of communication, these turrets have helped to strengthen the corner construction of the transept and served as buttresses to support its vault.

A corresponding buttress construction is still found on the west elevation of the transept. In the side elevation (fig. 59, Plate I) this can be clearly seen next to the transept, as a strengthening of the wall, from which a buttress is projecting, carried by an arch.

In the ground-plan we have indicated a corresponding construction on the south side of the nave and marked the buttress itself by the letter L.

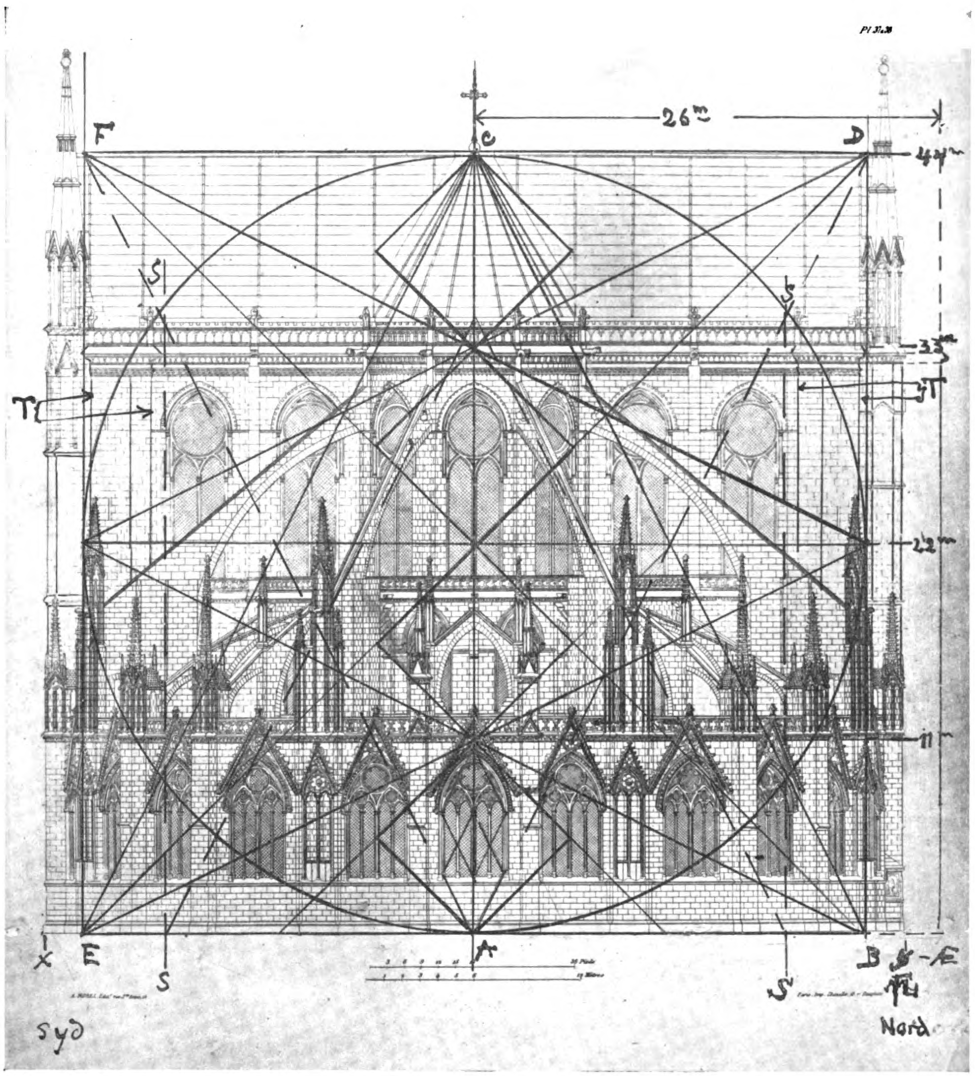


Fig. 64.—East elevation after Viollet-le-Duc and Lassus, analysis.

We find again this buttress construction in the elevation (fig. 63), taken from the book of plates, and giving a section through the present nave with the west elevation of the south transept. The section is on the line C—F in the plan. From the latter we transfer the circle having radius = 22 metres or two double bays, as well as half the square C—F—E—A.

The separate parts of the buttress construction are clearly seen. The strengthening of the wall is marked by K and the buttress by L.

Similar to what we saw in the plan, that the sides of the square fall just outside the stair turret on the east wall and coincide with the axis of the original gable wall, we find here also, in the west elevation of the transept, that the side E—F of the half-square falls just outside the buttress construction at the original corner, before the gable was moved outwards in 1257.

In other words: by means of the geometrical system of the cathedral of Paris, we have been able to mark clearly the division between the original parts and those added later.

Finally, we see in fig. 64 how the original east elevation of the transept falls within the square of four double bays. The stair turrets which strengthen the corners are marked by letter T.* We see moreover, how the two axes marked S in the outer walls of the second or outer aisle of the original church, fall just within the stair turrets in the corners of the transept, the original extension of which our analysis has proved to correspond with the demonstration of Viollet-le-Duc in his *Dictionnaire*.

We can therefore take it for granted that just as the theoretical diagram agreed with the original cathedral, so will our theory concerning the original extension of the transept give exact information in this respect. The conformity with Viollet-le-Duc is already a proof of this.

From our general survey it will be remembered how the system lines, introduced in the plan of the cathedral of Metz (fig. 19), make at once a distinction between what belongs to one design and what is added.

In the cathedral of Paris we make now the same experience. Just as the zoologist bases the various animal forms on the intellectual principles of science and can distinguish between various bones of fossils and be able to tell to what species they belong, so we can, by means of the intellectual principles on which sacred architecture is based, treat it in the same scientific manner and thus be able to distinguish the original from what is added. In other words, we have an exact method enabling us to form a distinct opinion regarding a building, architecturally and chronologically, whether it was really built by a master or by an imitator incapable of original thinking.

The kind of talk used in young ladies' high-schools, which tries to turn architecture into a matter of feelings, in other words, into the same mixture of sentimentality and stupidity peculiar to the art talk of our modern art historians, is here sent back to its proper place, to darkness and chaos.

The fact already that the diagram given above—being the theoretical result of our several separate analyses and isolated observations quite independently constructed from the small plan only—that this diagram should coincide with the authentic measured drawing of the side elevation, is already a sufficient proof of the correctness of the theory maintained by us. This proof becomes decisive when we remember further that our discoveries from the cathedral of Nidaros have found their full verification in the special analysis of the cathedral of Paris. By using these discoveries on other sacred buildings we have been able to prove a relationship irrespective of time and place.

These circumstances and these facts ought to render any other proofs superfluous to a

* The draughtsman has made the mistake of drawing the north transept equal to the south transept, while in reality the former extends to Æ, that is, X—A = A—Y instead of A—Y + Y—Æ.

Fig. 65.—West front, from a photograph.

thinking reader, were it not in the interest of our official mission to state from an historical point of view, not only the universal use of the rules for religious buildings, but also to show how, by a superior adaptation, they have been able to create and vary the most different architectural forms within the rules, or, more correctly, within the strict law of the rule.

We shall obtain proofs in the following chapter, when considering the cathedral of Cologne, that to achieve this, besides a thorough education, the architect must have intelligence and penetration, or, in other words, the imagination of a thinking man—in a far higher degree, or perhaps the reverse, of what is required in the modern gospel of "free art."

In the following chapters we shall satisfy ourselves that this talk about "free art," as concerns architecture, lacks every justification in history and in the philosophy of art; but now already, after the analysis of the cathedral of Paris, when looking at the perspective of its west elevation (fig. 65), we can satisfy ourselves that feeling is an uncertain guide when it concerns an architectural design. We have seen that the elevation up to the gallery over the open arcade in front of the triangle of the gable makes a square on the width of the front. To this is added part of the towers above this gallery with half a square.

Judging from one's feelings, one would believe that a square front seen in perspective would appear foreshortened in height, that is to say, it would seem considerably less in height than in width. As shown by the picture, the opposite is the case to such an extent that, had we not known it, we would have been convinced that the elevation of the cathedral of Paris up to the said gallery was considerably greater in height than in width.

Such is the difference of impression between the drawing on paper and the finished building in full perspective, when light and shadow are added to it.

It is a recognised psychological fact that a square is generally made too low when it is drawn purely from judgment. How much more deceptively the judgment must act when large buildings and big masses have to be realised according to drawings, carried out on paper without any scientific principle, and where we are not sure, moreover, of not being deceived by that very freedom, like certain ultra-modern building artists in Germany, in our time, with their uncritical and talentless as well as scantily learned imitators in Scandinavia, disporting themselves in "free creative exuberance," believing themselves inspired by their personal "feelings," and their "original imagination"!

CHAPTER V

THE CATHEDRAL OF COLOGNE

THE PLAN

ACCORDING to its plan and its elevation, this church belongs to the French type of cathedrals. On account of the fact that the end of its chancel has a plan which resembles the end of the chancels of the cathedrals of Beauvais and of Amiens, and having, besides, similarities with the latter as regards the plan of the chancel itself, it has been claimed that the cathedral of Cologne is an imitation of these. Even a real authority like Viollet-le-Duc, who, in consequence of his great intelligence and knowledge, never needed the commonplace phrases of our art historians, and who always had logical reasons for his statements, says as follows: that the plan of the apsidal chapels of Cologne cathedral seems traced over that of Beauvais. * This is quite wrong.

We shall prove that the cathedral of Cologne is a fully independent conception, which, at most, borrowed the idea or perhaps only the suggestion from the above-mentioned cathedrals, just as in its style it represents a further development of the style of those French churches not so very much older.

The cathedral of Cologne was planned in 1248, about ninety years after Notre-Dame of Paris, nearly thirty years after the chancel of the cathedral of Bourges was finished, twenty-eight years after the cathedral of Amiens, and twenty-three years after that of Beauvais.

For the sake of general documentation and of comparison it will be necessary to undertake an analysis of the plans of the two related French cathedrals; fig. 66 gives the plan of Amiens.

Like the cathedral of Reims, the cathedral of Amiens has a five-aisled chancel and a three-aisled nave, but they are both nevertheless planned perfectly regularly *ad quadratum*, just as they are perfectly according to the type which was developed in the 11th century, having the central tower as the centre of the church. The length of the transept is equal to half the length of the building; it is, therefore, in its entirety, designed within two large squares, one on either side of the transverse axis, through the central tower, which is shown by the diagonals from the transept to the west front, and by the two circles.

Considered as a three-aisled church, its length, with ambulatory included, equals four times the width of the front; as a five-aisled church, its total length equals three times the external width of the chancel, which is shown by the diagonals of 45°, D—E—F—G. Furthermore, we find the placing of the columns to be also according to the rule, because there are two double bays in the square in the three-aisled nave, and three double bays in the five-aisled chancel, as shown by the fully drawn circle in the nave, by the semicircle drawn with strokes in the chancel, and by the numbers written on the diameters of the circles.

Moreover it is of interest to mention that the height of the vault is also determined *ad*

* "Le plan des chapelles absidales de Cologne semble calqué sur celui de Beauvais" (*Dictionnaire de l'Architecture*. art. "Cathédrale," page 337).

quadratum. The inner width of the chancel is 50 metres and of the nave 32 metres. Therefore, if the church had been wholly built five-aisled or three-aisled, the height should have been either 50 or 32 metres. In order to avoid conflict between the height of the vault of the chancel and that of the nave, the architect has added the two widths, that is to say, $50 + 32 = 82$, and these determined the vaulting of the entire church *ad quadratum* from the average value $82 \div 2 = 41$ metres, which is exactly the height to the keystone, reckoned from the top of the plinth.

The height of the vault of the cathedral of Reims is determined in the same manner. The five-aisled chancel is there 44 metres, and the three-aisled nave 50 metres wide. The height of the vaulting of the entire church is then determined *ad quadratum* on the calculated average of 37 metres.*

A better proof of conscious scientific knowledge in the method of working of the medieval architect cannot be asked!

The general examination made this clear, the special analysis of the cathedral of Paris has illustrated it, and the cathedrals of Reims and Amiens underline this fact.

An analysis of the constructions of the cathedrals of Amiens, Beauvais, and Cologne will show how this scientific knowledge can lead to similarities and create similarities in variations—in other words, how it may allow the intelligent architect to work independently within one and the same thought. We take Amiens first, as the oldest within this order of thoughts.

In the design of this cathedral the architect has used the pentagram in accordance with the very ancient manner of constructing, which we have called above in an historical way "Hippocratic." We take a chosen width of the nave as radius, with this we draw three circles (see figs. 42 and 43), and by means of this again we find the radius of the circle in which the pentagon appears. With the help of this circle and of the circle inscribed in the pentagon—the last circle being divided into twelve parts—the architect has found the radii and the extension of the radiating pillars between which the chapels are determined. By means of a circle, the radius of which is the side of a pentagon, he has finally found the projection of the chapels and the interior width of the chancel. We must also notice that the square of the treble nave is divided into four *equally* large parts, while the square of the five-aisled chancel is divided into *unequal* parts—the first aisle being somewhat broader. This width is not arbitrary, but determined by the circle inscribed in the pentagon.

The construction of the plan of the chancel of Beauvais (fig. 67 †) is also founded on the "Hippocratic" formula of the pentagon; the chosen width of the nave is here also the radius of the three circles of the formula. The largest concentric circle in which the pentagon is found corresponds with the semicircular platform on which rest the apsidal chapels with their buttresses.

The second concentric circle, by means of which the projection of the chapels is determined, has as radius the side of the pentagon in the third concentric circle, which is inscribed in the pentagon inscribed in the largest concentric circle.

Finally, the radius of the fourth concentric but only half-drawn circle is the side of the decagon in the largest concentric circle. By means of this radius the ambulatory and its width are determined. In the cathedral of Beauvais also, as regards the placing of the columns, the square is divided into six unequal parts, as in Amiens.

* "La nef de la cathédrale de Reims a 37 mètres sous clef" (Viollet-le-Duc, see under "Echelle," p. 147). "La nef de la cathédrale d'Amiens a 42 mètres sous clef" (ibid., see under "Travée" and "Cathédrale"). The latter is given 42.5 m., but this is to the keystone of the diagonal arches, and not of the transverse arches. Viollet-le-Duc has measured from the floor.

† The black portions were built in the thirteenth century, those in hatchings in the fourteenth, while the nave is the supposition of Viollet-le-Duc, which our analysis shows to be right. Another of the many proofs of his knowledge as regards medieval cathedral architecture.

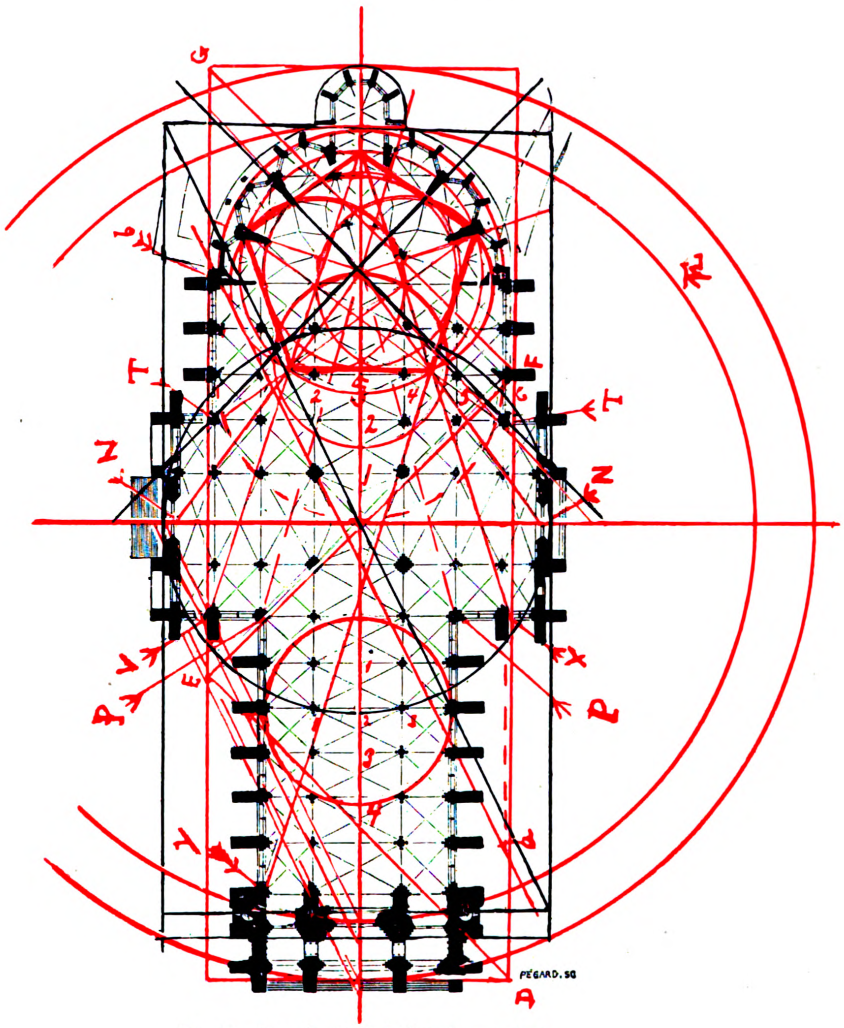


Fig. 66.—Plan of cathedral of Amiens, analysis.

This digression was necessary to understand the cathedral of Cologne and to determine its place in the development.

Fig. 68*a* gives the plan of the cathedral of Cologne. A glance at the extremity of the chancel shows at once a startling likeness with the two French cathedrals of Amiens and of Beauvais. Similar to the latter, the seven chapels of the apse fall within the same semicircular lines; but the similarity does not extend further with either of them. It may well be that

the idea of the construction of the chancel is taken from Amiens or Beauvais—and if it were the case, that ancient like modern architecture had been carried out without any leading principles, freely, according to taste, we would have considered that a direct imitation was here beyond the shadow of a doubt; the architect of the Middle Ages, however, worked in a different manner. By using geometry as his form-creating hegemonikon, he obtained through its self-creating activity the very entelechie of nature—an inexhaustible source of variations realising themselves; but this geometrical art had to be used by an imaginative thinker.

It goes without saying that this scientific imagination is quickened by impulses; but the influence does not go further in the superior and typical works of architecture of the Middle Ages—just as in every other independent superior production.

The construction of the chancel in Cologne cathedral is an illustration of how two productions similar to all appearances may be nevertheless the result of a fully independent method of working. The construction of the apses round the chancel, in Cologne, is also founded on the same ancient geometrical construction, the one we called above the "Hippocratic formula." But here it is used in a much more refined form from a geometrical and technically constructive point of view.

Instead of starting with the width of the nave, the German architect took as radii of the three circles the length of the furthest intercolumniation, or, what is the same in this case, the distance between the axes of the towers which he imagined to be on the west front. By means of these three circles and the distance between their intersecting points $h-g$ and $h-n$ (marked by an arrow on the right over the drawing), he has found the radii of the concentric circles marked III and II, according to the lines of which the ambulatory and the projection of the apses are determined.

By means of the lines from the intersecting points m and l through the centre h, and from f and g through h, or, if preferred, from k through g, he has succeeded in dividing the concentric circles in twelve equally large sections, of which seven fall on the apses. By using the side of a heptagon inscribed in circle II, he has found the radius of the red semicircle between circles II and III, through which the main shape of the apses has been determined.

The largest concentric circle is traced through the intersecting points m and l and the pentagon inscribed is found. Two of its sides are prolonged until they intersect each other and this determines the placing of the main porch.

It will be seen furthermore that the diameters of 45°, m—T and l—B, divide the largest concentric circles in four parts; in this way a square is inscribed which, in the drawing, is indicated by three of its sides, m—B—T—l. The sides B—m and T—l are lengthened towards west. It will be seen that these lines touch the circle of the "formula" with k as centre, and the red circle, the radius of which is the length of the farthest intercolumniation. These prolonged sides of the square get their length determined at points (A) and A, by being intersected by the lengthened diagonals of the dodecagon of the chancel. The rectangle of the design A—(A)—(B) is found in this way. As shown by the diagonal of 63° 26′ A—B and the diagonal of 45° B—C, it contains two and a half squares—similar to the rectangle of the design for Notre-Dame of Paris.

It will be seen also that the diagonal B—C touches the concentric circle II, the diameter of which gives the inner width of the church; three squares on this give the total inner length of the church from the zenith of the red semicircle to the inside of the west gable wall.

By making the distance between the axes of the towers the foundation of the construction of the chancel, the architect has been able to divide the square on the length of the diameter T—B into eight equal parts, of which six fall on the church and determine its width. In this way the distance between the axes transversally has become equally large, in opposition to

Amiens and Beauvais. The execution is also made easier, because the principle of equilibrium, which is the spirit of Gothic construction, can be realised unhindered, on account of this regularity.

The diagonal of the pentagon inscribed in the largest circle gives the side of the two large squares which form the rectangle into which the entire church is designed. The long sides of the rectangle of the entire design go through the angles β and γ of the pentagon.

The radii or, if preferred, the diagonals of the dodecagon of the chancel, which form 45° with the longitudinal axis of the design, determine the position of the large square, as, for instance, the diagonal through the point B and centre *h* which intersects the side of the rectangle of the design through the angle of the pentagon at point N. The side of the square towards west and the transverse axis of the design are found in this manner.

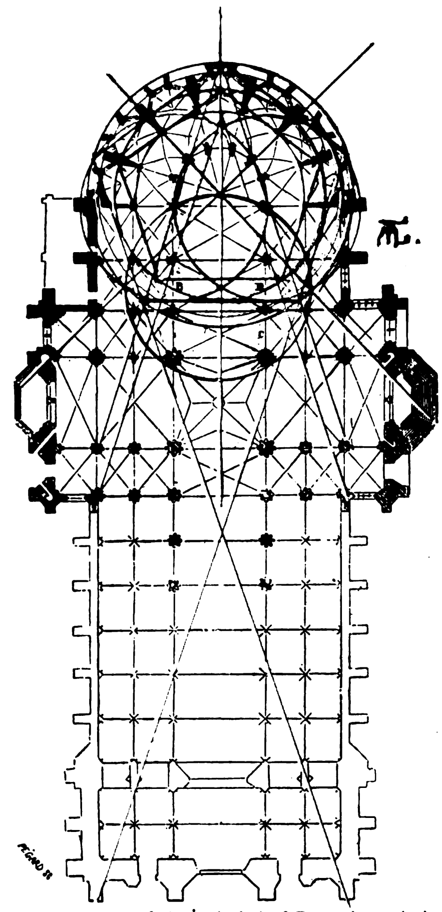


Fig. 67.—Plan of the cathedral of Beauvais, analysis.

The frame was thus finally formed, that is to say, the main frame, the rectangle of the two obligatory large squares within which this cruciform type of cathedral is designed—such as we have found it forcing its way in the eleventh century and becoming still richer and more developed in the thirteenth. For the sake of the analysis we have begun our construction from the interior, but it could of course have been made the reverse way. A careful reader will have noticed that the distance between the axes of the towers which we called the chosen radius ($f-g$) in the foundation of the "Hippocratic formula," or two double bays according to the proportion of the sectio aurea, is the minor of the diagonal of the large pentagon. In this way we can begin the construction by first drawing the large circle, and in this we inscribe the pentagon with its diagonals, by which points *f* and *g* are produced, and so on. Or the construction may be started with the large square, according to how any one well trained in designing cathedrals may find it convenient. In Cologne it seems as if the construction has been started from the large circle.

If we now compare Cologne cathedral to the one of Amiens, we see that they both lie within two large squares. The disposition within this is not arbitrary in either of them, but, helped by the intelligent imagination of the artist, they are produced by geometry, the entelechie of nature.

In Amiens the chancel is determined by the pentagon and the three circles, that is, the circle inscribed in the pentagon, the circle circumscribing the pentagon, and, lastly, the circle with radius = the side of the pentagon, by which the projection of the apses is determined.

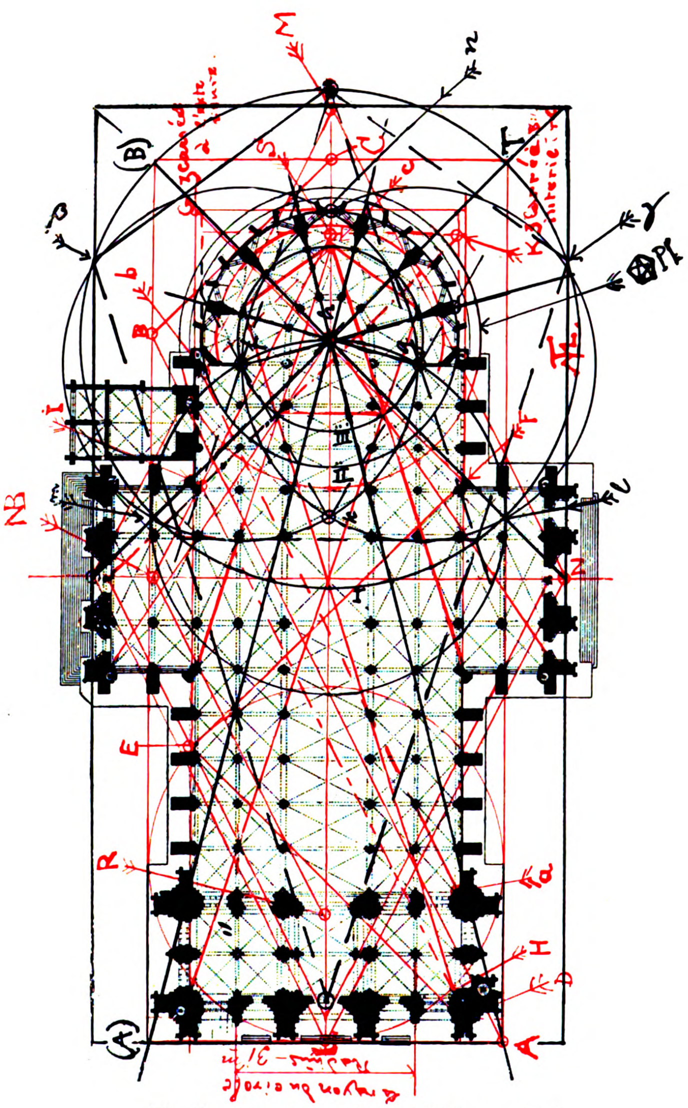


Fig. 68*a*.—Plan of the cathedral of Cologne, analysis.

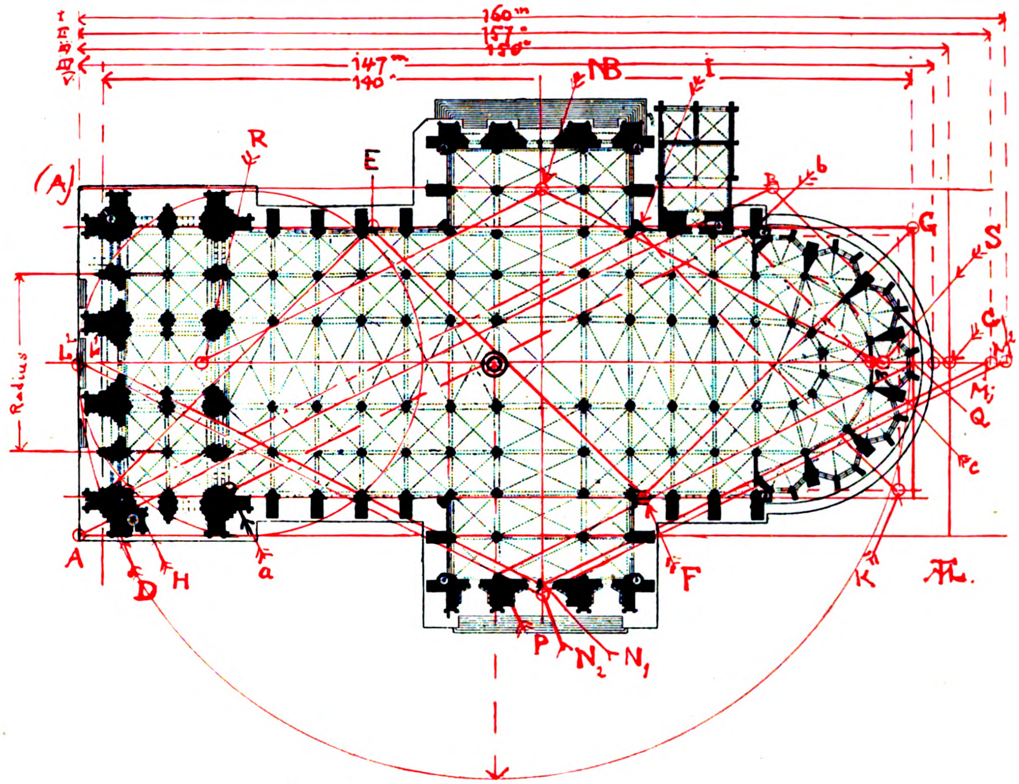


Fig. 68*b*.—Plan of the cathedral of Cologne, analysis.

The cathedral of Amiens is constructed *ad quadratum* and *ad pentagonum*, as shown by the black diagonals of the squares and the red ones of the pentagon.

In Beauvais the construction is based exclusively on the pentagon. Neither in Amiens nor in Beauvais are the seven parts of the circle equally large.

In Cologne we find the foundation to be the same very ancient Greek construction; but the architect has, at the same time, or rather within this, developed the construction *ad quadratum*. Its centre—*h*—is situated in the centre of the large east square, and in this way he has been able to develop a regular dodecagon inscribed in two concentric circles, the radii of which he found in the distance between the intersecting points of the three circles of the "hippocratic formula." The pentagon is used only for the semicircle which forms the platform of the construction of the chancel; the radius of this semicircle (marked by a small pentagon and the letters Pl) is the side of the pentagon inscribed in circle II, but in the very construction of the chancel the pentagon is not used. On the other hand, it is used in the conception of the entire church, but in a different and more important manner, as the construction of the chancel and the whole church *ad quadratum* seem to have sprung from the pentagon in the large circle.

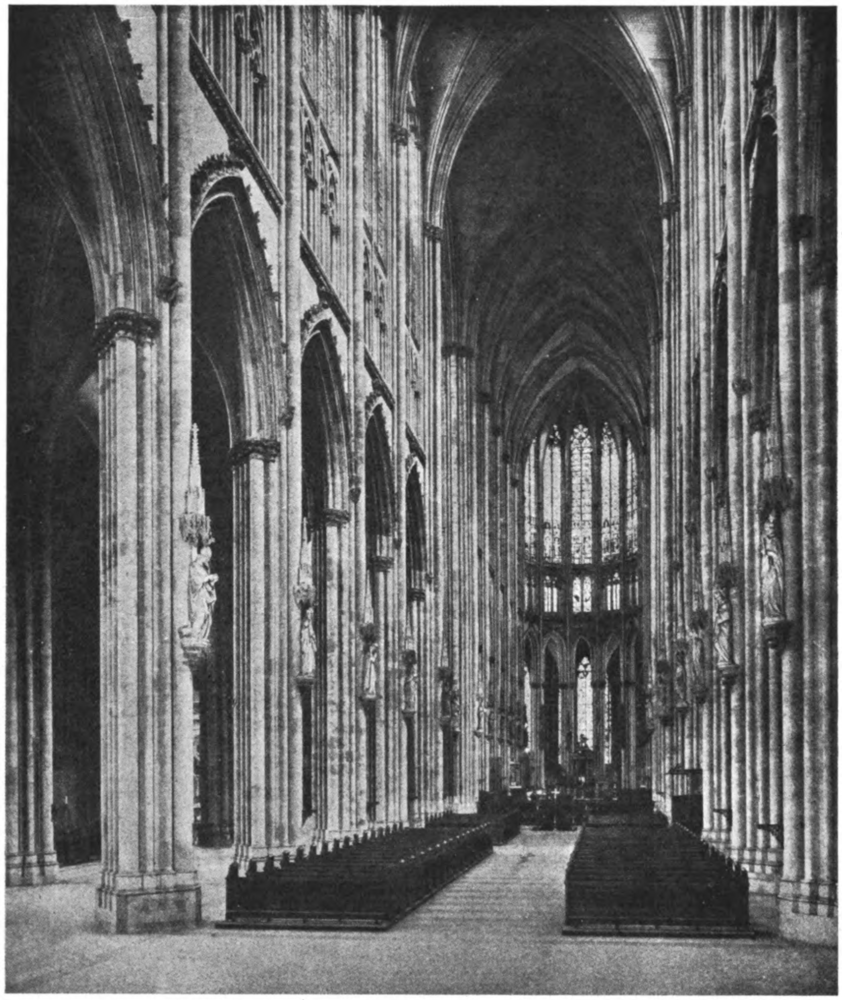


Fig. 69.—Interior of the cathedral of Cologne.

In all three cathedrals—although to a less degree in Beauvais—the polygonal construction of the chancel is related to the entire plan of the church. As regards Amiens, this is shown by the letters T, N—the porches of the transept V, P, and Y.

In Cologne we see the dodecagon corresponding with the principal points, for instance, with N—the porch of the south transept—and with the points (A) A, the corners on the front; similarly we see that the two lengthened sides of the pentagon in the circle which circumscribes the square on the width of the front correspond with the main porch of the west

front, and the diagonals from the top of the pentagon correspond with points I and F, where the chancel and the transept meet, while the corresponding diagonals of the pentagon inscribed in circle III correspond with the corners between the nave and the transept.

In Cologne the correspondence is more lively. The dodecagon and the pentagon correspond in the most varied manner, not only with the building, but also with the vital points of the auxiliary construction—as a result of the fact that everything is based on a constructive single idea, the square, for the construction of the polygon and for the church.

All three constructions do much honour to their designers. Especially noticeable is, therefore, the interesting manner in which in Cologne a dodecagon and a pentagon are constructed together, at the same time dividing a right angle into three, if the construction of the arch of 30° can be considered as offering such an example. As far as we know, these constructions have not been noted down in the history of mathematics. They are all flattering proofs of the geometrical manner of thinking in the Middle Ages.

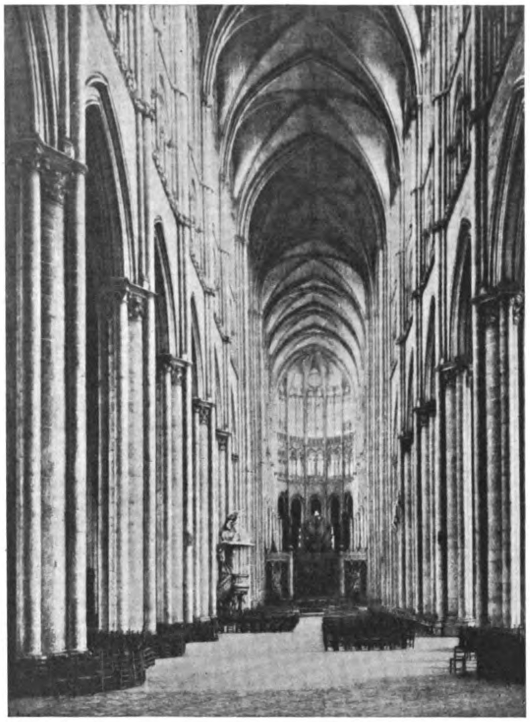


Fig. 70.—Interior of the cathedral of Amiens.

We see that in all three cathedrals the design has its starting-point at the place of the altar; that the proportioning of the building has begun here, with the altar as a centre, in the development of the idea of God's House. This is the case more or less in all medieval churches having an original construction, but we cannot stop to demonstrate this.

In Amiens and in Cologne the main altar is placed as focus—such as it was in the temples and the homes of Hellas—from where the diagonals of the polygon (several in Cologne) radiate to the principal and vital points of the building, not only the constructive but the ritual ones—as, for instance, where the corners of nave and chancel meet and meet also the porches. Later on we shall give proofs that this connection is neither an accident nor any geometrical jugglery, or some foolish invention of an immature mind, but that it is a spiritua symbolism, and like the rule *ad quadratum* an inheritance from the deep consciousness of life of antique philosophy. It is derived from a reasoned wondering and a religious reverence for the form-creating activity of nature itself, its entelecheia which for antiquity and for the Middle Ages, ignorant of chemistry, revealed itself through geometry.

However much Cologne cathedral might have been inspired from the cathedrals of Amiens and of Beauvais, it will be understood from our analysis that under no circumstances is it "traced" on either nor is it any slavish copy. They are all three tied to their special auxiliary constructions, developed from the same way of thinking, each being the result of

an independent conception under the command of strict geometrical reasoning, guided and controlled by the harmonising principle *ad quadratum*.

On account of the analysis which will be made later, of the medieval design of the front, we shall demonstrate further how the plan of this cathedral is determined *ad quadratum*. In fig 68*b* * the plan is undisturbed by the geometrical analysis made specially for the construction of the chancel.

We have already satisfied ourselves that the church, as a whole, is designed within the two large squares which are marked here by the diagonal L^2—N^2—M^2. We find furthermore by the diagonals L^1—P—Q that two squares on the inner length of the transept give the length from the outer wall line of the west front to the apse farthest east included. In relation to these extreme points, the line through the point marked by two black rings, which we may call the west wall of the central tower, is situated in the middle of the building. Moreover, the diagonals H—I—K show that the interior of the church, as regards the nave, has two squares, and as regards the chancel one square on the inner width. The diagonal *a*—*b* shows two squares from the east side of the towers to the beginning of the ambulatory and half a square for the ambulatory, a proportion quite similar to the rectangle A—B—C, according to the proportion of two and a half times the width of the front. Finally, the diagonals R—NB—S show that on either side of the transverse axis of the church itself, through the centre of the central tower, a square is situated on the width of the front, as in Notre-Dame (see fig. 51, where the corresponding diagonals are marked by the same letters).

We have thus found that everywhere the very ancient rules, first seen in the temple of Solomon, are in Cologne also the base of the proportioning—that is: the two squares.

We shall point out, furthermore, that the length from point L at the main western porch to point C = three sides of the pentagon inscribed in the largest concentric circle (fig. 68*a*), and that, as already remarked, two double bays are equal to the minor of the diagonal of this pentagon, the same diagonal which gave the side of one of the large squares into which the whole is designed. There exists therefore an inner geometrical continuity everywhere. We will explain the secret of this continuity when we treat of classic geometry and of the temple architecture based upon it.

It must be remembered from the analysis of the cathedral of Notre-Dame that as regards the plan and the front elevation, this church is designed within a rectangle containing two and a half squares or twice the distance between the axes of the towers. The length of this rectangle corresponds to the distance of three times the outer width of the front. This distance is also the length of the cathedral (see fig. 51, diagonals A—B—C and D—E—F—G).

As a result of this correspondence, a perfect harmony was produced in the elevation of the front and in the whole side elevation.

The cathedral of Cologne shows an attempt to create something entirely new, or more correctly, a variant within the old rules. This was not entirely successful.

We have seen how the place of the main altar, through new constructions, was pushed towards the west by being placed in the centre of the large east square of the design. If the cathedral of Cologne had been a copy of the cathedral of Amiens, the position of the main altar would have been at the ambulatory, and the chancel would have extended to point M instead of to point *n*. It would have then been perfectly in accordance with the large cruciform churches within the two large squares with the central tower in the middle, such

* The plan is taken from *Handbuch der Architektur*. Unfortunately it has been a little reduced in the reproduction. The scale of 1/1,000 mm. = 1 metre. The dimensions given are determined by the distance between the axes of the towers on the medieval drawing of the front reproduced in the same work; the distance is here 30 metres. It was only on revising the MS. before sending it to the printers that we discovered that the distance in the plan used was 31 metres. As regards the essential geometrical examination and its results, this difference is absolutely of no importance, therefore we made no alteration.

as they were developed in the eleventh century and of which we saw lately an example in Amiens.

On the other hand, if the cathedral of Cologne had been an imitation of Notre-Dame, it should have had its extreme east point at C and the width of the front would have been one-third of the length to this point. The rectangle of the design of the plan would have become in this way as long as the rectangle of the design of the front. Such an arbitrary imitation, however, is not feasible with the strict method of working used in the Middle Ages.

Through his innovation, the architect of Cologne has raised difficulties and produced a disharmony very detrimental to the whole church, which in the interior fully equals those of Amiens and of Beauvais.

In the analysis of the front we will indicate this disharmony and limit our present demonstration to the plan.

We have satisfied ourselves that this plan is designed perfectly *ad quadratum*. We shall find this principle carried out in an equally logical manner in the building of the church vertically.

Similar to the cathedral of Notre-Dame and other French cathedrals of the thirteenth century, the cathedral of Cologne rises *ad quadratum* in all the plans, transversally and longitudinally, the vault and the side-walls, from the plinth to the ridge, even to the finials on the summit of the spires. Each member is determined *ad quadratum*, by the diagonal of the half-square, according to 63° 26′, and consequently according to the diagonal of 45° of the whole square.

Transverse Section

Fig. 71 (Plate VI) shows the transverse section of Cologne cathedral.* We begin by drawing a horizontal line on the external plinth $A^1—B^1$, which corresponds internally with the top of the plinth of the aisles and the lower part of the plinth under the pillars of the nave. Then we draw the central axis of the church through point F^1 and the axes of all the walls. The distance $C^1—F^1$ is then half the width of the church, and forms, as we shall see, the base of half a square.

We draw now in red the theoretical auxiliary construction in accordance with figs. 39 and 40, by tracing a circle with a radius = half the width ($C^1—F^1$) of the church through points *a* and *b* on the plinth line ($a—F^1=F^1—b$ = one-half of $C^1—F^1$); the centre *c* is found by means of the equilateral triangle *a*, *b*, *c*. The regularity of the building, according to the principle demonstrated above, for the five-aisled cathedral, is revealed by the fact that the axes of the walls—which have already been mentioned in the analysis of the plan—divide exactly half the square of the church into three equally large parts, therefore the whole square on the entire width ($2 \times C^1—F^1$) of the church into six equally large parts, of which two fall on the nave and one on each of the four aisles.

If we lengthen the plinth line outwards from C^1 of one-third of the radius of the circle, or one-sixth of the side of the whole square, to point X^1, a vertical line through this point will form the outer limit of the front, the distance $X^1—F$ being half the basis of the square in which the total width of the front has been designed, equal to the distance between the intermediate pillars of the north and south aisles, or the distance between the axes of the towers—in other words, the same distance which in the plan (fig. 68*b*) is marked by the word "radius."

In accordance with our theory developed earlier, we make, moreover, this half side $X—F^1$ of the square the radius of a new circle through the intersecting points of the plinth line

* The drawing is borrowed from *Handbuch der Architektur*, ii, 4,c (2nd edition), p. 279.

with the axes of the intermediate pillars of the aisles (D^1 and the corresponding point on the right, which here, in the fractional drawing, is left out); the centre C^{111} is found by the equilateral triangle having as basis the distance between the two intersecting points mentioned.

Of these two circles of the theoretical auxiliary construction, the smaller one with centre c is inscribed in the *square of the church*, the larger one with centre C^{111} is inscribed in *the square of the front*. Half the side of the square of the church is divided at point a—on the line of the plinth in the middle of the first aisle—in two equal parts, as is the case for half the side of the square of the front at point D^1. This point D^1, moreover, is situated in the central axis of the north side tower.

The halves of the two squares are drawn by means of the tangents F^2—C^2 C^2—M^2, and M^2—H^2 of the smaller circle, and F^2—A^{11}, A^{11}—K^2, and K^2—L^2 of the larger circle. The side C^2—M^2 of the smaller half-square is situated in the axis of the outer wall of the aisle, while the side A^{11}—K^2 of the larger half-square is the limit of the front. The diagonals of the two squares and the diameters are drawn in red.

We observe now that the diagonal (of 45°) of the smaller square, runs from the corner C^2, into the middle of the capital of the pillar of the nave, while the line drawn likewise at 45° from nadir F^2 runs into the middle of the capital of the pillar between the aisles, in order to meet the diameter from centre c just above the top of the wall of the aisle, or the lower story of the church, and is deflected to zenith H^2 just under the keystone in the transverse arch over the nave.

We see that the diagonal (of 63° 26′) of the smaller half-square, from corner C^2 to zenith H^2, intersects the diagonal of 45° just mentioned from nadir F^2, exactly in the middle of the capital of the pillar between the aisles; it passes in the middle of the floor of the clerestory over the first aisle and stops under the summit of the transverse arch of the nave.

We turn now to the larger square. The diameter C^{111}—G^4 corresponds with the top of the window-sill of the clerestory and of the outer marks of the division of the stories on the buttresses, as shown by red rings. The story of the triforium falls accurately between the diameters through c and c^{111}.

There is an interplay between the larger and the smaller circle which will now be apparent.

We see that the zenith of the smaller circle marks the transverse arch and does not, as in our theoretical construction, come beyond the top of the walls of the nave. In order to reach this last-named height, in Cologne cathedral, the basis of the square of the small circle had to be lifted from F^2—C^2 to the top of the plinth F^1—C^1. When this lifting is done (see the construction in black), it will be observed that zenith H^1 of the circle reaches exactly to the top of the walls of the nave.

That such a lifting of the net of the construction up to the plan of the plinth has really taken place, is shown by the fact that zenith L^2, after the lifting of nadir F^2 to F^1, coincides at point L^1 *exactly with the top of the ridge*, as shown by the dotted roof line. We see here one of the advantages of dividing the square into six equally large parts, such as we explained above to be the result of the construction of the chancel; through this mode of dividing, the roof could be constructed *ad quadratum* on the width of the nave.

Starting from the absolutely right data that a building only begins above the plinth, which is only the finish of the foundations, the builder of Cologne cathedral has therefore lifted the theoretical auxiliary construction to it, and developed it from here, although not exclusively, but in connection with the theoretical auxiliary construction, the circles of which have then respective centres at c and c^{111}, and intersect the line over the plinth at points a and D^1 respectively. He has built the whole interior, to the top (H^2) of the transverse arch and to the division of the stories, which we noticed were determined by the diameters through

c and c^{111}, *ad quadratum* from base F^2—C^2; the exterior, on the other hand, is built *ad quadratum* from base F^1—A^1.

The diagonal x—L^1 (of 63° 26′) of half the larger square determines the ridge, while the corresponding diagonal C^1—H^1 of the smaller half-square over the plinth determines the place of the capital under the groin of the vault of the nave (marked by a small black ring). It then stops at zenith H^1 and the tangent through this point determines the height of the walls of the nave. The distance between zeniths H^1 and L^1 = one-fourth of the side of the large square, x—F^1. It will then be understood that the small square is a measured part of the larger one. The side of the square of the front and of the exterior = eight times the distance between the axes of the columns of the church, the side of the small square of the interior = six of these distances. In other words, the large square is divided into eight equally large parts, of which six fall on the smaller square. Thus the geometrical play between the two squares is obvious.

The diagonal x—L^1 (of 63° 26′) of the larger half-square intersects diameter G^1—c^1 of the smaller circle at its intersecting points with the central axis of the second aisle, while the corresponding diagonal C^1—H^1 of the smaller half-square intersects the same diameter at a corresponding point of the central axis of the first aisle.

The diagonal of 45° of the larger square from corner x intersects the same diameter at its intersecting point with the axis of the wall of the nave just under the capital of the column in the window of the triforium. A diagonal of 45° from nadir F^1 in the middle of the plinth line and common to the larger and the smaller black circle passes first through the extreme point G^1 on the horizontal diameter of the smaller circle, and is deflected at the corresponding point on the tangent of the large circle, G^2, up to point L^1 at the top of the ridge.

A diagonal of 63° 26′ from the top of the plinth on the surface of the outer row of columns supporting the vault in the outer or second aisle, passes through the projection of the coping over the triforium and the top of the parapet on the floor of the external gallery of the clerestory in order to end exactly under the keystone of the diagonal arches. We have, in this way, four diagonals of 63° 26′ which are to be especially noticed: (1) in the smaller red square, from C^2, which determines the *height of the capitals* of the main pillars and the *summit of the transverse arch*; (2) from the top of the plinth of the wall pier, which determines the *height under the diagonal arches* and marks also the square of the *interior of the church*; (3) the diagonal of the square of the small auxiliary construction lifted to the top of the plinth (therefore C^1—H^1), which fixes the *height of the walls of the nave* and the *height of the capital under the groins*; and (4) the diagonal of the large square lifted to the plinth (therefore x—L^1), which determines similarly the *height of the walls of the nave*, and also the height of the ridge.

We have thus undeniable proof that Cologne cathedral was designed *ad quadratum*, both inside and outside, and also in the division of the stories, the architectural distribution, and the heights of the capitals.

It would lead us too far, after what has already been explained, to continue the demonstration so as to point out all the marvellous details of the interplay. We shall only remark that just as the proportion between width and height of the rectangles of the theoretical construction, the large one of the nave and the small ones of the aisles, is as 1 to 3, so is the proportion between the width and the height of the interior as 1 to 3. The inner width of the nave (which, by the way, equals the side of a regular decagon inscribed in the circle having as centre c^1) is marked in the drawing by arrows on the horizontal diameter through this point; exactly three times this width is the height from the plinth line to the top of the transverse arch, or the distance from the top of the upper division of the plinth

to the lower part of the keystone of the diagonal groins. The inner width of the aisles goes exactly three times into the height, measured from the plan of the plinth.

This proportion in the nave is marked by the dotted red diagonal from point Y on base C^2—F^3, and in the aisles similarly by a dotted red diagonal. It will be seen how these diagonals fix the dimensions of the pillars. In the rectangles lifted to the plinth line, the corresponding black dotted diagonal is found, going from the intersecting point between the plinth line and the axis of the wall, to stop upwards exactly at the beginning of the vault.

The angle formed by these diagonals of rectangles with the base is, accurately calculated, 71·564°, that is to say, very nearly corresponding to the angle formed by the base and the diagonal of a regular pentagon, 72°. The difference is so insignificant that the angles of the rectangle, here in the drawing, are introduced by means of a parallel ruler from the constructed diagonal of a pentagon.

This proportion of 1 : 3 is therefore a result belonging to the five-aisled church, where it creates proportions in harmony with the whole fabric; it is created here everywhere, in the church itself, in the raising, longitudinally as well as transversally, by dividing the square into six. In Cologne cathedral we find the same division carried out to its extreme limit, in the vertical design, in length, both inside and outside, such as we saw in Notre-Dame.

We have already remarked that the level of the capitals of the triforium is placed just over the diameter through centre c^1 of the smaller circle over the plinth line, or the horizontal centre line in the square of the church (marked by the letters A—B), a line which becomes a common level in the following drawings, from where we transfer it to the drawings of the longitudinal plans, which we are about to consider.

Longitudinal Plan

(*a*) *Longitudinal Section*

We start from this level line A–B which has been transferred to the longitudinal plan, the longitudinal section (fig. 72*a*), and the side elevation (fig. 72*b*) (Plate VII).*

We begin naturally from the interior so as to see how the raising of the side elevation is carried out as a genuine expression of the interior.

On the level line A—B in fig. 72*a* we have point G^1 on the left. From here the diagonal of 45° of the square goes to points $H^{1\,(1)}$ situated in the vault to the right, and $F^{1\,(1)}$ down to the right on the plinth line, respectively. We see further that the diagonals of 63° 26′ of the half-square go from point G^1 to H^1 at the top of the vault and to F^1 on the plinth. All these points correspond to those marked by the same letters in the geometrical analysis of the transverse section.

Figs. 72*a* and 72*b* give the same two and half bays internally and externally. With the vertical axis through point G^1 we add half a bay to the longitudinal section on the left. In this manner we obtain two whole bays, and one half-bay on either side of them—altogether three whole bays. By the diagonals of 45° going from the mentioned point G^1 on the left, upwards to point $H^{1\,(2)}$ above the vault, and down below to point $F^{1\,(2)}$ on the line over the plinth, we have already made sure that the three bays lie within a half-square. The longitudinal section, therefore, is drawn *ad quadratum*, and the square, as in the transverse section, is divided into six, with one-sixth of a square for every bay; consequently the length of the bay goes six times into its height—this is shown by the network of lines at an angle of 45°. From the floor to the vault there are also six squares in the length of the bay. The

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proportion between length and height in the lights is characterised by the diagonal of 72°, similar to what happens in the transverse section of the aisle and of the nave.

It will be remembered that in the monastic church of Vezelay and in Notre-Dame of Paris we saw examples of how the architectural proportioning was determined *ad quadratum*. In the later, and therefore more richly developed Cologne cathedral, the culminating point of Gothic art, we find an excellent specimen of how the complicated distribution is produced by the harmonising help of this principle of the square. This is clearly seen in the drawing.

We can begin by showing the correspondence between the principal points of the wall taken as a whole. We see, for instance, that the diagonal of 45° of the square coming from $F^{1\,(2)}$ (down to the right of the plinth) after having first determined the level of the bases of the columns in the windows of the aisle at point *b* and the level line of the canopies over the statues on the pillars of the nave, determines the level of the centre of the rose in the windows of the aisle; it determines the height of the lower story; and finally, midway up the height of the wall, at G^1, it marks the level of the capitals of the triforium. When deflected from here the diagonal gives the level of the capital under the transverse arches of the vault, the level of the centre of the rose in the windows of the clerestory, and finally it determines the interior height of the church *ad quadratum*, at point $H^{1\,(2)}$.

The diagonals after the angle 63° 26′ correspond obviously with these fixed places; but besides this, it is interesting to see how principal and secondary features and the entire architectural punctuation between these features fit into each one separately, and mutually into one another, within every bay and the bays mutually, from the base to the top like sentences and periods.

If we begin, for instance, with point M, capital No. 1, of the window which we indicated farthest on the left in the aisle, we shall find that this capital corresponds with the one in the clerestory windows which is also marked No. 1. Furthermore, we could start from the plinth of the column under point M of the above-mentioned window of the aisle. This point is also marked No. 1. The diagonal passes through capital No. 4 in the same window, through plinth No. 1 of the windows of the triforium, and stops at capital No. 4 of the windows, of the clerestory. Or the diagonal from capital No. 3 of the windows of the aisle passes through capital No. 1 of the windows of the triforium, through capital No. 3 of the windows of clerestory, and ends at the top of the same window.

Let us take an example of the correspondence within an architectural period consisting of a principal and a secondary feature—it is as the proportion between the windows of the clerestory and those of the triforium, which are made like treble windows, seemingly three and three together. We take the middle window of the clerestory—the capitals of which are numbered from 3 to 7—and we begin, for the sake of convenience, from the centre of the rose, following the diagonal of 63° 26′ downwards to the left. It passes through the capital under the transverse arch of the vault, then through the middle column of the window of the clerestory and of the triforium, where it intersects the horizontal division between these windows to end at the plinth on the left of plinth No. 1 of the windows of the triforium. We find in a similar manner the connection between the triforium and the lower story of the nave. We take, for instance, the diagonal of 63° 26′, which starts from the plinth of the pillar of the nave farthest to the right; it passes through the corner of the window of the aisle, on the right of plinth No. 14, and runs up at the height of the capital of the middle column of the window of the triforium.

Lastly, in order to return to the wall as a whole, we could take the diagonal from the intersecting point between the central axis and the line over the plinth of the bay indicated farthest to the left; it passes through plinth No. 4 in the window of the aisle, through the canopy over the statue on the pillar of the nave, through capital No. 7 and the centre of the

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rose tracery in the window of the aisle, through plinth No. 4 of the triforium, it intersects the axis of the pillar of the triforium at the height of the capital, passes through capital No. 7 of the window of the clerestory, and ends at point $H^{1(2)}$. If we had drawn the other half of the square and from this point $H^{1(2)}$ we had drawn a diagonal downwards to the right, to the continued line of the plinth, we would have obtained correspondence with the three new bays and from here we could have continued through the whole length of the wall.

(*b*) *Side Elevation*

Fig. 72*b* represents, as already mentioned, the exterior of the part of the side-walls of which fig. 72*a* shows the interior. We see that the side elevation is drawn strictly and without compromise, *ad quadratum*, in genuine correspondence with the longitudinal section. After examining the latter, the analysis of the side elevation speaks for itself. Here also we find all the architectural divisions determined by the system—stories, string-courses, lights and their divisions, position of plinths and capitals, centres of rose tracery, main cornice, and height of parapet above this, as well as divisions of finials. We find, besides, the ridge and its crest to be also determined *ad quadratum* on the length of a double bay, as shown by the diagonals from the ridge O^1—Q and O^2—P. When demonstrating the transverse section, we found that the ridge—when reckoned from the top of the walls of the nave—was determined *ad quadratum* on the width of the nave, or two-sixths of the square of the church, or two-eighths of the square of the front. The two bays in the vertical transverse plan are then as long as the bays in the vertical longitudinal plan. In Cologne, therefore, equally large divisions of the square have been used in all three plans, the horizontal and the two vertical ones, transversally and longitudinally, in opposition to Paris, where the divisions, transversally, are not equally large, the nave being somewhat broader than two-sixths of the square. In Amiens the first aisle of the chancel is somewhat broader than one-sixth of the square. As a result of this thorough equal division of the square of the design in all the plans, the roof of Cologne cathedral has been able to rise *ad quadratum*, not only on the length of two bays in the longitudinal plan, but also on the width of the nave in the transverse plan.

We found that in the transverse section the summit of the ridge, point L^1, was also *ad quadratum* on eight bays. It is evident that this—as a result of the equal division in the transverse plan and in the longitudinal plan—is also the case in the longitudinal plan. We can satisfy ourselves on this point by following the red diagonal of $63^\circ\ 26^1$ from point O^1, on the summit of the ridge, downwards to the right, to point N^2. It passes through the top of the parapet at point R, through the centre of the rose of the clerestory window; it meets the axis of the pillar at point N^2, where it is deflected; it passes through a capital in the window of the aisle, to end at point K on the axis of the pillar and in the projection of the string-course, which runs under the windows of the aisle.

There are two squares having the length of two bays between the top of the crest and a level line through point N^2, situated just in the middle between the ridge and the plinth line. If we suppose the diagonal from O^1 continued through point N^2 downwards on the right to a point on the above-mentioned string-course under the windows of the aisle, we get the half-square complete, and in this way we obtain altogether the length of four bays within this. Consequently the top of the crest is situated *ad quadratum* on the length of four double bays or eight single ones—just as it was the case in Notre-Dame.

Just as in Notre-Dame also, we find in Cologne that the principle *ad quadratum* has guided and determined the architectural design and its divisions; but in Cologne, this scientific method is more developed. We found this to be the case already in the plan, and we saw it also in the somewhat older cathedral of Amiens. Here we shall see a new proof of the

shrewdness of the medieval architect in the employment of this method, which, instead of being an obstacle, always helped to overcome difficulties and solve knotty points.

In the side elevation (fig. 72*b*) it will be noticed that the diagonals of 63° 26', from F^1—N^1 and from N^1—O^2, are drawn right up to the ridge. This is due to an oversight. Point O^2 must naturally be as much lower as regards point O^1 as point F^1 is lower as regards point K, through which is situated the level of the base of the square of the whole design. We could have expected this to correspond with the line over the plinth, instead of with the string-course under the windows of the aisle, the level line through K. This is no contradiction or any breach of the method; on the contrary, it is an outcome of it, being a consequence of the construction of the chancel.

* * *

As mentioned when demonstrating the plan, we found that in the medieval drawing of the front the distance between the axes of the west towers was 30 metres = the distance x—F^1 in the transverse section (fig. 71, Plate VI). The width of the front becomes in this way 60 metres. As this contains eight bays, the length of each bay is 60 ÷ 8 = 7.m.5. From the plan in *Handbuch der Architektur*, the width is 60 metres; 62 ÷ 8 = 7.m.75 for every bay. In figs. 72*a* and 72*b*, which give a part of the vertical section of the chancel, the length is from 7.m.25 to 7.m.3. The maximum difference is then 7.m.75 ÷ 7.m.25 = 0.m.5 for the length of the bays of the chancel and of the nave. It will be remembered from the examination of the plan that the length of the chancel became shorter because the construction starting from the high altar was made to correspond with the longitudinal axis of the transept through the porches of its gable walls, just as the entire plan was determined by the position of the altar. The chancel with the altar is thus moved towards west. In order to retain the division of the square into six in the placing of the columns, the architect has been obliged to make a compromise within the given shorter chancel. In this way the bays have become shorter here than in the nave. It is of course difficult, not to say impossible, on account of the small scale of the drawings reproduced, to show with arithmetical accuracy how great this difference becomes between a square of eight bays in the nave and a square of eight reduced bays in the chancel. With the material at our disposal we must content ourselves with the geometrical difference in the size of the square. This difference is the distance between the points F^1 and K.

As the ridge is now determined *ad quadratum* on the length of eight bays in the nave, both in the transverse section and the side elevation, the bays of the chancel become as much higher as the distance between K and F^1. In the nave, on the other hand, the length of eight bays is equal to the height from the top of the plinth to the top of the crest.

All these circumstances put together—the shortened chancel and the shortening of the length of the bays resulting from it, moreover the fact that the roof of the chancel, erected in 1322, is nevertheless constructed *ad quadratum* on the length of eight bays of the nave, only built in the nineteenth century—are some of the many sure proofs that Cologne cathedral was produced in a single conception, and planned in one design already in 1248; this is the general opinion with a few exceptions, as for instance Hasak, who has doubted and even denied it, being ignorant that a scientific principle forms the base of the religious architecture of the past. The unscientific "history" of art has been content to judge in vague phrases and to guess at the chronology. The cathedral of Cologne is similar to all other important cathedrals of the Middle Ages: it is the result of one thought, one conception, and it cannot be otherwise.

Not any more than in Notre-Dame have we found here in Cologne any arbitrary method,

no " amusing " inventions; everything is carefully calculated, determined, and produced by the entelechie of the geometrical system.

The same is the case with the west elevation from the plinth to the summit of the spire.

The West Elevation

Fig. 73 (Plate VIII) [1] gives the original medieval drawing of the west front of Cologne cathedral.

After the previous analysis of the front of Notre-Dame and of the plan of Cologne cathedral, the present analysis should not require any elaborate explanation.

The distance between the central axes of the towers is marked by points D[1] and E; an equilateral triangle on this will give the centre, point C of the large red circle. From the square A^2—B^2—L^2—K^2 circumscribing this circle, it will be seen at once that the front is designed *ad quadratum*. The vertical sides of the square are continued upwards, and their extreme points are found by the diagonal of 63° 26'—as, for instance, the diagonal from point A^2 on the left of the line of base. This diagonal intersects the central axis of the church at the summit of the gable, point I^2; it is deflected at point N^2 on the lengthened side of the square of the front, and intersects finally the axis of the south tower at point S^2. A horizontal line through this point gives level O^2—P^2, and the limit of the rectangle of the design of the front. It will be seen that the line passes through the top of the finial of the spire. This rectangle contains two and a half squares: the first is the square of the front, which has its limit marked L^2, the second is limited by N^2, and the half-square by P^2.

It can be seen that the length of the base line is marked underneath, 60 metres, and 2.m.5 × 60 metres = 150 metres.

This rectangle with proportion of 60 : 150 or 1 : 2½ within which the front is drawn, corresponds of course with the rectangle of the plan, fig. 68*b*, marked by letters A—B—C, and the length of which is given by the line of dimension III.

The analysis of the plan has shown us that the length of the church from the outer wall line of the west gable wall through point L^1 to the outside wall of the extreme east apse at point Q is = three times the exterior width of the church. The three squares on this width are marked by the diagonals of 45°, D—E—F—G.

We have transferred this proportion on the drawing of the elevation (fig. 73, Plate VIII) in hatched lines.

The width of the church on the line over the plinth is indicated by the distance from points *a* and *b*. The circle is drawn with a radius of half that width, the distance a—F^1. The square which circumscribes this shows that the level of the top of the parapet on the wall of the front is determined *ad quadratum* on the exterior width of the church. By means of the diagonals a—d, d—e, and e—g^1 of this square we produce three squares which form the rectangle of the church. It will be seen that the limit of the rectangle g^1—g^2 corresponds exactly with the level of the construction, and therefore with the real extremity of the spire, at the height of 140 metres, while the top of the finial comes somewhat above the level O^2—P^2 at the height of 147 metres above the plinth. From the lines of dimension IV and V (fig. 68*b*) of the plan it will be seen that 140 metres give the length of the church itself, from L^1—Q, while 147 metres are the length of the platform on which the cathedral rises, similar to an ancient temple rising from its krepis.

It is of fundamental interest to our documentation to be able to state: first, that the medieval drawing of the front of Cologne cathedral, similar to the front of Notre-Dame with

[1] Reproduced from *Handbuch der Architektur*, by permission of the publisher.

the spires added by Viollet-le-Duc, agrees fully with the plan; secondly, that, including the spire, it is entirely drawn *ad quadratum*; and thirdly, that the height to the summit of the spire is exactly equal to the length of the church.

In connection with this, it is also of importance to notice that one-half of the height of the tower falls on the tower itself and the second half on the spire—similar also to Notre-Dame. It will be seen that the diagonals from the two intersecting points of the central axis of the front, with the line over the plinth, which is point F^1 in the central porch, and with the level line g^2-g^1 over the constructive extremity of the spire, point h, the first diagonal upwards and the second downwards to the right, intersect each other at a point marked by two red rings. A horizontal line through this point divides the central axis of the front from F^1 to h into two equal parts. The base of the pyramid of the spire is on a level with the line called on the drawing: la base de la flèche.

The outlines of the spires, like those of Notre-Dame, are determined also by the system; the theoretical base of the pyramid lies on the diameter of the circle of the front. In Notre-Dame, on the other hand, it lies on the base line of the theoretical auxiliary construction over the plinth.

The medieval architect therefore has strictly followed the medieval cathedral rule: to build the towers with their spires equal to the length of the church. It is also worthy of notice that he has reckoned the height from the top of the plinth, as shown by the experiment just carried out, by means of the diagonals from points F^1 and h.

In Notre-Dame we found the length of the cathedral to be twenty-four bays = three times the width of the front, that is, eight bays. Therefore, it was easy to make the front and the height of the towers and their proportioning in agreement with thc whole of the church.

In Cologne the planning is much more elaborate. We have already seen that the plan of this cathedral shows a new departure in the west European cruciform type of the eleventh century. As such it is designed within two large squares. The nave and the transept, as regards their length, are quite according to the western square; but in the new cruciform construction the chancel became shorter. In this way the total length of the church is shortened so as to be equal three times the exterior width, instead of either three or two and a half times the width of the front.

In his endeavours to follow the cathedral rule, the medieval architect has only throught of realising the length of the church in height; but he has forgotten the given condition—the two large squares according to which his entire cruciform church is proportioned. He has also come to grief with the auxiliary system and committed an error which brings the towers out of harmony, not only with the lower parts of the front but with the whole building.

Already in the planning of the rectangle of the front, consisting of two and a half squares on the width of this front, the architect obtained for the spire and finial a height of 146 metres above the plinth level o^2-p^2, and through this also a satisfactory accordance with the length of the platform of the church, as, by adding only 1 metre, he got its full length, just as by the three squares on the external width of the church he obtained the height of the spires *without* the finial—reckoned from the plinth—to be exactly equal to the length of the church, buttresses not included. However, a comparison with the transerve section (fig. 71, Plate VI) will help us to understand what is faulty in this final result.

When drawing the transverse section, the architect has used the auxiliary construction below the plinth line for determining the principal points of the interior, while, as concerns the exterior, he lifted it on the line over the plinth: in this manner he determined the level of the main cornice on three double bays and the level of the ridge on four double bays.

We have introduced this proportion in the drawing of the front, after having placed the

axes in the walls of the nave. The distance between these is one double bay. It will be seen that while the height to the main cornice equals three double bays, the top of the triangle of the gable reaches only to point I^2 instead of to I^1, the level *ad quadratum* on four double bays, to which the summit of the gable must reach, as shown by the transverse section and the side elevation.

It will be seen that the diagonals of the half-square from point I^1 go downwards on both sides and stop at points A^1 and B^1 on the line over the plinth, while the diagonals from I^2 stop at A^2 and B^2 on the line under the plinth. In his endeavour to keep to the cathedral rule and not to exceed the height which he had already obtained with the three squares on the width of the church, the designer forgot to lift the auxiliary construction on the line over the plinth, which, we remember, he had made the base of the proportioning of the exterior of the entire cathedral, in length and in width, in the transverse section, in the longitudinal section, and in the side elevation. In this way the drawing of the front becomes the result of two different suppositions.

The gable wall to the top of the parapet is proportioned from the line $A^1—B^1$ over the plinth, and the whole part above the triangle of the gable and the towers with their spires are proportioned from the base line of the auxiliary construction, $A^2—B^2$, under the plinth.

In Notre-Dame we obtained already the proof that the geometrical system *ad quadratum* had been used by the architects of the Middle Ages; the medieval drawing of the front elevation of Cologne cathedral has not only confirmed this fact, but, through the mistake pointed out, *we have obtained an undisputable proof, and we can note triumphantly that the system has been employed exactly as we have done*, that is to say, by beginning with a supposed line over the plinth, and with a chosen radius, find the centre of the circle whereby the tangent through its nadir, for instance, point F^2 in fig. 71 (Plate VI), becomes the base of the auxiliary construction. *Through this only and in no other way could he succeed in placing the summit of the triangle of the gable exactly at point I^2, and in making the projection of the coping to correspond with the diagonals down to the extreme points of the base under the plinth—and ad quadratum* on this.

It is evident that the able medieval architect who, throughout his designs, has shown such knowledge and shrewdness must have quickly discovered his important mistake, which would soon reveal itself in the fact that the ridge of the chancel, which had already been drawn in the front elevation, reaches much higher than the summit of the gable. This fact is illustrated by the addition of the transept on the right. Thanks to the excellent method of drawing of the geometrical system, it was unnecessary for him to start again the great and troublesome work which the wonderful and beautifully executed drawing must have necessarily cost him.

As we know, he did not live to correct his error. The church was far from completed in the Middle Ages. As mentioned, the chancel was finished in 1322, but, strangely enough, it remained isolated with the east wall of the transept, with no connection westwards with the towers. The south tower, in which the bells were hung in 1439, had not reached higher than to the main cornice of the nave, that is to say, where the nave ought to have been, because this and the north tower had only reached the height of the walls of the aisles. The work stopped entirely in the sixteenth century.

The drawing of the front elevation disappeared; it reappeared only in 1814, when one of Napoleon's soldiers found it in the garret of an inn in Darmstadt.

The time known as the Renaissance allowed the architecture of North Europe to be forgotten, and it was despised as belonging to a crude and barbaric period. It is therefore significant that even Gerhard Schöning, the well-known Norwegian historian, who, in 1762, published his large work on *Trondhjems Domkirke*, seems to apologise for the church in his

Reise gjennem en Deel av Norge ten years later, when he says: "Although a Gothic building, it is one of the finest of its kind and most according to the rules."

In his commemorative writing of 1772 on Erwin von Steinbach, the architect of the cathedral of Strassburg, Goethe had the honour of being the first—in the whole of Europe, we believe—to rouse the feeling of appreciation of Gothic architecture. When this feeling broke through during the kind of national renaissance, which a reaction had brought against the dry rationalism and the worship of enlightenment of the eighteenth century, the building of the cathedral of Cologne was resumed (fig. 74 shows its appearance in 1842 when this took place).

The drawing of the front elevation came again to the fore. But the conditions were changed. The decline of intellectualism in the fourteenth century, as a result of the plague,

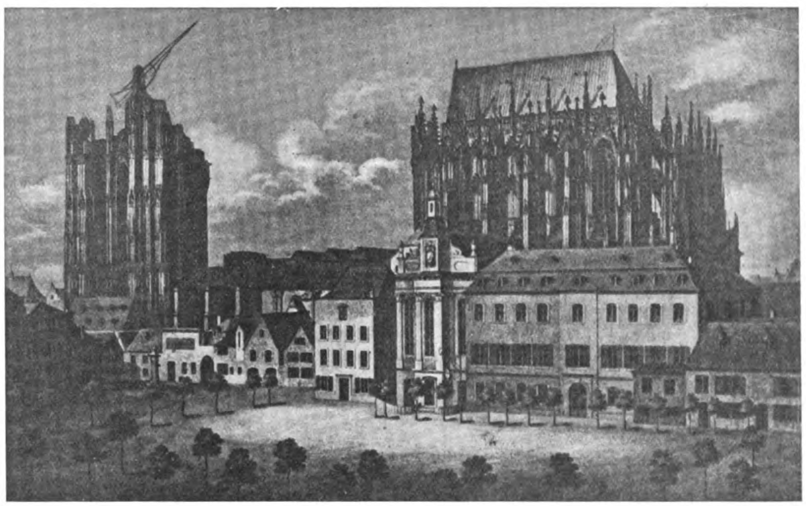


Fig. 74.—Perspective drawing of Cologne cathedral at the beginning of the nineteenth century.

and continued in the fifteenth, with the Renaissance, with Lutheranism denying all authority, with the hatred of dry Calvinism for symbols, all worked towards taking away from mankind the finest part left of antique civilisation, which in its deep consciousness of life had accepted its laws humbly and had used them as a guide in its religious architecture. The Renaissance and Protestantism have vulgarized humanity; they have depraved it and made it self-worshipping.

The nineteenth century lacked the talent, and we have lacked it to this day, not only to be able to read critically the medieval drawing of the front elevation, but altogether to understand the architecture of antiquity and of the Middle Ages.

The mistake of having made the triangle of the gable too low could be corrected by being compared with the existing roof of the chancel; but apart from that, the drawing was used such as it stood—with the difference only that the height of the spires was altered from 140 metres + 7 metres to 150 metres + 7 metres for the finials. The line of level through h and Q^2 (in fig. 73, Plate VIII) became, so to speak, lifted up to Q^1 and Q^3. We do not know what reasons the architects could have had for heightening the spires. It is in itself

correct, but by being done without altering other parts of the drawing it becomes wrong, as it only serves to emphasise what is wrong already, namely, the proportioning of the front resulting from two different suppositions: firstly, the part above the parapet on the main cornice, that is to say, the towers from the base line of the auxiliary construction *under* the plinth; secondly, the part below the parapet, which is the front itself, from the line *over* the plinth. By heightening the spires to 150 + 7 metres, their height is again determined by this line, while the towers are carried out unaltered according to the proportioning given by the line under the plinth!

In order to show the incongruity thus created in the proportioning, we have made, on the front, an experiment with the proportion of the sectio aurea.

As this shows, the part from the plinth to the top of the parapet on the main cornice, which is the front proper, is according to this proportion; that is, the first main part which is the first story, is to the second main part, which is the story of the triforium and of the clerestory combined architecturally, as this second portion is to the entire part under the parapet. The part from the plinth to the faulty summit of the gable, point I^2, is also in proportion to the part between points I^2 and Q^2, which are the summit of the spires, according to the sectio aurea. The part of the spire which lies between the top of the central axis, Q^2, and the cornice of the octagonal story or the level through the point marked by a double cross on the central axis, is also in proportion to the part from here down to the plinth according to the sectio aurea. We find it evident, therefore, that the proportioning according to the sectio aurea is the aim, a point on which we shall be convinced later as regards other parts of the church.

But the architect was not able to realise his intention because of the mistake he had made. The important part, the third story of the tower, is out of proportion. It is too low and crushed, and it brings the whole out of harmony.

There is no correspondence either between the front as a whole, that is to say the entire elevation to the summit of the spires, and the church as a totality, such as we see had been done in Notre-Dame, in the horizontal and in the vertical plans, with such strict logic. But that this connected correspondence has been aimed at, will be shown by a further examination. With this object in view we have added on the right of the drawing of the front elevation the south arm of the transept. The red dotted line signifies the outer wall of the gable, and the part in hatchings outside this means the projection of the plinth under the buttress. On the left the north spire is indicated, and on this is marked the height of the spire in the medieval drawing. It will be seen that the diagonal drawn with an unbroken red line downwards to the right from point R^2, which is the top of the finial in the medieval drawing, runs down to the point marked by a small square on the line indicating the gable wall of the transept; from here it is deflected down to point F^2 in the middle of the line of the auxiliary construction. If we try now with a diagonal—the red dotted one—from the constructive extremity of the spire, at level g^2—g^1, this does not give any correspondence with the auxiliary construction.

On the other hand, if we draw the two black unbroken diagonals from the corresponding points in the correct spire construction, that is to say, the summit of the spire on the left, at the height of 157 metres and at 150 metres, the correct height of the spire which is the constructive extremity of the spire at level O^1—P^1,—it will be seen that the first diagonal comes down to the intersecting point of the top of the parapet, with the surface of the gable wall of the transept, from where it is deflected down to the line of the plinth, to the intersecting point of the latter with the axis of the wall of the nave, indicated on the left, while the other is deflected on the line of the gable at the point marked by two rings and meets point F^1 in the central porch.

By transferring these observations to the plan (fig. 75) we shall, moreover, become convinced that the present height of the spires is correct in opposition to the drawing of the front elevation.

The dotted line $A^1—B^1$ represents the line over the plinth and the unbroken one $A^2—B^2$ represents the base of the auxiliary construction on the plinth. The medieval tower construction, as well as its auxiliary lines, which come from the line under the plinth, are drawn with black unbroken and thicker lines, while the correct construction and its auxiliary lines are drawn in broken black lines coming from the line over the plinth. For the sake of clearness, the transept and its roof are also put in. We see that the unbroken thick auxiliary line from under the plinth intersects the central axis at a point marked by a small black ring—the wrong summit of the gable—and after its deflection from the side of the rectangle on the width of the front, it intersects finally the central axis of the tower. The line of dimension VII on the right of the drawing shows 146 (147) metres from the line $A^1—B^1$ over the plinth. The intersection point of the auxiliary line with the central axis of the tower gives the top of the finial. The black unbroken diagonals of 45° show how the three squares on the width of the church make the extremity of the spire construction correspond with the length of the church, except the west buttresses (see line *v*, 140 metres). These heights, 147 and 140 metres, are transferred to the spire on the left, and the unbroken black diagonals, going downwards to the right from these points of height, show that there is no correspondence with any vital point in the whole design.

The outlines of the pyramid of the spire are put in red in the demonstration of the medieval drawing of the lower, as they are at present, with their constructive theoretical base, marked by small black rings on the half-diameter of the semicircle drawn fully on the line under the plinth. The base used in the nineteenth century is the same as the one used by the medieval designer, and nevertheless the spires have been made higher!

On the left we see the correct drawing of the tower, with its auxiliary lines coming from the line over the plinth. This is drawn in black strokes. We see that the summit of the gable is marked by two small black rings on the central axis; it falls on a level with the ridge line of the transept. We find that the height of the constructive extremity of the spire coincides with the boundary line of the rectangle on the width of the front, when it is lifted to the line over the plinth. The line of dimension III on the left of the drawing shows a distance of 150 metres, while the top of the finial appears to fall exactly on the boundary line of the large square of the design and, according to line II, to come at a height of 156 (157) metres.

The diagonal from the top of the finial is drawn in a red broken line. It will be seen that it is deflected from the surface of the south gable wall, at the top of the parapet on the transept, and stops at the line over the plinth on the axis of the south pillar of the north tower. The diagonal from the extremity of the pyramid stops after its deflection from the line of the gable wall, on the wall of the front in the middle porch.

The theoretical base of the pyramid of the spire is marked by two black rings on the half-diameter of the semicircle drawn in strokes and lifted to the line over the plinth.

Finally, it will be seen that the axes of the towers fall at the intersecting point of the large red circle with the east boundary line of the rectangle formed by the two large squares into which the whole cathedral is designed. We have now continuous correspondence between all parts of the cathedral—in all the plans, as it has been the chief aim in all our earlier analyses.

In fig. 76 (Plate IX) the reader will find all our collected observations, taken from the plan, the transverse section, the side elevation, with the transept added in its entire length, from the front with the wrong medieval drawing of the tower, and with the correction of it, in order to show how the tower should have been drawn if the architects of last century had understood the medieval drawing.

It would take us too far to examine the whole drawing. The interested reader will certainly be able now, without any tiring explanation, to read the analysis and convince himself that the towers are in full agreement with the entire edifice and its spirit, as it appeared clearly and distinctly everywhere without exception in every analysis.

We shall limit ourselves now to pointing out some main features.

The diagonal of 45° from the intersecting point of the central axis of the church with the main cornice down to point B[1], on the plinth of the buttress, which strengthens the corner of the transept towards south, shows that the transept in its elevation is built on two squares.

The part above the parapet mentioned in the demonstration of the tower, on the left, as a preliminary to the proportioning, has been lifted on the line over the plinth. The third story of the tower has been visibly raised and the pyramid of the spire gets its base at the proper level, in the middle of the tower, where the medieval architect would also have put it, if he had been able to carry out his design, because, as remarked, in this also the platform of the pyramid of the spire lies in the middle of the collective height of the tower.

It only remains to show how the entire elevation stands in the proportion which we found intended and which is realised in the front itself, that is, in the proportion of the sectio aurea.

With this purpose in view we have drawn, farthest to the right, a line which shows a proportion already pointed out according to the sectio aurea, between the height to the ridge and the height from there to the top of the finial. Moreover, we find on the left, five lines marked in Roman figures.

Line I shows the same proportion between the height to the top of the parapet on the octagonal story of the pyramid of the spire, and the height from here to the top of the finial.

Line II shows the same proportion between the height to the cornice on this story of the spire, and farther, from here to the constructive extremity of the spire under the finial.

Line III shows that the height to the top of the parapet on the main cornice of the front stands in the same proportion to the top of the parapet on the third story of the tower; while—

Line IV shows the proportion between the main cornice of the front and the main cornice of the tower, which is its third story.

Line V shows the earlier-mentioned proportion of the sectio aurea in the architectural distribution of the front.

Finally, there exists the same proportion between the octagonal story of the spire and the third story of the tower, because the story of the octagon from the top of the parapet to the platform of the pyramid, which, seen from below, corresponds with the top of the parapet on the main cornice of the tower, stands in the proportion of the sectio aurea to the third story of the tower, reckoned from the main cornice of the front.

It will be fully recognised, consequently, that the correction made in the drawing has produced a perfect equality in the proportioning of all the parts of the entire front from the summit of the spires—just as the entire front is brought in relation with the whole conception. Unity has been produced; there has been one mould—harmony.

There is no break at a single point, because there is unity in the complicated variety of the mighty building.

But this is not the case with the edifice as carried out, because the towers, in the nineteenth century, have been erected on the basis of the mistake of the medieval architect. Through the partial heightening of the pyramids of the spires, their bases have come to be placed considerably beneath the horizontal centre line of the towers. The already too low third story of the tower is, through this, still more crushed. The total height and the proportioning of the front itself to the top of the parapet on the main cornice are determined

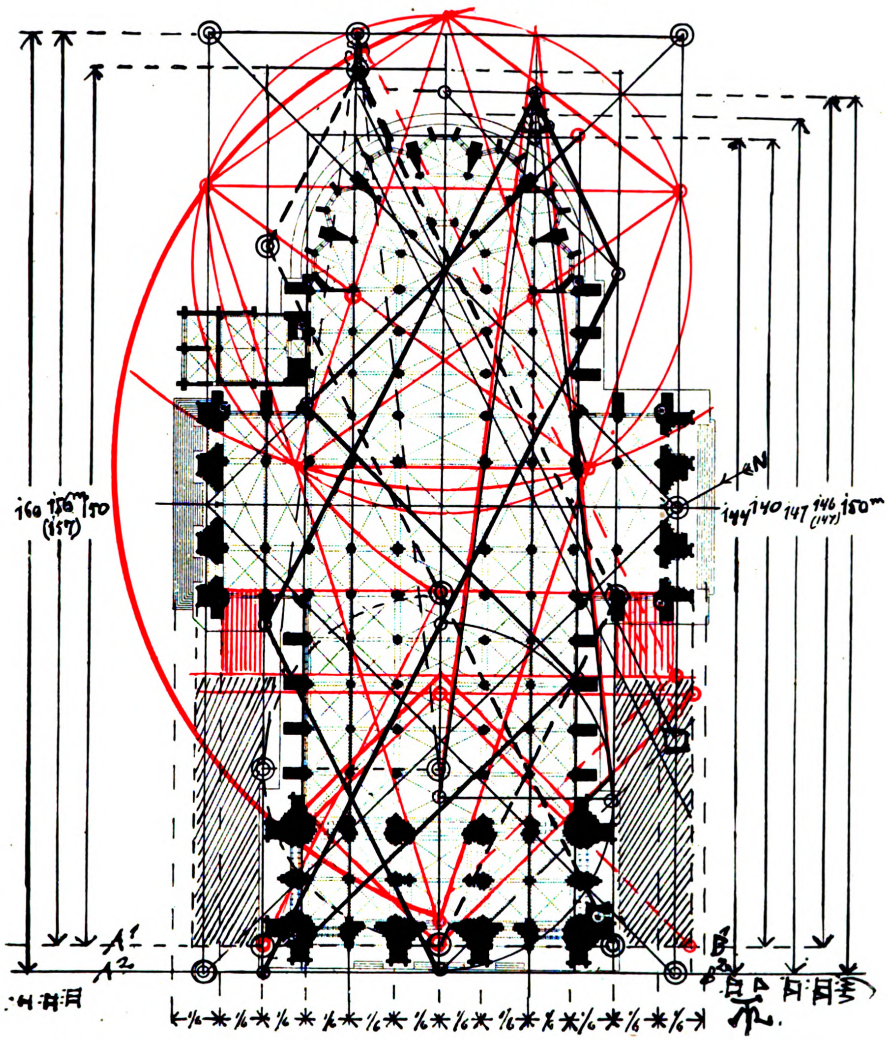


Fig. 75.—The construction of the tower in proportion to the plan of Cologne cathedral, analysis.

from the line *over* the plinth, while the part between the parapet and the summit of the spire—the third story and the octagonal story of the spire—is proportioned from the line *under* the plinth!

Equality and harmony are broken; there is disharmony because the medieval drawing has not been understood. The west elevation in Cologne makes no beautiful impression, therefore. The exaggerated decoration which covers restlessly all the surface adds also to this impression, and so does the exaggerated emphasis of the vertical lines of Gothic architecture which attract

the eye in an upward direction *only*, without pause and without rest; it is not musical moreover, because it lacks the correct intervals (fig. 77).

According to the corrected drawing, the towers rise harmoniously, one story above another. Quietly and majestically one part develops from another, in one tone, without offending the critical eye in its travelling upwards, because there exists the just rhythm between the horizontal divisions.

We recommend the reader to give this drawing a special study. It is as interesting as it is instructive.

Finally, we must mention the pentagram transferred from the plan. We see, firstly, that just as the chancel and the radiating construction of the apse were produced from the large square, so does the spire develop within this large square, the side of which is the diagonal of the large pentagon. We mentioned that the minor of this diagonal is equal to the distance between the axes of the towers.

As the minor of the diagonal of a pentagon is the diagonal of a new pentagon of second order, the distance between the axes of the towers becomes the diagonal of a new pentagon. By introducing the diagonals in this new pentagon, a pentagon of third order and a pentagram which is emphasised in hatchings are produced.

It will be seen that the base of the smallest pentagon goes through the summit of the spire over the crossing of the nave with the transept. It will be seen, furthermore, that we have marked the summit of the pentagram inscribed in this pentagon with a small red ring. By lengthening two of the sides of the pentagon downwards we obtain this wonderful play: the pentagram has theoretical points of support in the intersecting points of the central axis of the spires with their real base, marked by a red ring under the spire on the left, in the extreme points of the theoretical base of the pyramid of the spires, and also in the plinth under both the gable walls of the transept.

Moreover, the lengthened sides, going downwards, of the first and largest pentagon stop in the lintel over the main porch, similar to what has happened in the plan, where they stop on the threshold of the main porch.

In order not to disturb the other analytical lines of the small plan (fig. 75), we have not carried out the progression of the sectio aurea downwards or inside the pentagon of second order in this development, but it will be understood that here also the same play develops itself.

The high altar is situated in the middle, and with the host as centre, pentagones seem to ignite from one another between the spires in a harmonious geometrical progression, according to the proportion of the sectio aurea.

When the pious medieval worshipper stepped inside the porches of the transept or followed the long processions through the main porch, he came into a wonderful world of imaginary pentagrams, in front of him and over him, round the altar, in the sanctuary, high up between the spires and over the roof of the church where chancel and church meet—the symbol of creation and of the miracle of life in ancient and Christian times.

When treating Greek philosophy we shall prove that this is an entirely Greek thought, which determines absolutely the art of classic temple-architecture.

* * *

Now it only remains to show that the architectural and decorative distribution of the front is carried out in the same manner as in the longitudinal section and the side elevation. It is evident that, as it was there, so it is with the front—that is to say, *retificata*, drawn in a net *ad quadratum*, to quote the Milanese document.

For this demonstration we have made a new drawing (fig. 78, Plate X).

The Fig. "Ad Fig. 76 (Plate IX) and Fig. 77" on next page are given as a supplement to Fig. 76 (Plate IX) and Fig. 77, and also as an illustration of our statement, pages 75 and following. Side by side with the medieval front we give here a sketch of it as it ought to have been if the medieval architect had not made the mistake we pointed out, when using the auxiliary construction. The difference in the lofty appearance and in the harmony is obvious.

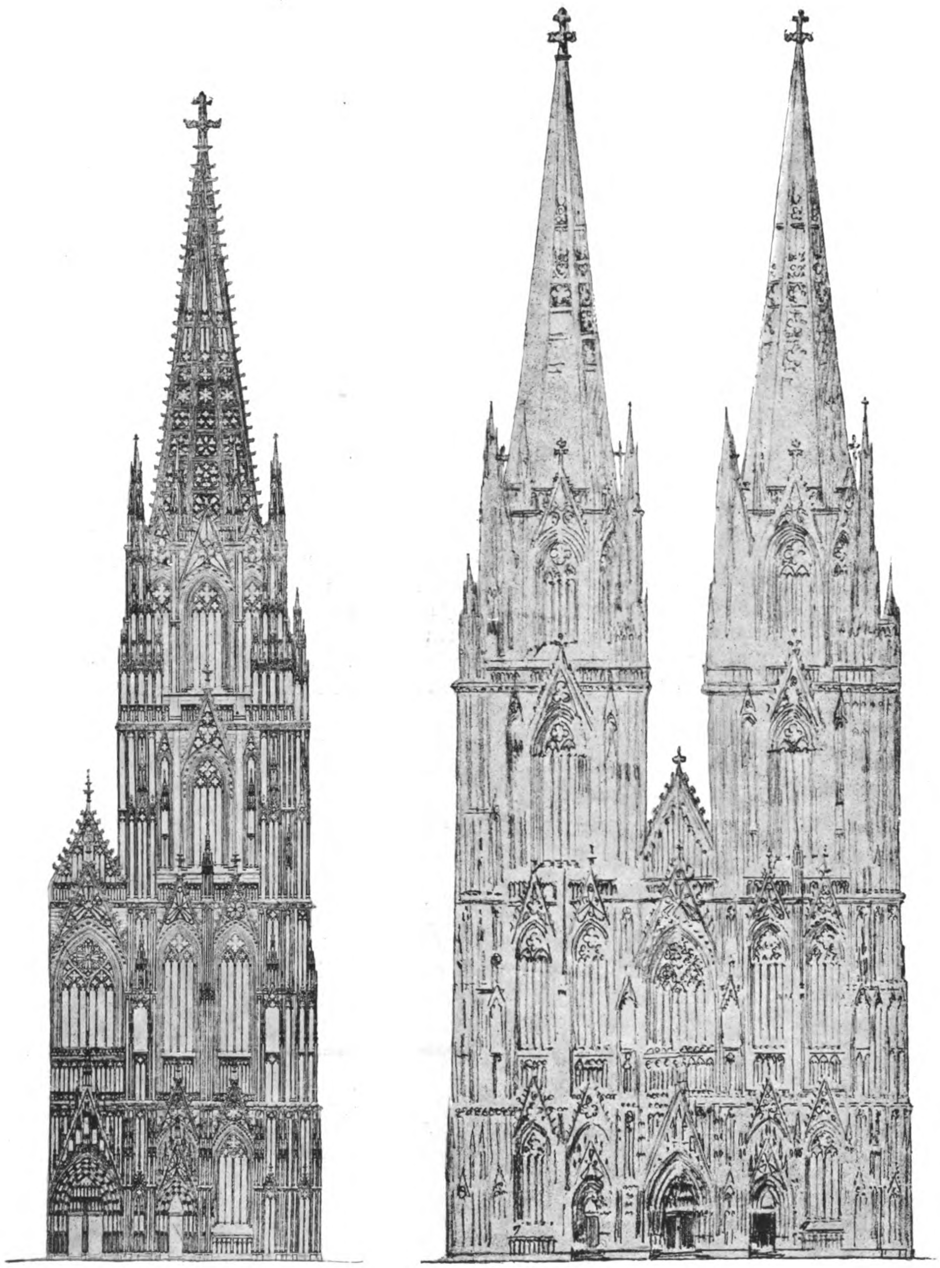


Fig. "Ad Fig. 76 (Plate IX) and Fig. 77." The Cathedral of Cologne. The medieval drawing of the front, corrected as it would have been if the architect had not made the error in design, pointed out by us.

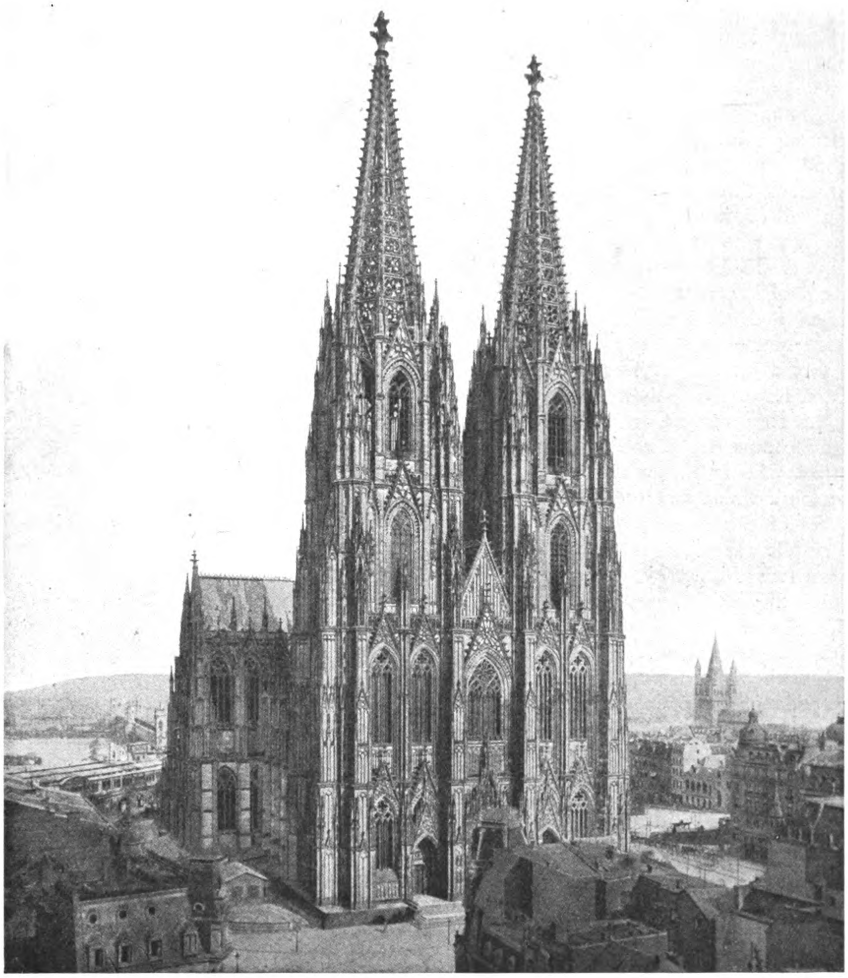


Fig. 77.—Cathedral of Cologne, seen from N.W.

We could begin, for instance, with the north corner of the front, at point A, on the line over the plinth. As it will be seen from the corresponding point B, at the south corner, there exists at point A, the base of a pilaster in the panel work. A diagonal of 63° 26′ from here passes through the following points: through capital 8 in the second row of windows, through the top of the finial on the gable, through a base in the parapet over the platform of the south tower, through the centre of the rose tracery in the canopy over the window of the octagonal story of the spire; it is deflected from here down to a capital in the panelling on the octagon of the spire, it is again deflected from here down to capital 3 on the middle mullion of the third row of windows, it passes successively through two bases in the panelling, through a capital in the panelling on the central axis of the buttress on the story beneath, through the capital marked 6 in the large central windows, through the intersecting point of the column under capital 3 with the top of the parapet in front of the large window, through the extreme north base of the window-sill, and, finally, down to base 6, from where it is deflected to capital 2 in the lowest window on the north side; it ascends further to the height of the capital marked NB (see NB on the south side) in the panelling on the buttress, from where it is deflected upwards to capital marked 3 in the third row of windows in the north tower—see corresponding capital in the south tower. From here it is deflected to capital 2 in the large central window, it passes through the intersecting points of the top of the parapet with the fifth column in the same window. After having passed through other points marked on the drawing, it reaches the base, at the plinth line, marked 19², from where it begins a new endless play of angles and lines.

In the same manner we could begin over the plinth of the middle mullion of the central porch and choose, for instance, the diagonal which goes obliquely southwards.

It fixes the height of the porch, runs through the cap of the middle mullion in the extreme south window of the second row, it intersects the corner of the buttress which frames the square of the front, it corresponds upwards with various important main divisions, such as the middle capital in the window of the third row, in the south tower, and from there, with numerous features. If we follow the line downwards from the capital in the south window of the second row, we find that it comes down to C and is deflected towards base marked 23 over the plinth, from where it is deflected and passes through the base under capital 5, through capital marked 1, through the summit of the baldaquin over the window on the left, through the base farthest down to the left in the row above, through the projection of the main cornice on the central axis of the front, it determines the position of one of the crotchets on the gable, to meet, at last, the base under capital 4 in the window of the octagonal story of the north spire.

Or we may take the diagonal from base 21 over the plinth line. It determines the distribution of the panelling under the window, it passes the base under capital 1, through the summit of the canopy over the side porch, through the capital in the panelling on the central axis of the buttress, through a division in the panelling in the corner of the buttress, it passes through capital 6 and the centre of the rose of the central window, where it corresponds to both sides, upwards and downwards.

It is unnecessary to quote other examples. The reader interested in the subject may himself choose other starting-points, and he will be struck everywhere by the wonderful connection which exists and the endless correspondence between all vital points of the purely architectural as well as of the decorative distribution.

In order to emphasise more clearly this "retification" of the front *ad quadratum*, besides placing a net formed of lines of 63° 26′, we have introduced the diagonal of 45° of the square, which in its geometrical function is identical with it.

The play of these lines is also worth a study.

The drawing needs no explanation.

We must point out, finally, the stroke and cross diagonal which starts upwards on the left from capital No. 4, on the middle mullion of the large central front window and stops at a point marked by three small red rings in the central axis of the north tower. This point marks the place of the raised capital (No. 3) on the middle mullion of the window, on the third story of our corrected drawing of the tower (fig. 76, Plate IX).

In this drawing the correspondence has become now more lively, because the related points seen in the example mentioned stand in mutual relationship, which was not the case in the faulty medieval drawing.

We leave the reader to the instructive reflections which the play of lines will naturally suggest.

There have been differences of opinion as regards the date of this medieval drawing of the front.

Hasak believes it to date from about 1300. After having examined the transverse section, the longitudinal section, and the side elevation by means of lines according to an angle of 60°, which he thinks erroneously to have determined the proportioning, he says of the drawing of the front: "Diese Westansicht ist aber bedeutend später gezeichnet; sie stammt vielleicht von 1300 und zeigt andere steilere Richtungslinien als das Schiff" * ("This west front is drawn considerably later, however; it dates probably from 1300 and indicates other steeper lines than the nave").

As our drawings are from Hasak, the reader will be obliged to admit, after seeing these lines of 60° in *Handbuch der Architektur*, that they are placed quite arbitrarily, and after our irrefutable demonstration he will at once be convinced that they have not the slightest relation to the church for the simple reason that it appears to be designed *ad quadratum* throughout, and not *ad triangulum*.

The west elevation is therefore not at all designed according to "other steeper lines," but exactly according to the same principle as the rest of the church, its plan, its transverse section, its longitudinal section, and its side elevation.

Our several analyses have proved without contradiction that the entire church is produced by one conception, one thought, and probably designed by the same hand, although the west elevation may possibly date from some years after 1248, as its ornamentation appears to be somewhat later.

We follow the same method here as we did for Notre-Dame—that is to say, we collect our observations in a theoretical diagram (fig. 79, Plate XI).

From this it will be seen how the proportioning of all the main parts of the cathedral is determined from the plan, as the two large squares on the length of the transept which form the frame for this, form also the frame in which the front, the transept, and the side elevation are designed. We see, furthermore, that the rectangle of two and half squares on four double bays, or the width of the front, lifted on the line over the plinth, together with the superior limit of the rectangle of the two large squares, determine the constructive finish of the spire and the top of the finial respectively. The construction of the spire falls within one large square and the construction of the tower itself within the other.

A glance at the side elevation shows that this also is designed within the two large squares on the length of the transept.

From the fact that there are two large squares in the height, as there are two large squares in the length of the design for the side elevation with the tower, the whole church,

* Max Hasak, "Kirchenbau des Mittelalters," *Handbuch der Architektur*, iv, 3, p. 278, etc., 2nd edition (Leipzig, 1913).

like Notre-Dame of Paris, falls within a single large square. This large square contains then four squares on the length of the transept.

The diagonals of 45° drawn in the side elevation of the nave, marked by a small red ring, show that the height of the wall of the nave is determined *ad quadratum* on three double bays and the height of the ridge on four double bays, reckoned from the top of the plinth, while the diagonals of 63° 26′, drawn in the side elevation of the chancel, marked by two red rings, show that the square on four of the shorter double bays of the chancel does not reach from the ridge farther down than to the string-course under the windows of the aisle.

As the church is as high as the front is wide, it follows that the side elevation with the roof to the end of the ridge is designed just as the plan itself to the position of the high altar, viz. within two large squares on the width of the front. This proportion is shown by the slanting diagonal of 63° 26′ from the point marked by a small red ring on the plinth line under the buttress of the west front to the mentioned point on the ridge line. The steep diagonal at the same angle from here down to the line over the plinth shows that the elevation of the ambulatory is designed within a half-square.

The diagonal from the extreme east point of the plinth line to the point marked by a small ring at the top of the parapet and the slanting diagonal from here down to the point mentioned under the west elevation show three squares on the external width of the church; these are the same conditions as in the plan.

It will be noticed, furthermore, that we have indicated how the architectural and decorative distributions are produced by the system, as by means of it we have placed and drawn the windows of the aisle, of the triforium, and of the clerestory.

The spire drawn over the crossing differs in height from the real spire which was built in the last century having a height of 109.m.8, reckoned probably from the ground and not from the plinth, in analogy perhaps with French churches, not according therefore to a conscious *reason*, or any principle. According to the system, the height, as far as the small plan can give it, should be 111.m.5 to 112 metres above the plinth, and therefore it expresses a certain part of the church, just as the west tower is the expression of the entire church. The diagonal of 63° 26′ drawn in strokes from the summit of the spire downwards on both sides shows consequently that the height is fixed *ad quadratum* on the distance between the axis of the east wall of the west towers and the axis of the wall of the ambulatory —therefore the real length of the church. The diagonals of 45° downwards on both sides from the summit show, furthermore, that the latter is situated *ad quadratum* over the parapet, on the length, from the central axis of the church to its extreme east limit, and from the central axis to the axes of the towers, just as the slanting diagonal, according to 63° 26′ from the summit, shows its correspondence with the tower itself towards west exactly at the level of the base of the spires.

In connection with this phenomena we must, finally, notice the pentagon. This is now drawn in the plan also. It will be seen here how its diagonals, the length of which equals the side of the large square of the design, form pentagons and pentagrams towards and around the high altar. We have already pointed out the same play taking place between the towers, similar to the fact which called our attention there: that not only did the base of the pentagon of the third order go through the summit of the spire over the crossing, but that the lengthening of the diagonal, or the two steep sides of the pentagram in this pentagon, corresponded with the axes of the towers in their intersecting point with the line of level of the real base of the spires.

Now that we have had an opportunity of drawing the church in its entire length and shown in that way also that the height of the spire over the crossing is the result of the

geometrical system of the church, it may be of interest to underline that we have found here, in the drawing of the front and by means of the pentagram, the same correspondence between the summit of the spire over the crossing and the base of the spires of the west towers which we found in the side elevation through the slanting diagonal of 63° 26′.

It might be of further interest to indicate the extremities of the lengthened sides of the pentagram on the spire above the crossing in the side elevation: as shown by the projection below on the plan, the one on the right ends in the middle of the large west square of the design, and the one on the left ends at a corresponding point in the east square, just at the very place of the host on the high altar.

We find, then, an inner continuity everywhere; there is no conflict because the thought of unity has been carried through all the plans, more consequently than in any other building of the thirteenth century. It has been our good fortune to realise the thought of the medieval architect when he designed his plan—which the nineteenth century was not able to understand —through our rediscovery of the system of the religious architecture of the past.

Fig. 79 shows Cologne cathedral in its corrected state, as it would have been if carried out in the Middle Ages, and not as it is now through a misunderstanding and through "analogies." We see the front stripped of its exaggerated decoration; it appears elegant and quiet, each part grows out of another, just as in the side elevation, which is perfect in its proportion and in this respect, therefore, not surpassed by any cathedral of the five-aisled type.

Cologne cathedral is designed by a great thinker. It seems obvious to pick out the most learned citizen of Cologne at that time, the scholar *Albertus Magnus*. The medieval art of building cathedrals is based on Greek thought; but as regards the logical carrying out of intrinsic unity and its resulting organic continuity, the cathedral of Cologne is, more than any other, related to the Parthenon, whereof we shall convince ourselves later on.

We have now made a special analysis of two typical cathedrals: Notre-Dame of Paris from 1160–1220 and Cologne cathedral from 1248 onwards. We have found that they are both the result of the same form of thought expressed in a primitive manner in the description of the temple of Solomon in the Bible. We found it again in Vitruvius and in the oldest churches, and we have been able to follow its development from the three-aisled church until, in the thirteenth century, the best time of Gothic art, it was realised in the cathedral of Cologne, to its extreme limit, without a flaw.

We have given here an exhaustive proof that the principle *ad quadratum* is the base of the art of cathedral building on the Continent.

We turn now to the conditions in England.

CHAPTER VI

ENGLAND

THE CATHEDRALS OF LINCOLN, SALISBURY, WELLS, YORK, AND WESTMINSTER ABBEY

(1) The Cathedral of Lincoln

IN the previous demonstration and exhaustive analysis of two typical churches, the cathedrals of Paris and Cologne, we have explained the principle of cathedral building so clearly that we consider sufficient now to point out the main features only of English cathedrals in order to prove that these are built according to the same method as those of the Continent.

Fig. 80 gives the plan of the cathedral of Lincoln, which, in its present state, is supposed by English archæologists to date from 1190–1235, 1256–1320.

The diagonals *e—f—i* show the original Gothic plan of the church from about 1256, when, by extension towards east, it obtained its present form, as shown by the diagonals *e—g—h*.

Both before and after the extension, the design of the plan was kept within two large

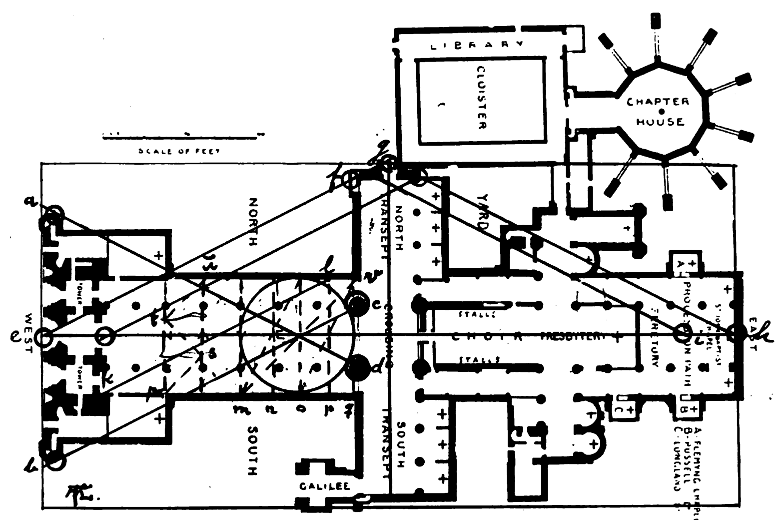


Fig. 80.—Plan of the cathedral of Lincoln, analysis.

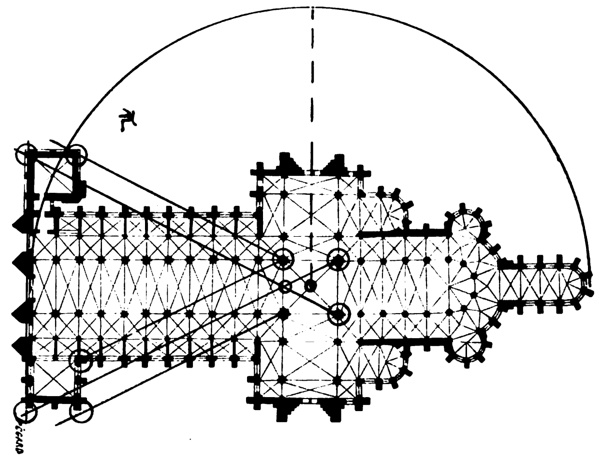


Fig. 81.—Plan of the cathedral of Rouen, analysis.

squares on the length of the transept. Before the extension, the line of division was placed in the west wall of the transept, but *after* it was situated perfectly according to the system, in the centre line through the central tower, according to the rule of West European cruciform churches.

We find, therefore, that the placing of the front towards west is fixed *ad quadratum* on the length of the transept.

The front, which is an addition of the first half of the twelfth century, and is placed like a mask outside an older front, projects, as it will be seen, with its great width, outside the aisle. Nor is this width chosen accidentally, but it is only a logical use of the diagonal 63° 26′, as we have seen it used everywhere else on the Continent, to fix the position of the vital points.

It will be seen that the diagonals after this angle, starting from the axes of the two west pillars under the central tower, marked respectively *c—b* and *d—a*, cut the part *a—b* in the west side of the square of the design. The front finds its width within this part. The south corner of the front is therefore determined by the diagonal coming from the north-west base of the tower, and the north corner by the diagonal from the south-west base of the tower.

The proof that this is no accident is furnished by the cathedrals of Wells and of Nidaros, where the width of the front is determined in a similar way; this will be shown in its proper place.

As a third example we may also add the cathedral of Rouen (fig. 81). The placing of the front towards west is here also determined *ad quadratum* on the length of the transept; but reckoned from the west wall; the north-west corner of the front is determined by a diagonal from the south-east, and the south-west corner by a diagonal from the north-east base of the tower. In this way the front has become correspondingly broader and extends even beyond the transept.

We are not able to give more examples, as, besides the four cathedrals mentioned, no other church is preserved having a front projecting beyond the aisles.

As the width of the front is determined in all four cases similarly, we have, however, a valid proof that this is not arbitrary. English churches are often badly proportioned. They are also built *ad quadratum,* but their architects in the Middle Ages do not seem to have understood quite fully the real meaning of the theory of the principle, its real function, which is to create unity throughout all parts of the building, and that this unity can only be obtained by being carried out in all the plans, the horizontal and vertical, transversally and longitudinally.

Lincoln cathedral is in this respect a typical example.

We have seen that, as a whole, it is designed within the two large squares, but already in the proportioning of the nave, the principle is broken, this being made somewhat larger than the sum of the two squares on the inner width of the church, as shown by the diagonal *k—l*. The effect of this fault could easily have been neutralised by a compromise

in the placing of the columns; but another fault is made just here, which is disastrous to architectural harmony.

It will be remembered from the analysis of the five-aisled cathedrals of Paris and of Cologne, that in these, harmony was obtained by a logical division of the square into six, in all three plans. There are therefore six bays within the square in the length, as there are the length of six bays in the height of the square, on the width of the church.

It is true that we have not had any opportunity yet to undertake a special examination of a three-aisled church, but we have already explained that it is produced by dividing the square into four. We have already seen that in the nave of the cathedral of Amiens, which is three-aisled, there are four bays within the square on the inner width of the nave. Moreover, in the church of Vezelay (fig. 20, p. 13) we saw an example of the division of the square into four, longitudinally, as there are two bays within the half-square and consequently four squares on the length of a single bay in the height of the whole wall.

In the plan of the cathedral of Lincoln (fig. 80) we have marked the square on the inner width of the church, by the circle introduced, and indicated it furthermore by the diagonal of 45°, *m—v*. By dividing this square into four, we have indicated the distance between the axes of the pillars as it should have been according to the regular placing; the four bays are marked *m—n*, *n—o*, *o—p*, and *p—q*. But, as will be seen, there are only three bays within the square. This arrangement of three bays in the square is not, however, carried out in the whole nave; the two columns farthest west are placed according to the rule, but the others have too short intervals. The result is, that the regular division into four, as shown by the inscribed circle, has been used transversally, but could not be realised in the raising of the wall in height. The height of the wall itself and the transverse section as a whole have, of course, been able to *ascendere ad quadratum*, as shown in fig. 82, but within this given height, there are only three squares on the length of every bay, instead of four. This is demonstrated in the plan by the diagonals *r—s—t—u*. It has not been possible to realise the raising of the bays in height, according to the principle which governs the transverse section, as the great distance between the pillars demands a larger square and therefore higher walls than the square on the width of the plan has been able to allow. Two dissimilar factors are therefore introduced: a square divided into four transversally and one divided into three longitudinally. The cathedral of Lincoln has thereby become a provincial deformity, architecturally speaking.

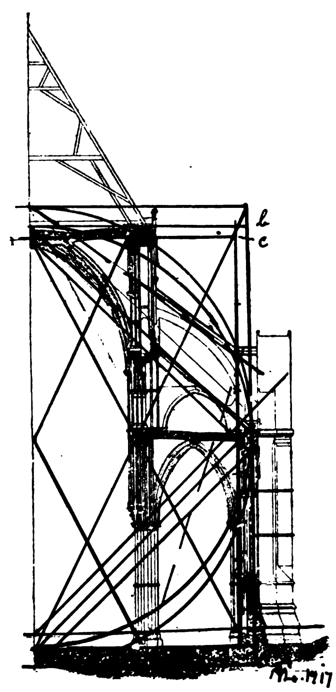


Fig. 82.—Transverse section of the cathedral of Lincoln, analysis.

Fig. 83 convinces us on this point. The architectural distribution is determined *ad quadratum*, but instead of a rectangle of two squares, each double bay contains only one and a half square in height. The proportion is not of 1 : 2, but of 1 : 1½, and in every single bay of 1 : 3 instead of 1 : 4, according to the principle of a three-aisled church.

This painful misproportion is still more pronounced in the retrochoir (fig. 84). The rich architectural distribution is here also logically fixed *ad quadratum*, as in Cologne, but the composition lacks equality, because it is robbed of a quarter of its natural development, and, through it, of a

well-balanced termination. The raising is compressed, and through it not only the proportioning, but the scale also, are wrong. The raising is therefore a movement without unity of cadence, without equality, without clearness, and the rich growth of flowers rather emphasises than hides this lack of harmony.

It will also be noticed that there is no accordance in the use of the system of proportioning internally and externally; it begins internally on a level with the floor, externally above

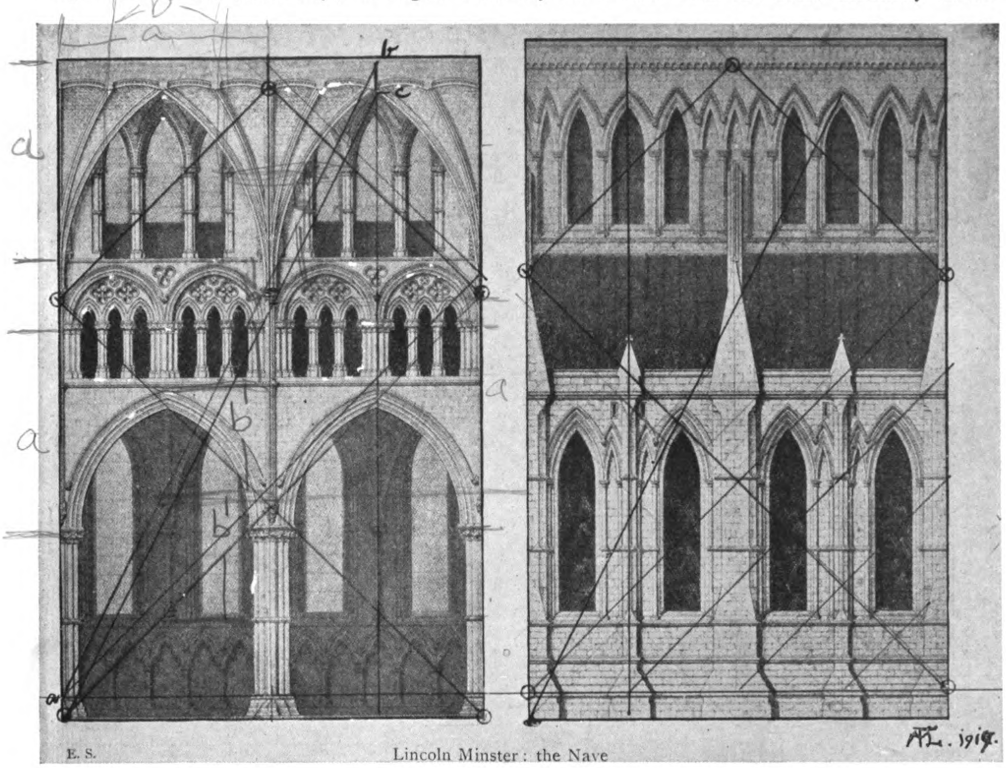


Fig. 83.—Part of longitudinal section and side elevation of the cathedral of Lincoln, the nave, analysis.

the plinth. The interior is, therefore, as shown by the longitudinal section, figs. 83 and 84, reduced in height by being lowered from level *b* to level *c*; this is also a consequence of the great interval between the columns, which would otherwise have exceeded the height limit indicated in the transverse section (see fig. 82, *b—c*).

The whole disharmony of the cathedral of Lincoln would have been avoided if the architect had understood the theory of the method, and given the columns the correct distance according to the division of the square into four, as shown in the plan by the letters *n—m—o—p—q*. He was hampered by a given data, the two west towers, and the central tower in the east remaining from Norman times, and he has not been able to free himself from these forced conditions.

The cathedral of Lincoln is no original work, the architect has not understood the prin-

ciple of its construction, and he has lacked the logic without which all art sinks down to imitative provincialism. In its design it is a provincial misunderstanding, in its decorative splendour a copy of the cathedral of Nidaros.

But, with all its magnificence, it is just an excellent illustration of the words of the French architect "Ars sine scientia nihil est," or, in other words, that flowers, foliage and decoration,

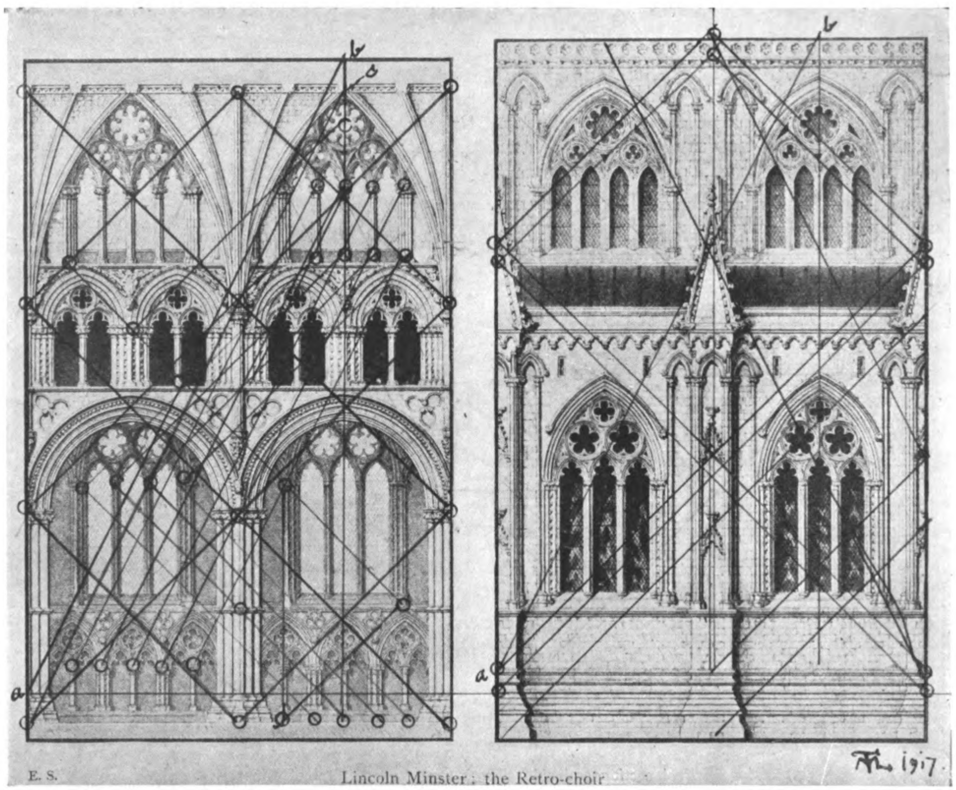


Fig. 84.—Part of longitudinal section and side elevation of the cathedral of Lincoln: the retrochoir, analysis.

and what is called free artistic feeling, are nothing if they are not guided by the firm command of the principle of harmony,—or balance and unity.

What is of chief interest to our purpose, however, is to have proved that even this irregular church is designed, or is an attempt to design, according to the principle *ad quadratum*, by an architect who only copied without having a full understanding of it.

In the cathedral of *Salisbury*, however, we shall find the principle correctly used in all the plans.

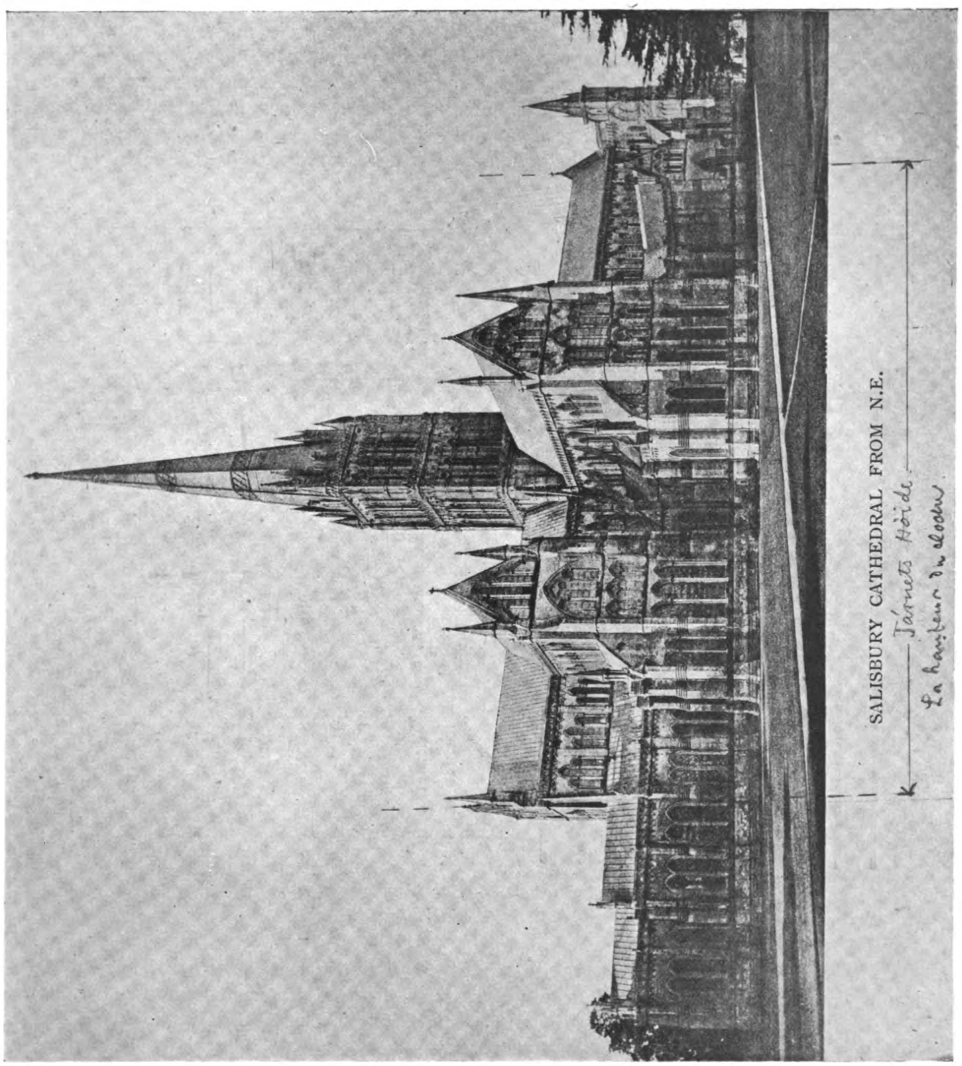


Fig. 85.—Cathedral of Salisbury, from N.E.

(2) The Cathedral of Salisbury, 1220–1225–1258

This church is superior to all others in England, on account of its unity of design, which could be carried out without any hindrance, as it was built entirely new, that is without rising from some older building. Only Beverley, and Westminster Abbey especially, as we shall see later, can equal Salisbury cathedral as regards unity and design. There is, indeed, something dry and barren in the details, but there is a virile decision, an almost Greek severity and purity in its planning, and in the whole building. We are conscious of an impressive quiet

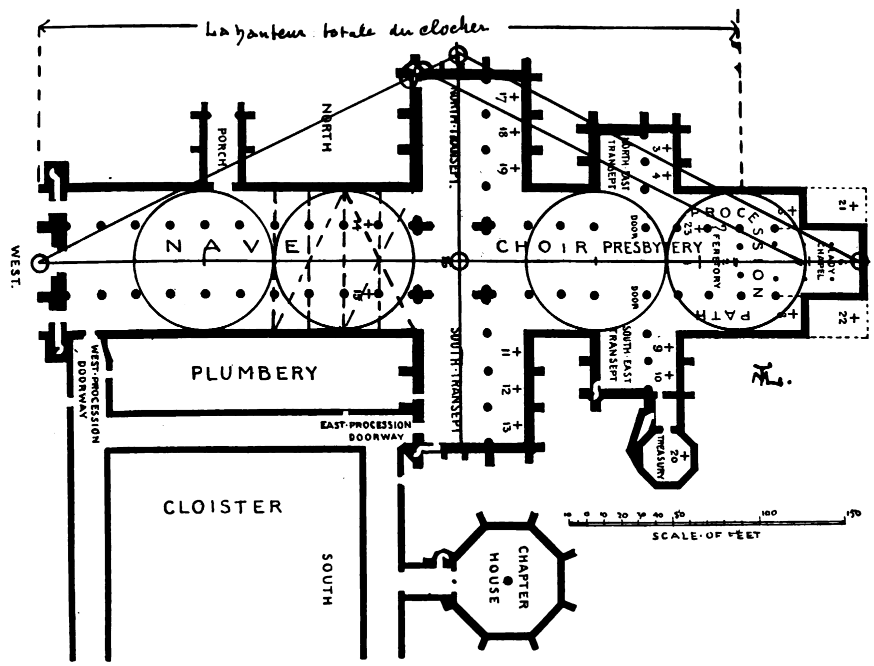


Fig. 86.—Plan of the cathedral of Salisbury, analysis.

greatness in its effect, the whole mass appears as if raised by some internal living power, culminating in the towering spire. The same greatness prevails also in the interior (fig. 85).

We see that it is designed within the two large squares of the cruciform church, on both sides of the centre line of the church transversally—as we saw the cathedrals shape themselves in France and England, and as already mentioned, in Nidaros at the end of the eleventh century. The nave is here also longer than twice its width, but, as compared with Lincoln cathedral, its plan is designed perfectly according to the principle of the three-aisled church. The square is divided into four for the placing of the columns as can be seen by the introduced dotted lines in the nave. Consequently four bays go into the square on the

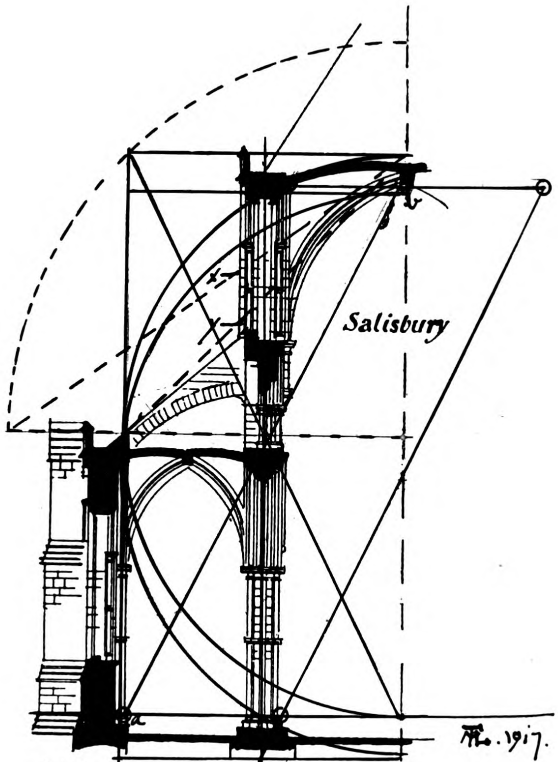


Fig. 87.—Transverse section of the same, analysis.

Fig. 88.—A bay of the longitudinal elevation of the nave, do., analysis.

width according to which both the transverse section (fig. 87) and the side elevation are built (fig. 88).

The cathedral of Salisbury is fully realised *ad quadratum*.

(3) The Cathedral of Wells, 1180–1242

This cathedral is a variant of the cruciform type of church of West Europe. As shown by the diagonals *c—d* and *d—e*, drawn in the plan (fig. 89), the central tower is situated in the middle; but the transept is considerably shorter than half the length of the church. As it is three-aisled, however, and as, in its width, it extends farther towards west and east than with a one or two-aisled transept, the length of the church could nevertheless be determined according to the rules, because the surface of the wall of the west front is placed exactly *ad quadratum* on the length of the transept, while the position of the extreme east altar in the Lady

Chapel is determined in a similar way, as indicated by the diagonals *c—f* and *g—e*. We have the obligatory two squares in the nave, west of the transept, and in the chancel the two squares east of it, as shown by the diagonals. It is also deserving of notice that the position of the main altar (marked by two small rings) is determined *ad quadratum* on the length of the transept through the west wall of the central tower just as the east wall (marked by point *a*) of the west towers is placed in a similar manner.

Finally we find that the width of the front is determined by the diagonals from the axes of the pillars of the central tower, just as in Lincoln and Rouen.

The main lines or the frame of the somewhat different plan of the cathedral of Wells is, therefore, also designed according to the continental rule, *ad quadratum*; but the architect of this cathedral does not seem to have understood either that the principle had to be carried

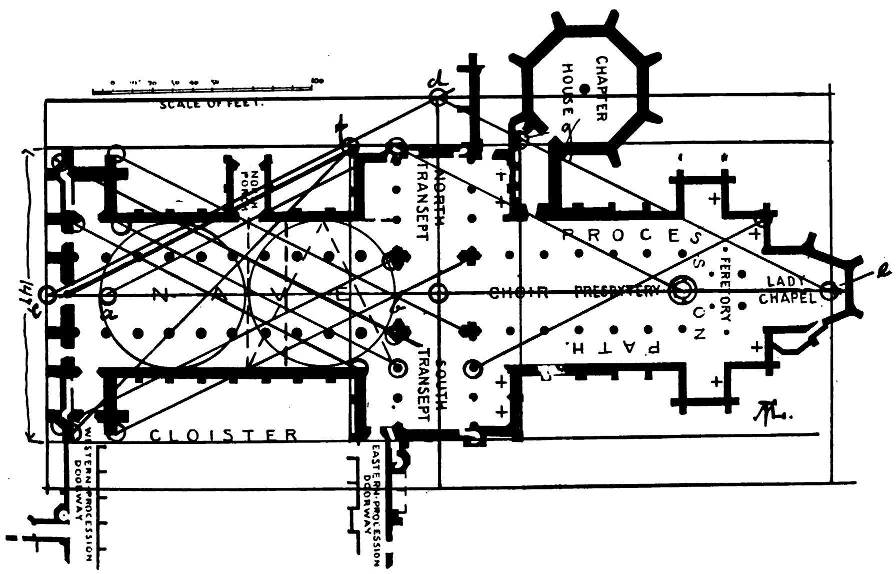


Fig. 89.—Plan of the cathedral of Wells, analysis.

out equally within the main lines, because, in the placing of the pillars, he has committed the fatal error of breaking away from the principle of the three-aisled church—the division of the square into four. As shown by the circles drawn in the nave, the square is regularly divided transversally, as two-fourths of it fall on the nave and one-fourth on each of the aisles. In the longitudinal direction, the square is, however, divided into four and a half parts. There are then four and a half bays within each square instead of four, the opposite of the fault made in Lincoln. We saw there how the architect had lost within the given height *ad quadratum* on the width of the church, one quarter of the height necessary to the harmony, in proportioning each single bay. This harmonic height could not be realised without heightening the walls of the nave, and, thereby, exceeding the main frame. The height of the bays in proportion to the width became 1 : 3 instead of 1 : 4.

At Wells he has created the opposite condition. Here was an opportunity of realising the proportioning of the bays within the given height of the nave *ad quadratum* on the sum

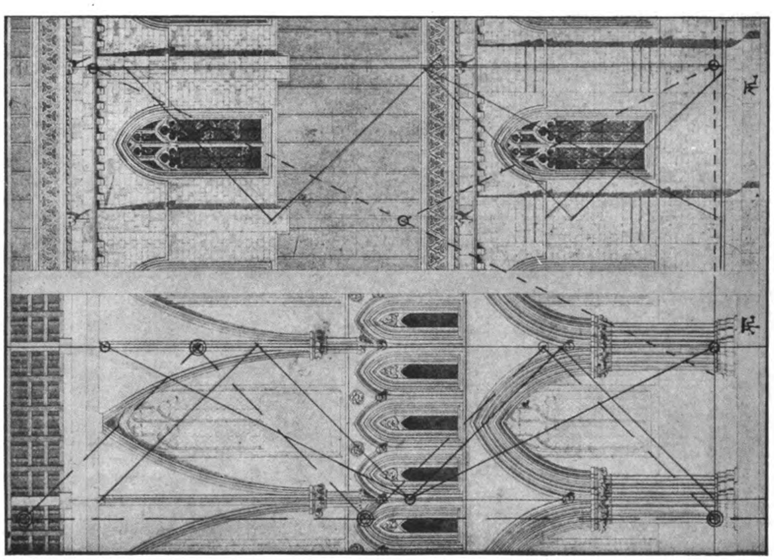


Fig. 90.—A bay of the nave inside and outside, do., analysis.

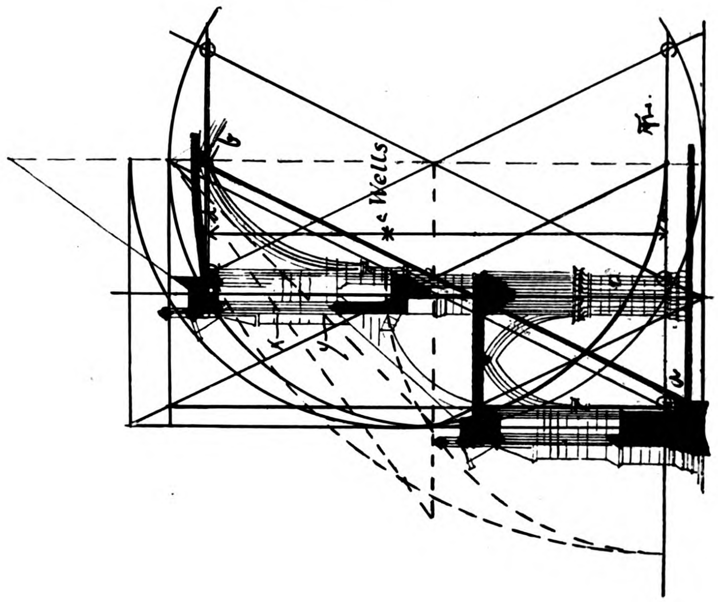


Fig. 91.—Tranverse section of the same, analysis.

of the length of four and a half bays or the width of the church according to the same proportion which had divided the square in length, that is, 1 : 4.5; but suddenly he remembered the rule which is that the dividing of the three-aisled church into four has also to be done in height as well, and to each bay he gave the proportion 1 : 4 as shown by the fully drawn diagonals in fig. 90. But thereby the height of the wall of the nave to the vault is determined *ad quadratum* on four bays, and not determined according to the square on the width of the church, which in the plan contains four and a half bays. The height of the wall of the nave to the vault has thus become half a bay lower than it should have been if the square on the length had been divided into four. This fact, which is demonstrated in fig. 90 by the diagonals of 45° traced in dotted lines and furthermore marked by two small rings, has obliged the architect to lower the square of the design of the interior, down to the line of the floor, as shown by the diagonal $a-b$ of the half-square in the transverse section (fig. 91).

In the meantime these deviations have for us a secondary interest only compared with the fact shown in our analysis, that Wells cathedral also is constructed according to the continental method *ad quadratum*.

In order to complete our examination, we give in fig. 92 an analysis of the transept and central tower. It will be seen that the transept, in its length, is designed within two squares, one on either side of the central axis of the tower, just as the height of the tower, to the main cornice, is determined *ad quadratum* on the length of the transept. The height of the small corner towers also is determined according to the geometrical auxiliary system. The same refers to the front (fig. 93). In the absence of an orthogonal drawing of the elevation, we had to be satisfied with a photograph of it. This is taken straight in the middle of the front, and therefore it can be used to show that this is constructed *ad quadratum*.

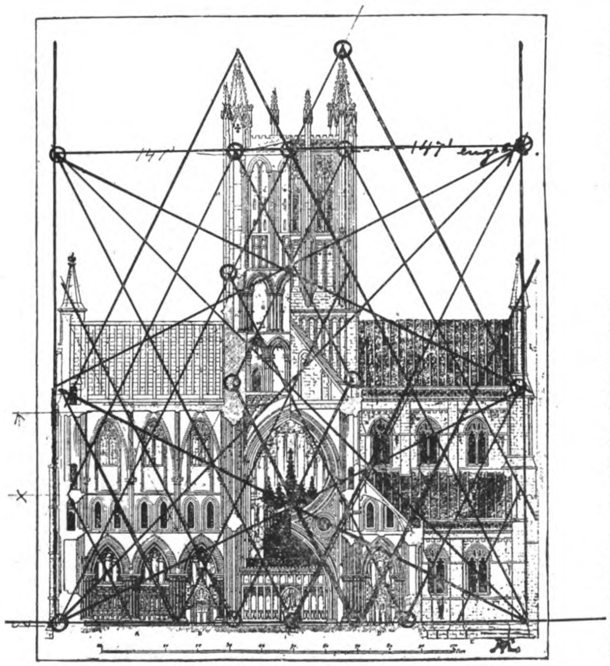


Fig. 92.—Transept and central tower of the cathedral of Wells, analysis.

(4) The Cathedral of York

We have already shown, in the general examination, that this is equally the case with the cathedral of York (fig. 32), as regards architectural distribution. It will also be seen, from fig. 94, that this applies to the decorative distribution of the details. The front of the cathedral of York, which was completed in 1338, and therefore considerably later than the drawing of the front at Cologne, is decorated entirely according to the same principle, the same rule, as the decoration of the elevation of Cologne Cathedral. In York there

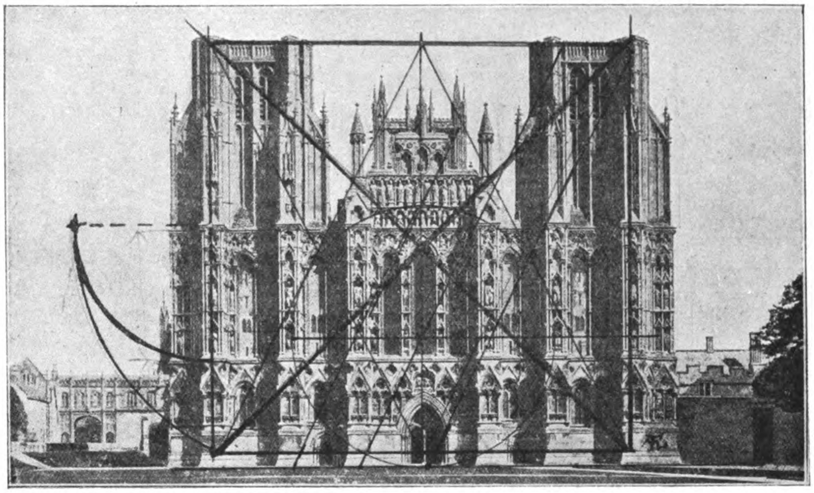


Fig. 93—West front of the same, analysis.

is the same relationship between the decorative main points, although not in such a perfect way as in the other.

After the detailed examination of the drawing of the elevation of Cologne cathedral, we consider it sufficient to point out the apparent relationship between these two fronts, as regards the manner in which the architectural decoration is distributed and carried out.

(5) Westminster Abbey

Our analysis of the art of church building has fixed, as an irrefutable fact, that the regular church types, that is, the five and the three-aisled churches, are the result of the division of the square into six and into four, and that each one has its genuine harmonic proportion as a result of this division.

Besides these two regular types there exists, however, a third to which, for practical reasons, we can give the name of *Westminster type*. It is really a hybrid creation, being produced by a combination of the regular five-aisled and the regular three-aisled church.

This hybrid church originated in France in the twelfth century. As examples, we will give the cathedrals of Noyon, Chartres, Amiens, and Reims. These churches, up to the transept, have a five-aisled chancel, but a three-aisled transept and nave.

The genesis of the new type is given in its history. We find it first in Noyon and in Chartres, where, without heightening the existing three-aisled nave after its raising *ad quadratum*, they built a new five-aisled chancel, which in its height was fixed by the older church.* The other case is where a new church has been built with a five-aisled chancel with the genuine raising of the five-aisled church *ad quadratum*, but where the transept and the nave, from economic considerations, have been built as in a three-aisled church, according however to the height of the five-aisled chancel.

* When the church was restored in the thirteenth century, the vault was built *ad quadratum* on the average width of the chancel and of the nave, viz. 45 m. 25 + 31 m. : 2 = 38 m. 3.

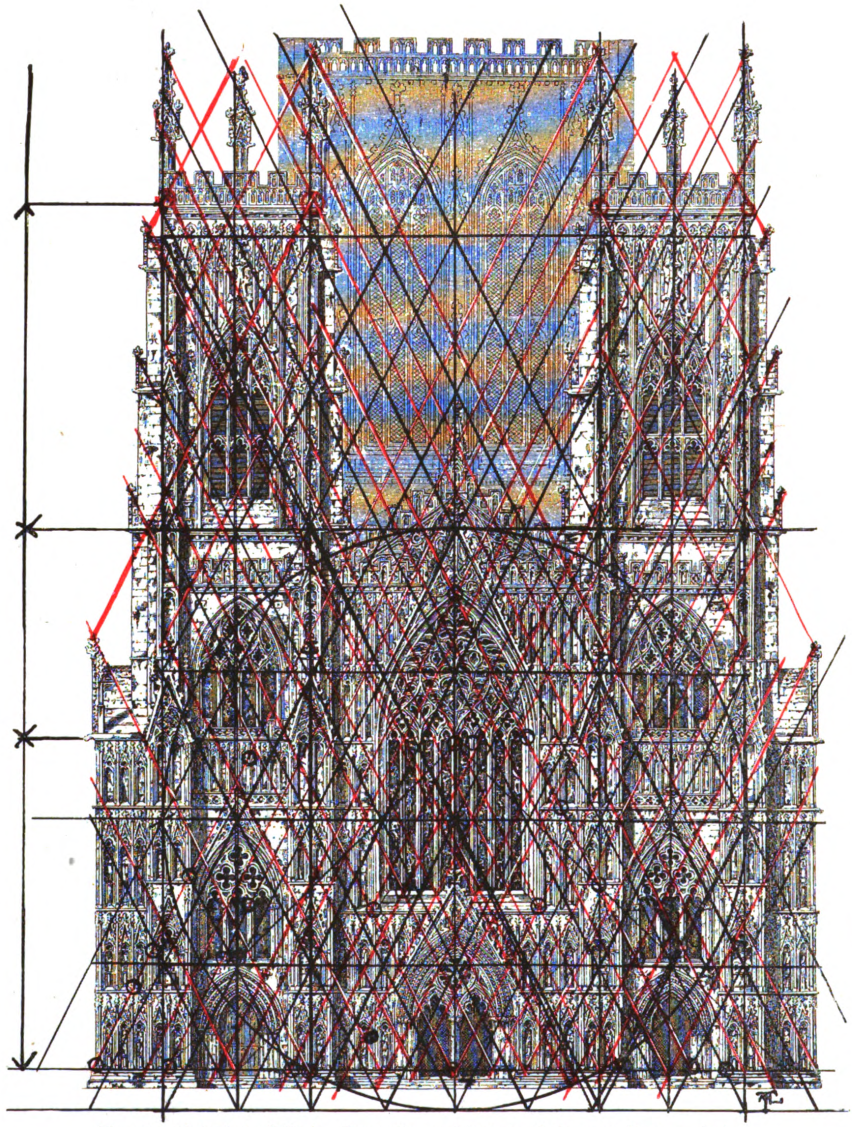


Fig. 94.—Cathedral of York. Decorative architecture of the west front, analysis.

We regret to say that we lack material to give examples, but we have mentioned the case as a necessary feature of the theoretical sequence.

It is evident that by such a mixture one of the components will have to suffer and to give up its own natural and genuine proportioning.

The consciously scientific architects of the Middle Ages must, of course, have noticed this at once.

It is an interesting proof of this consciousness of art that, through an entirely new planning, they tried to avoid the conflict which arises in the mixing of the two regular types, by making mutual concessions.

The cathedrals of Amiens and of Reims are examples of this. Both these churches have been planned as mixed types, with a five-aisled chancel and a three-aisled transept and nave.

We have explained how the problem was solved, by determining the height of the vault *ad quadratum*, according to an average of the different breadths of the church, the three and five-aisled parts. We have pointed out similarly that the length of the bays was a compromise in order to realise the raising of the walls of the nave *ad quadratum* on the determined average height of these walls.

Here we have a proof that the cathedrals of Reims and of Amiens were planned with a five-aisled chancel and a three-aisled nave, and it gives us, at the same time, the indisputable evidence of a fully conscious art which, even in a *chosen* accommodation, in a calculated compromise, does not allow any caprice and free feeling, "aus dem Tiefen des Bewustseins," "from the depth of subconsciousness," but was led by the harmony-creating power of geometry, as a result of philosophic and artistic consciousness.

This hybrid form consisting of a three-aisled and a five-aisled church, by which the former had to give up its natural proportioning—as in Amiens and Reims—was imitated.

England possesses, as far as we know, no five-aisled church, but in Westminster Abbey, founded in 1245, it possesses a three-aisled one, which, as far as concerns proportioning, is in every respect an imitation of the five-aisled.

On account of the fact that no one has so far succeeded in finding the abstract principle out of which the different types of churches have risen, French, German, and English archæologists have all taken Westminster Abbey to be a French type, being misled by its great height and its resulting proportions, which they believed to have been produced exclusively out of preference and free choice, instead of being the result of geometrical functions following an esthetic and philosophic requirement of unity in construction.

Ecclesiastical archæology has been a turmoil of commonplace talk, where, without making themselves ridiculous, people discoursing on art could hold forth and "make discoveries" by means of big words and sentimental phrases.

The great talkers on art in France, Germany, and England have not been able to "discover" anything more interesting in Westminster Abbey than that it is a direct copy of the cathedral of Reims, which it is least of all.

We proved above that Reims cathedral represents a conscious compromise in the mixing of the two regular church types. In Westminster we find a three-aisled church in which is employed the height-proportion of a five-aisled one. *Westminster Abbey* could much rather be considered an imitation of Bourges cathedral, or of Cologne, if this had not been of a later date.

The case is, however, that none of these churches are copies. Reims and Amiens are each individually beautiful evidences of a superior mastery of the principle of church building, in the manner which was employed to unite these two types.

And in the same way, Cologne cathedral in Germany is not a copy, but a fully independent and logical product of the idea of the five-aisled church. Nor is Westminster Abbey an

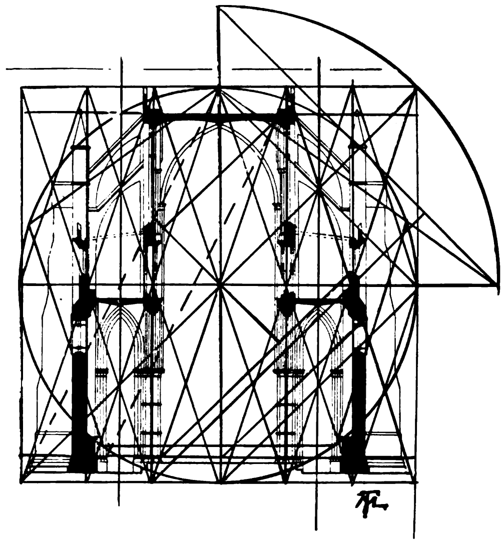


Fig. 95.—Transverse section of Westminster Abbey, analysis.

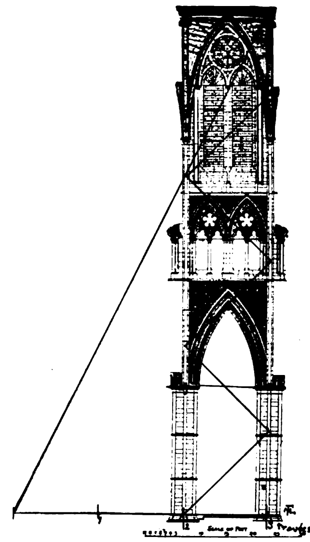


Fig. 96.—A bay of the chancel, the same, analysis.

imitation of any single church; it is indeed a hybrid, but it must be considered as a fully independent attempt to produce the heaven-reaching grandeur of the five-aisled church in a three-aisled one, and, seen from the transept, where one does not notice that it is out of proportion—that is to say, far too narrow in relation to the height—this effect is attained. Besides the cathedrals of Cologne and Amiens, there is hardly any church which possesses a more imposing interior than Westminster Abbey.

In the transverse section (fig. 95) it will be seen how this three-aisled church is built according to the system of a five-aisled one, through the division of the square into six. The drawing needs no explanation.

The case is the same in fig. 96, in which we see how the side-wall is built according to the same principle.

The diagonal of the half-square according to 63° 26′, as well as the diagonals of 45°, show that it is raised *ad quadratum* on the length of six bays. This square is thus divided into six, and, as there are twelve bays in the nave, the longitudinal wall in its totality is designed within two squares.

Westminster Abbey, the cathedral of Salisbury, and probably Beverley Minster are the best designed churches in England; but Westminster Abbey, and with it probably Beverley Minster, are the only ones which are correctly constructed—that is to say, Gothic.

We have mentioned earlier that English architects do not seem to have understood fully the principle *ad quadratum*, and the static formula according to which the Gothic churches had to be constructed.

The main advantage of the Gothic style is not only its pointed arch or its sharp mouldings under a splendid efflorescence, but it consists in the fact that it is built according to scientific constructive principles so as to obtain the greatest possible effect with the minimum of effort.

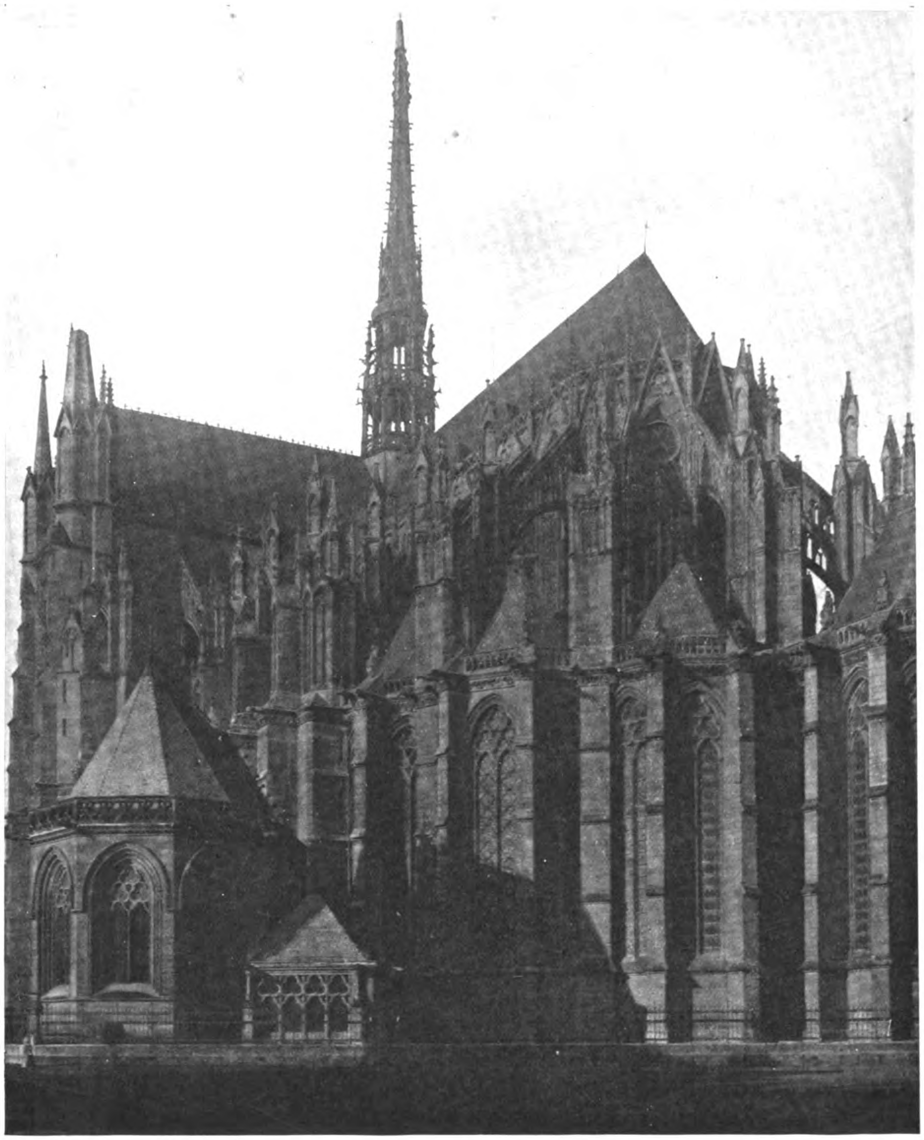


Fig. 97.—Cathedral of Amiens seen from S.E.

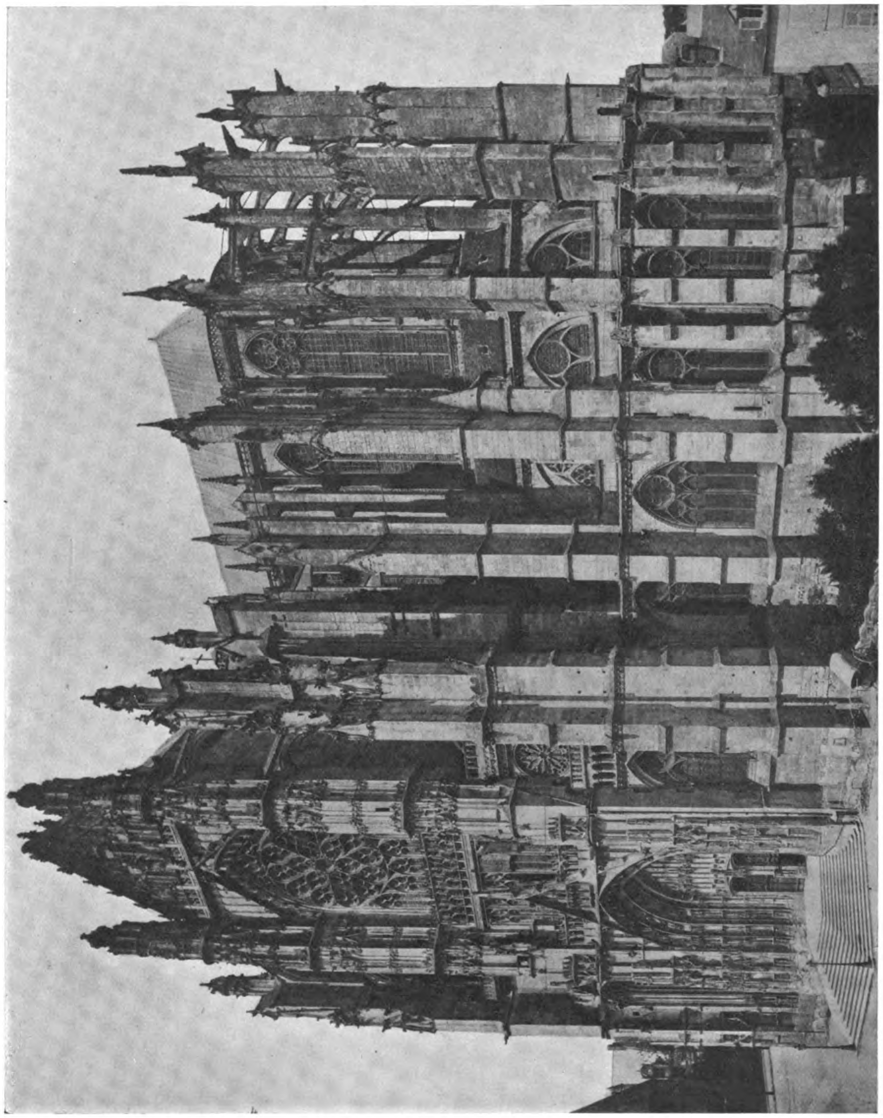


Fig. 98.—Cathedral of Beauvais seen from S.E.

Norman churches lavished stones in their massive pillars and walls. The Gothic style made everything thinner by collecting all dynamic forces in static centres and supporting these at the right points. The whole became an intellectual system, the victory of the spirit over the weight of matter, not been surpassed even by the great art of engineering of our times. They placed their buttresses like a guard of giants, stretching out flying buttresses like strong arms around the tall house of God to guard and defend it.

The construction is so living that one can almost see the static functions of the various parts of the building and its growth. The richness of flowers is spread upon it; there is the harmony of music, it is a dream of static art and of decorative imagination where the poetical feeling of nature and the mysticism of Gothic art can revel in this intellectual marvel which Gothic art is in itself.

English Gothic style, with the exception of Westminster Abbey, lacks this wonder, because it is not constructed in a Gothic way.

Apart from Westminster, and perhaps Beverley, there is scarcely a church which has a correct system of buttresses; their vaults would have collapsed if stones had not been lavished in the thick walls.

Salisbury cathedral, which is quite correctly designed *ad quadratum*, is a characteristic example (fig. 87); the flying buttress, which should have reached against the wall between x and y, is placed too low to be of any use. It is a pure misunderstanding on the part of workmen without theory. The case is the same in Wells cathedral (fig. 91). In both of these, stones were lavished in the thick walls just as in the Norman churches.

But it has also happened that churches *have* collapsed.

A good illustrative example of this lack of science in English Gothic is the already-mentioned cathedral of Ely (fig. 41). The system of buttresses is absolutely useless.

What most Gothic churches in England lack, therefore, is just the spirit of Gothic art, its constructiveness. They are really practically only Norman churches in which the builders have put pointed arches, and which they have clothed in the outer garment of Gothic architecture, using its mouldings, and, with the passion for flowers of Gothic times, they adorned it with a splendid and beautifully carved ornamentation—a dream-like magnificence.

Just as in all the countries on the Continent—not only France and Germany—we see that the principle *ad quadratum* was also used in England, everywhere and without exception, both during the Norman and Gothic periods, more or less consciously and correctly at times.

CHAPTER VII

OTHER CHURCHES OF CENTRAL EUROPE

WE shall add to the above given documentation a few more examples, in order to prove that it is not only cathedrals which are built according to the method *ad quadratum.*

These examples are taken from France, Germany, Austria-Hungary and Bohemia. Besides cathedrals they also represent abbeys and parish churches from the beginning of the eleventh to the middle of the fifteenth century.

The drawings need no comment, but we shall make a few remarks on some of them.

Of the five French churches, the church of *St. Philibert of Tournus* is the oldest (fig. 99). It dates from 1009. The latest is the cathedral of Laon (fig. 103), from the end of the twelfth century, while the elevation which is given in connection with the demonstration of the *sectio aurea*, in the cathedral architecture of the Middle Ages, dates from the period between 1200 to 1220. Both the transverse section and the elevation show that the system *ad quadratum* has been used.

The church architecture of Central Europe is represented by seventeen churches, dating from the beginning of the eleventh to the middle of the fifteenth century.

In addition to our remarks in the general examination of the oldest romanesque churches of Germany, we shall point out the fact that the plan of several of these—some of which are older than the west European cruciform cathedrals—is also designed within two squares on the length of the transept. Besides the *cathedrals of Mainz and Bamberg* mentioned here (figs. 105 and 106), this applies to the cathedral of Spire (fig. 36) and to the two very ancient churches of Hildesheim (figs. 37 and 38); these three plans are older than any other in Western Europe.

It is no less interesting to notice how the elevations of these beautiful German romanesque churches are designed strictly *ad quadratum.* On account of their great age, we mention first the *cathedral of Fünfkirchen* in Hungary (fig. 117), *of Gurk* in Austria (fig. 118), both from the eleventh century, and the *cathedral of Spire* (fig. 104), from the eleventh and twelfth centuries.

Among the elevations we attract attention to figs. 115 and 120, which will both show that the arched towers have also been proportioned *ad quadratum.* The tower of the *minster of Freiburg* (fig. 115), was built in the course of two or three periods; the first square from 1270 onwards, the second from 1301 onwards, and the third towards about 1340. The difference of time is expressed clearly in the style, but unity is not destroyed, thanks to the scientific method.

The design of the open spire of Freiburg is therefore later than the design of the spire

of Cologne cathedral, but the spire itself is of its kind, the oldest in Europe. We will not lose the opportunity of quoting the remark of the Baurath Hasak concerning it:

"These Masters understood their art admirably. Who would dare, in our days, raise up to the clouds these masses of stones cut out like lace-work, in the same conditions then existing in the Middle Ages—that is to say, when preceding generations had not left anything similar behind them? Only a precise calculation of the thrust of this mass, as well as a perfect knowledge of the supporting power per cubic centimetre, could have made such a juggling trick possible."

("Es ist staunenswerth, wie jene Meister ihre Kunst beherschten. Wer würde es heutzutage wagen, diese Steinmassen wie Spitzenwerk durchbrochen bis in die Wolken zu türmen, wenn wir in der Lage des Mittelalters wären, dem frühere Geschlechter nichts derartiges hinterlassen hatten. Die genaueste Berechnung des Gewichtes dieser Steinmassen und die Kenntnis, was sie jedem Quadratzentimeter ihres Steines an Last zutrauen durften, kann allein solche Kunststücke ermöglicht haben." *)

As regards the *cathedral of Prague*, from 1244–85 (figs. 123, 124) we will mention that it is planned and partly built by the French architect *Mathias of Arras* from Avignon, but completed by the German architect *Peter Parler* from Würtemberg. Here, as in Freiburg, we have one of the numerous examples of how different architects succeeded each other without disturbing the work of the previous one in trying to be "original"; but with real artistic sense submitted themselves humbly to one and the same law.

* Hasak, *loc. cit.*, p. 238.

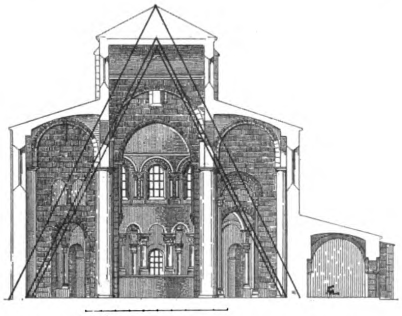


Fig. 99.—St. Philibert of Tournus, transverse section, analysis.

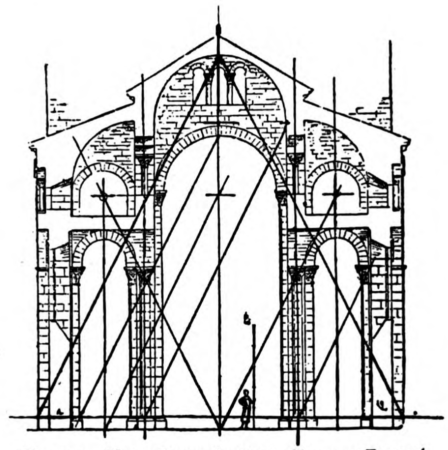


Fig. 100.—Notre-Dame du Pont, Clermont-Ferrand, transverse section, analysis.

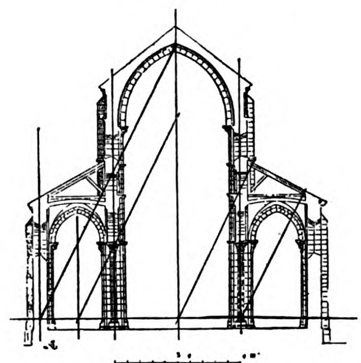


Fig. 101.—Cathedral of Autun, transverse section, analysis.

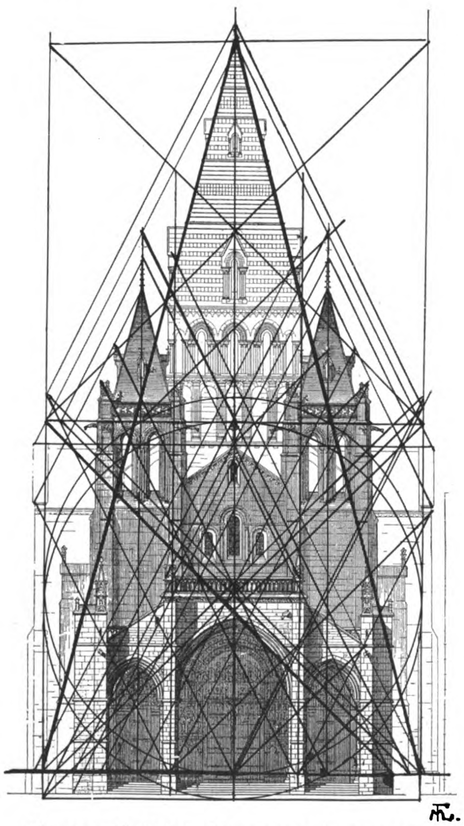


Fig 102.—Cathedral of Beaune, the front, analysis.

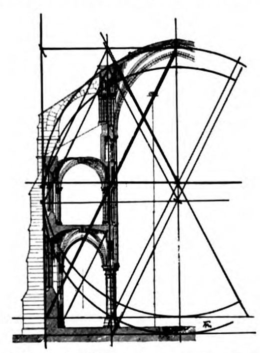


Fig. 103.—Cathedral of Laon, transverse section, analysis.

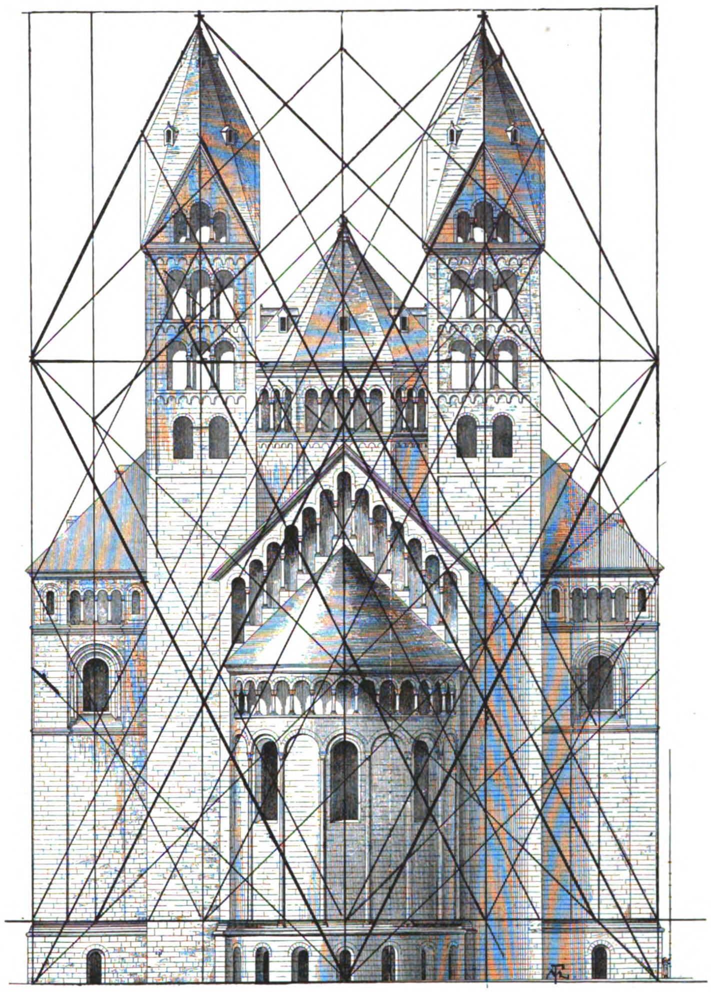


Fig. 104.—Cathedral of Spire, the east front, analysis.

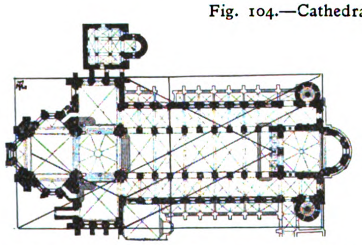


Fig. 105.—Cathedral of Mainz, the plan, analysis.

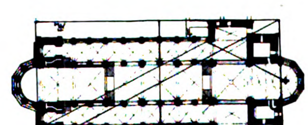


Fig. 106.—Cathedral of Bamberg, the plan, analysis.

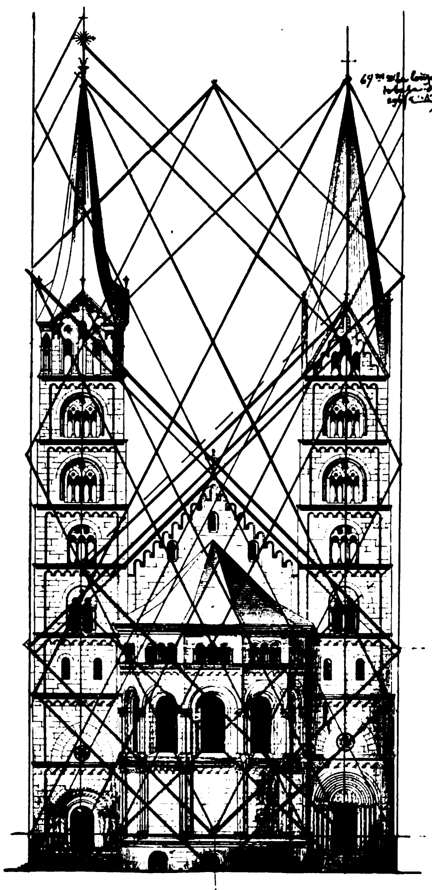


Fig. 107.—Cathedral of Bamberg. Elevation of the chancel, analysis.

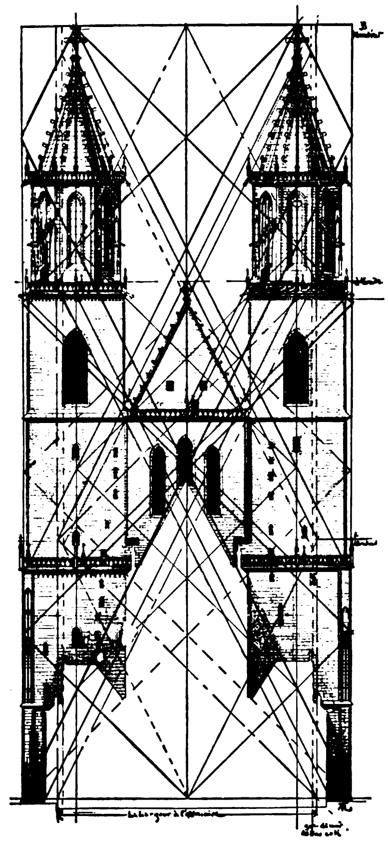


Fig. 109.—Cathedral of Magdeburg. The story of the octagon with the spire, analysis.

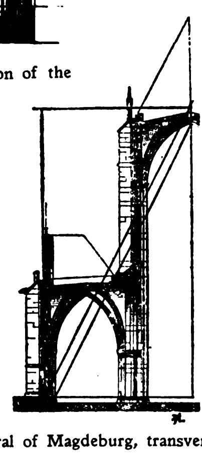


Fig. 108.—Cathedral of Magdeburg, transverse section, analysis.

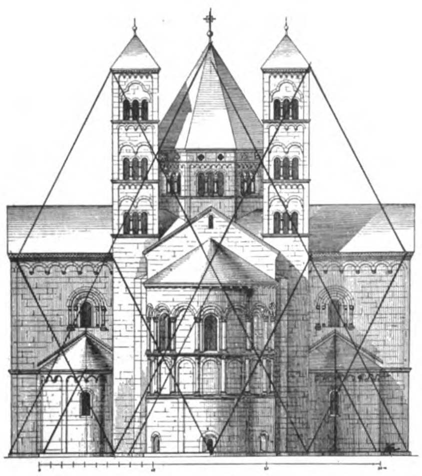


Fig. 110.—Abbey church of Laach. Elevation of the chancel, analysis.

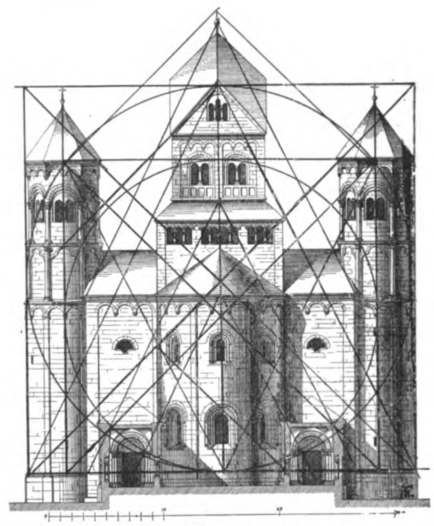


Fig. 111.—Abbey church of Laach. The west front, analysis.

Fig. 112.—Abbey church of Maulbronn. The west front, analysis.

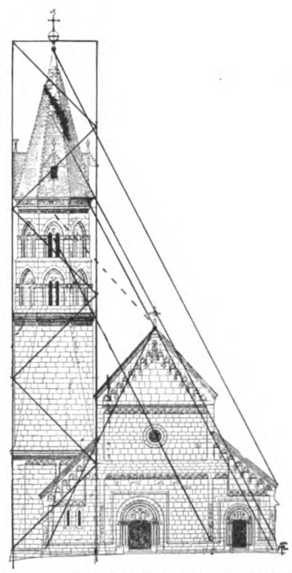


Fig. 113.—Church of St. John in Schwäbischgmünd. The west front, analysis.

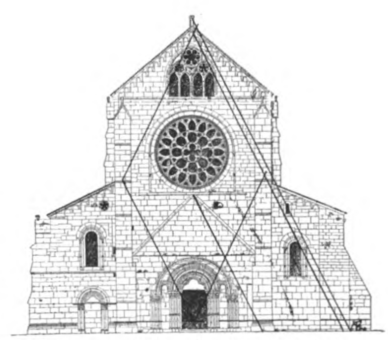


Fig. 114.—Abbey church of Otterberg, analysis.

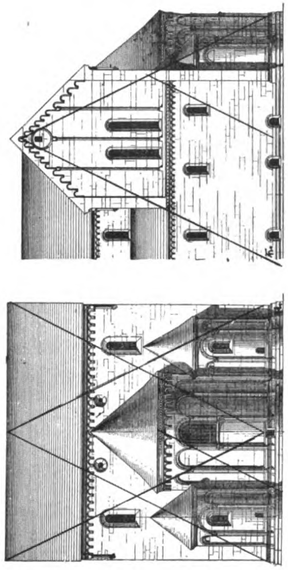


Fig. 118.—Cathedral of Gurk. Elevation of the chancel and part of south wall, analysis.

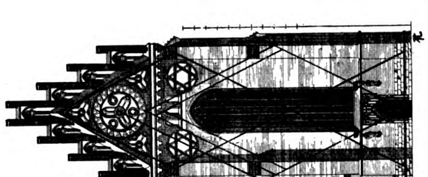


Fig. 116.—Cathedral of Stendal. Altmark. West front, analysis.

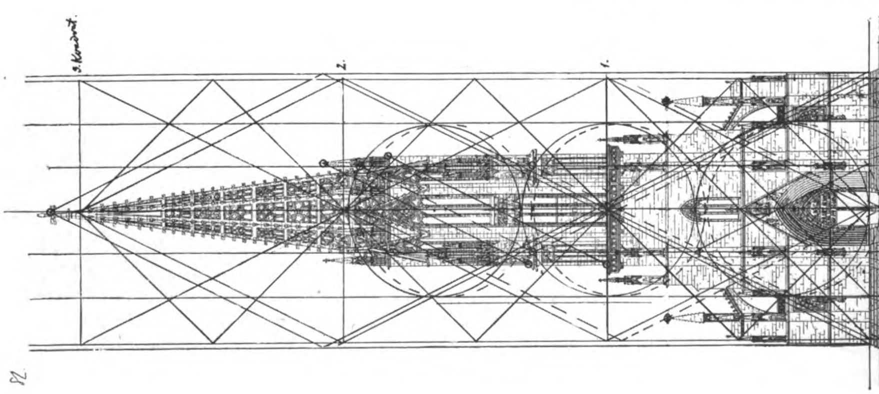


Fig. 115.—Minster of Freiburg in Baden, the west front, analysis.

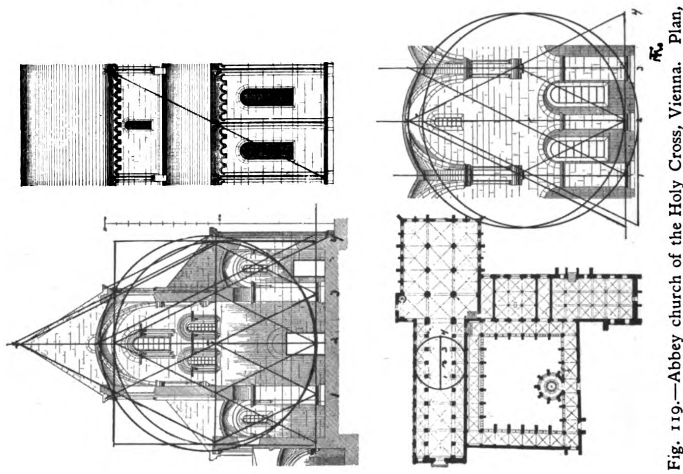


Fig. 119.—Abbey church of the Holy Cross, Vienna. Plan, transverse section, and part of longitudinal elevation inside and outside, analysis.

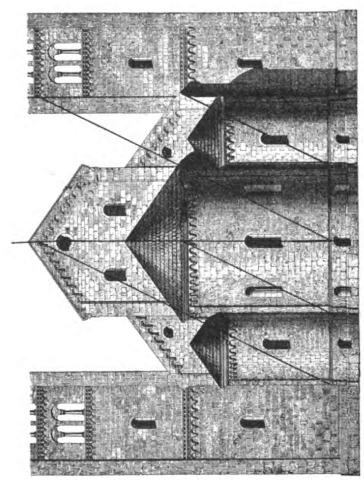


Fig. 117.—Cathedral of Fünfkirchen. East front, analysis.

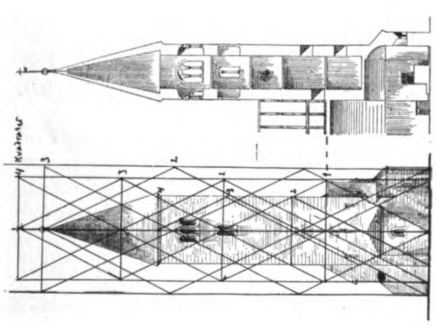


Fig. 120.—Tower of St. John, near Bosen, Tyrol analysis.

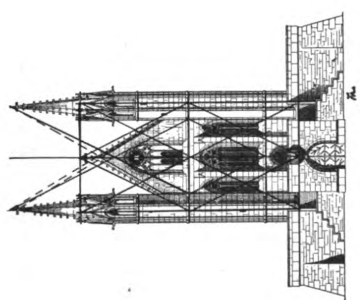


Fig. 122.—Mortuary Chapel of Zedlitz, Bohemia, analysis.

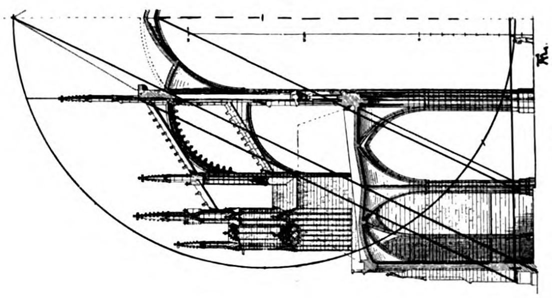


Fig. 124.—Cathedral of Prague. Transverse section, analysis.

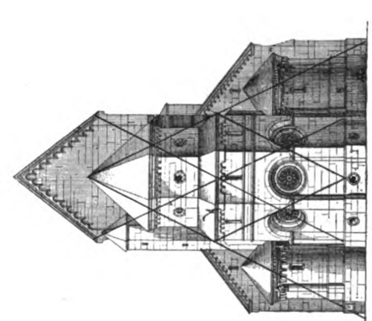


Fig. 121.—Abbey church of Trebitsch, Moravia, analysis.

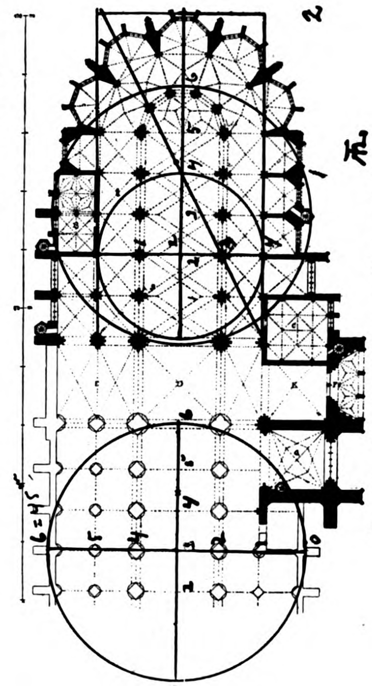


Fig. 123.—Cathedral of Prague. Plan, analysis.

CHAPTER VIII

DENMARK

WE take four specimens from the religious architecture of Denmark, which is more especially related to the German, both as regards the romanesque and the Gothic styles; they are the cathedral of Roskilde and three parish churches, one from the country and two from towns.

The cathedral of Roskilde dates from about 1220. The analysis of the side elevation (fig. 125) shows that this also is built according to the same rules which fixed the proportions of

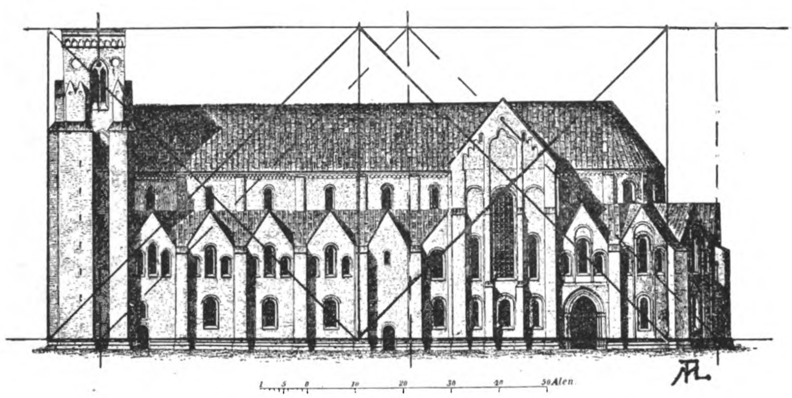


Fig. 125.—Cathedral of Roskilde, longitudinal elevation, analysis.

the cathedrals of Paris and Cologne. The distribution of the interior walls of the nave is badly proportioned; this will always be the case when there is a breach of the rule allowing in a three-aisled church the length of three bays instead of four bays in the square on the inner width (see plan, fig. 126). As it will be seen, this cathedral has an entirely French character, and forms an exception in Danish religious architecture.

Fig. 127 gives *the parish church of Gumlösa* in Skaane.* As the analysis shows, the height of the vault, the walls, the gables with the ridge, as well as the chancel, and the tower, are determined *ad quadratum* on the length of the church—that is to say, in the longitudinal section and not in the transverse section, which would be an impossibility for a one-aisled church, as it would make it a freak. With a divider, the reader will be able to satisfy himself on that point. The church was consecrated in October 1191 by Archbishop Absalon of Lund,

* Ceded to Sweden in 1658.

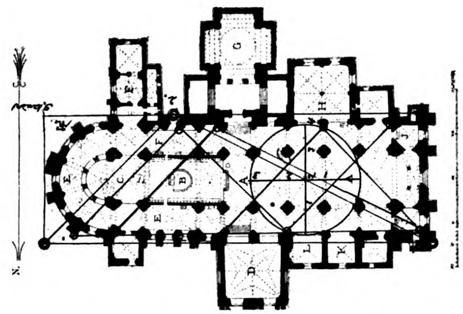


Fig. 126.—Plan of the church of Our Lady, Helsingborg, analysis.

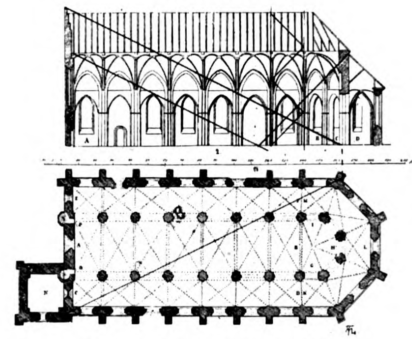


Fig. 128.—Church of Our Lady, Helsingborg. Plan and longitudinal section, analysis.

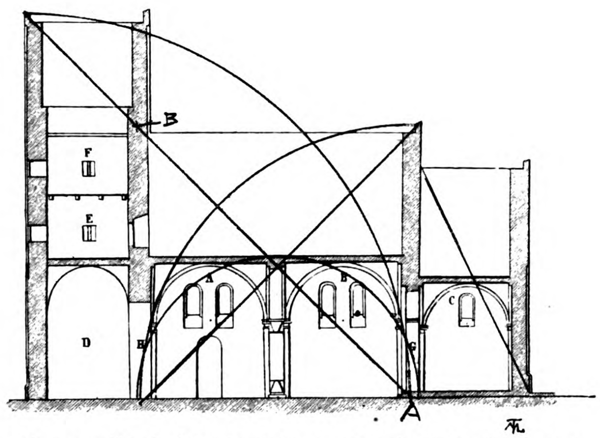


Fig. 127.—Church of Gumlösa in Skaane. Longitudinal section, analysis.

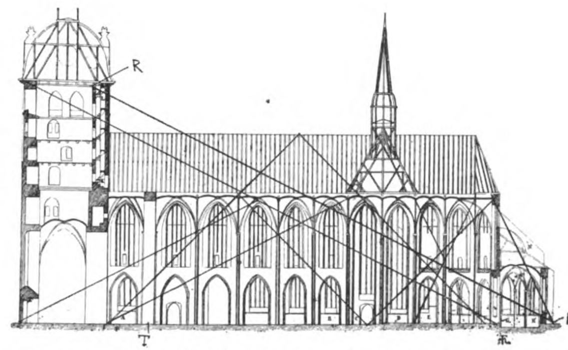


Fig. 129.—Church of St. Peter, Malmö. Longitudinal section, analysi.

in the presence of the exiled Archbishop Eirik of Nidaros—a fact that it may be of interest to notice.

Fig. 128 gives the plan and longitudinal section of *the church of Our Lady, Helsingborg,* from the thirteenth century, and fig. 129 gives the longitudinal section of *the church of St. Peter, Malmö,* founded in 1319.

The drawings need no comment. They show that here also the principle *ad quadratum* has determined the proportioning. The tower of the church of St. Peter dates from the fifteenth century, the original one having collapsed. The slanting diagonal of 63° 26′, P—R, shows the western side of the original tower, while the thick pillar marked T shows the eastern side.

The examples given are sufficient to prove that the churches of Denmark have been built according to the same principle as everywhere else in Europe. The same applies to Sweden, where religious architecture is related to the Danish, and belongs to the German group, wherefore we consider it superfluous to take any examples from that country.

CHAPTER IX

NORWAY

IT was through our study of the cathedral of Nidaros that we made the first discovery of the system *ad quadratum*, which proves to be the basis of all religious architecture from the earliest classic time to the later Middle Ages. It is obvious that the cathedral of Nidaros is no isolated phenomenon in Norway. An examination of Norwegian churches shows, moreover, that this system has been the basis of the planning of churches throughout the whole Middle Age—from the cathedrals to the parish churches, indeed even down to the smallest stone and wooden churches; we shall make it clear in the following examples. As far as possible these are given in chronological order, each within its group: parish churches, cathedrals, and wooden churches.

The "Fylke" or "shire" churches are classified among the parish churches as before the creation of fixed bishoprics they represented, within every fylke (or shire), an intermediate group between cathedral and parish church. They collected tithe, not only from the parish, but from the entire fylke, and received from all other parish churches and chapels of ease in the district a contribution which corresponds to the *cathedraticum* of the cathedrals erected later. In the fylkes of Borgarthingslög and of Eidsivathingslög there were respectively two or three fylke churches, or principal churches, as they were called in the later jurisdiction. The children had to meet at these churches to be confirmed by the bishop, who once a year came there to celebrate mass, and to receive the baptismal vows of the catechumens. These churches were similar to the cathedrals; they were planned for a more extensive ritual, with accommodation for a larger ecclesiastical assembly. They distinguished themselves in plan and elevation from the ordinary parish church, which, as a rule, was small, and had a single aisle. In order to show the development we shall begin with:

PARISH CHURCHES

of which we first mention two with a single aisle.

(1) The Church of Mostr, Sunnhordaland

This is the oldest stone church in Norway, and dates from the year 995. Fig. 130 gives its longitudinal section. From this it will be seen that the height of the ridge and walls is determined *ad quadratum* on the length of the church, similar to the one of Gumlösa.

(2) The Church of Dale in Lyster, Sogn

It dates from the thirteenth century. Fig. 131 (the plan) and fig. 132 (the longitudinal section) show that this church is proportioned in the same manner as the previous one.

We now take the three-aisled churches.

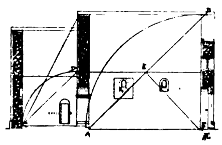


Fig. 130.—Church of Mostr. Longitudinal section, analysis.

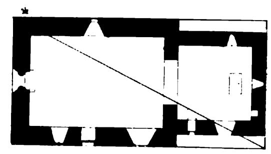


Fig. 131.—Church of Dale in Sogn. Plan, analysis.

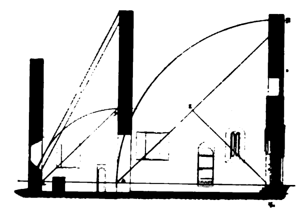
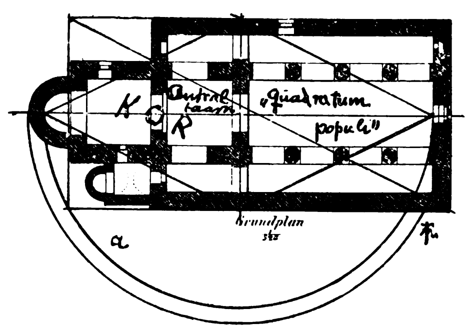
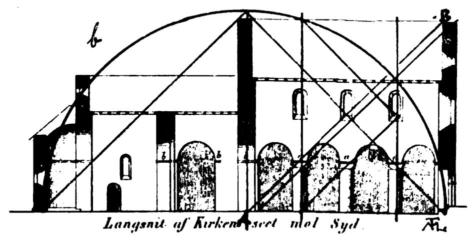


Fig. 132.—The same. Longitudinal section, analysis.

(3) The Church of Akr, near Oslo

This church is one of the two fylke churches in the province of Vingulmörk, and it is one of the oldest in Norway. It is mentioned at the time of Sigurd Jorsalafare, consequently before 1130, but it dates from the early part of the eleventh century—it may even be contemporary with the church of Mostr. We reproduce in figs. 133*a*, 133*b*, and 133*c* its plan,





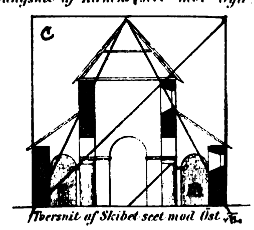


Fig. 133 *a*, *b* and *c*.—Church of Akr, near Oslo. Plan, longitudinal and transverse sections, analysis.

longitudinal section and transverse section. This three-aisled church, with central tower and low wall for the nave, without triforium, represents an ancient type, which appears early in small churches of Normandy and Brittany. The plan is perfectly regularly designed within two squares, one for the chancel, the other for the nave, or *quadratum populi*. This applies to the longitudinal section (fig. 133*b*), which shows that the level of the ridge is determined *ad quadratum* on the length of the nave. It will be seen equally that the height to the ridge represents also four bays of the arcade. The square has also been the determining agent in the transverse section. We find consequently that the rule *ad quadratum* has been used logically in all the plans, horizontally, longitudinally, and transversally.

(4) The Church of St. Nicolai in Gran, Hadeland

Here we find a larger parish church, probably one of the three principal churches of Hadafylke. Figs. 134, 135, and 136 give the plan, transverse section, and side elevation. This three-aisled church has also a central tower, low walls for the nave without triforium, and belongs therefore to the same type as the church of Akr near Oslo. Similarly it is among the very oldest churches of Norway. The drawings show that the principle *ad quadratum* is strictly carried out in all the plans.

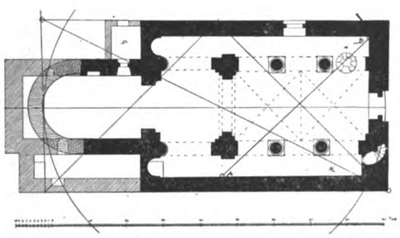


Fig. 134.—Church of St. Nicolai in Gran, analysis.

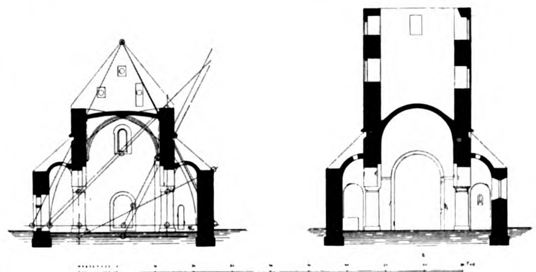


Fig. 135.—The same. Transverse section of nave and central tower, analysis.

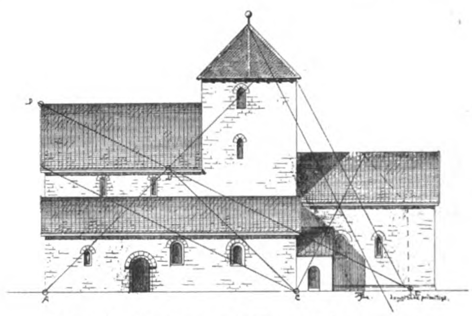


Fig. 136.—The same. South elevation, analysis.

(5) The Church of St. Mary, Bjorgvin

What applies to the church of Akr near Oslo and the church at Gran, applies naturally to a parish church in Bjorgvin, the largest trading town of Northern Europe in the Middle Ages. This church, which dates probably from the first half of the twelfth century, is also built *ad quadratum*, and is related to the smaller Norman churches. Its plan (fig. 137) has been designed within two squares.

Like the above-mentioned three-aisled fylke churches, the church of St. Mary in Bjorgvin could, as a three-aisled church, have allowed the height of the ridge to correspond with the square on the external width (see figs. 138*a*, 138*b*, 133*c*, and fig. 135). But, apart from this, it will be seen that in the longitudinal section all the three-aisled churches of this type are proportioned in the same way as the oldest single-aisled church; see diagonals A—B and C—D (figs. 139, 130, 132, and 133*b*), and also F—H—L (in fig. 139), as well as A—B—C (in fig. 136) and A—E—F (in figs. 130 and 132).

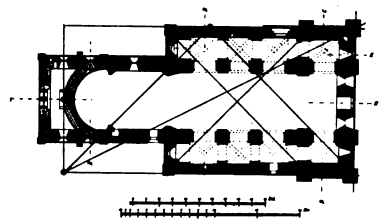


Fig. 137.—Church of St. Mary in Bjorgvin. Plan, analysis.

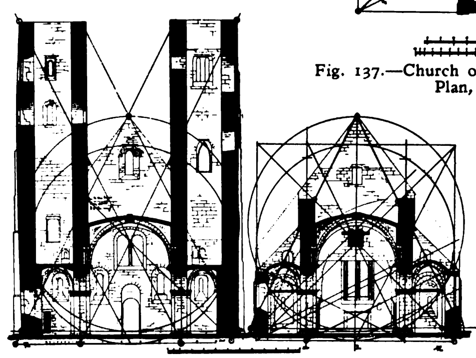


Fig. 138 *a* and *b*.—The same. Transverse section towards W. and E., analysis.

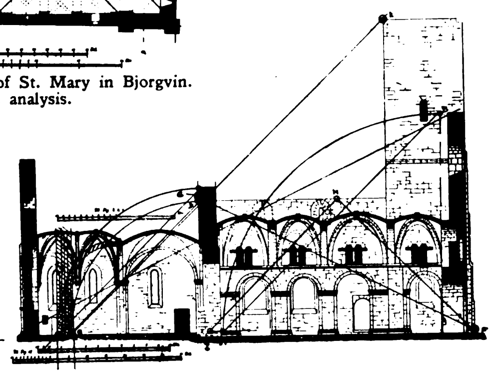


Fig. 139.—The same. Longitudinal section, analysis.

CATHEDRALS

(1) Cathedral of St. Halvard, Oslo

This cathedral was pulled down towards 1650 to furnish material for the walls of the fortress of Akershus; only the foundations are saved. These are reproduced in fig. 140, and show a Norman cruciform church with central tower. The oldest part dates probably from the time of King Olaf Kyrre, 1066–1093. According to the saga of Sigurd Jorsalafare, it was completed before 1130, when Sigurd was buried in the transept. This oldest part is marked by the diagonals A—B—C. Later, its chancel was extended, by which the transverse axis of the church was moved from the western wall of the transept to the central axis of the central tower, as shown by the diagonals A—B—E. It is designed within two large squares on the length of the transept. The plan of the extended chancel is put in darker hatchings. Its shape is not Gothic, but entirely Norman; and the extension must, therefore, have taken place in the twelfth century. The church in its entire length lies within three squares on the width of the front (see diagonals F—G—H).

As shown by the circle, there are four and a half bays within the square on the width of the church. The nave is very short because the transept is short; consequently it contains scarcely one and a half square instead of two. In spite of its provincial peculiarities, the plan has

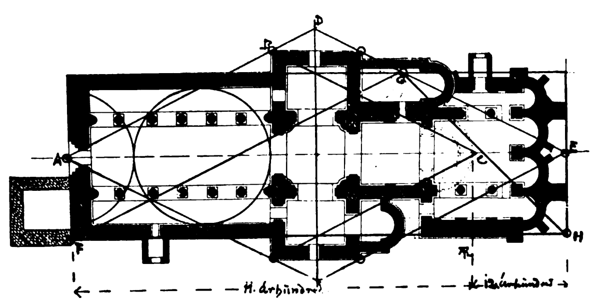


Fig. 140.—Cathedral of Oslo. Plan, analysis.

been strictly designed *ad quadratum*. From it, it is easy to form a correct idea of its elevation.

The plan of the cathedral of Hamar shows also a cruciform church, perfectly *ad quadratum*.

Of the cathedral of Bjorgvin, dating from before 1160, there remains no trace; but according to its rent-roll, Bjorgyngar Kalfskinn, and the account of the sagas on the royal burials, it seems as if the plan of this also was cruciform.

(2) Cathedral of St. Swithun, Stavanger

The Norman nave of this cathedral is previous to 1128, when it is mentioned in the saga of Sigurd Jorsalafare. The character of the Gothic chancel points to about 1250, and not, as supposed, to the time of its rebuilding after the fire of 1272. King Magnus Lagaböter, who held the revenues of Rygjafylke, resided a great deal in Stavanger during the life of his father,

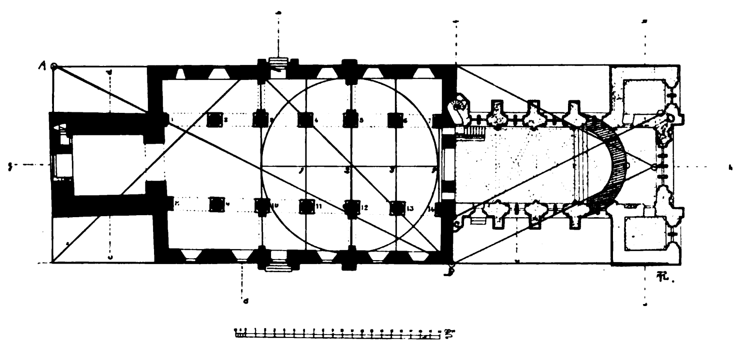


Fig. 141.—Cathedral of Stavanger. Plan, analysis.

King Haakon Haakonssön; this place had a royal residence, and he probably contributed to the Gothic extension of the church.

The plan (fig. 141) after the extension lies within three squares on the external width of the church, two squares of which fall on the Norman nave. As shown by the diagonal C—D, the chancel lies within two squares. If the aisles had been carried forward to the front, the entire nave would have contained two squares internally; but, as shown by the square and the inscribed circle, it only contains one and a half square, like the cathedral of Oslo. The distribution of the church is otherwise perfectly regular, as four bays lie within the square on the internal width. But, like all churches of this type without triforium, the side-walls of the nave do not rise *ad quadratum* on the length of the four bays (see the transverse section, 142*a* and 142*b*, also the longitudinal section, fig. 143). The conditions of height of the church are determined from the longitudinal section, like the church of St. Mary in Bjorgvin, the churches of Gran and of Mostr; they are marked in the longitudinal section by the diagonal A—C. This line corresponds to the diagonal A—B in fig. 130 (the church of Mostr), fig. 133*b* (the church of Akr) and fig. 139 (St. Mary's Church in Bjorgvin), fig. 132 (the church of Dale) and 127 (Gumlösa). Similar to the church of Akr, the level of the ridge is situated *ad quadratum* on the length of four bays (see fig. 133*b*).

It appears from this that the cathedral of Stavanger is proportioned *ad quadratum* in the same way as the parish churches, with the sole difference that, in the longitudinal section

of the cathedral, the interior consists of a rectangle of two squares instead of only one square in the small parish churches. This is the only one of the five Norwegian cathedrals within the boundary of Norway proper which has not the character of a cathedral but of a parish church.

From this we think it probable that the cathedral of Stavanger has been a large parish church—*a fylke church*—before it was honoured with the episcopal throne. Stavanger has been, from earliest times, a market-place, and the centre of Rogaland, where the wealthiest families, not only of Gulathingslagen, but of the whole of Norway resided. The family

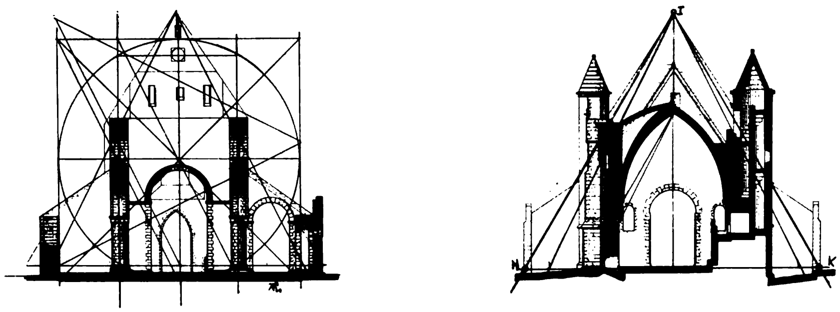


Fig. 142 *a* and *b*.—Cathedral of Stavanger. Transverse section, analysis.

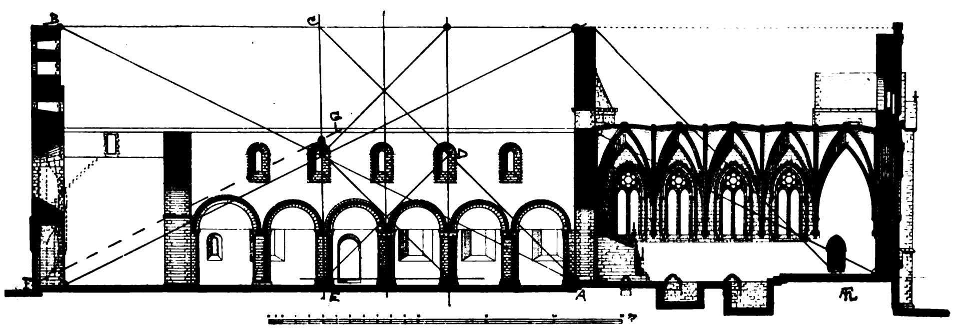


Fig. 143.—The same. Longitudinal section, analysis.

of Erling Skjalgssön, besides being related to the Anglo-Danish and Swedish royal houses, was closely related to the Norwegian throne, and his son, Aslak Erlingssön, was a cousin of St. Olaf. It seems natural that such a rich and powerful province should have taken pride in building a large and beautiful church at an early date. The entire architecture of the nave points to a great age. Figs. 144*a*, 144*b*, and 144*c* give details of it. They have all the oldest Norman forms in their mouldings, as well as in their ornamentation and point to a period preceding 1100.

In the churches of Akr and Gran, and in St. Mary of Bjorgvin, we found the top of the ridge to be *ad quadratum* on the external width (figs. 133*c*, 135, 138*a* and 138*b*). In the cathedral of Stavanger it lies *ad triangulum* on the same width; see the diagonals H—I—K, fig. 142*b*, giving the transverse section through the chancel facing west.

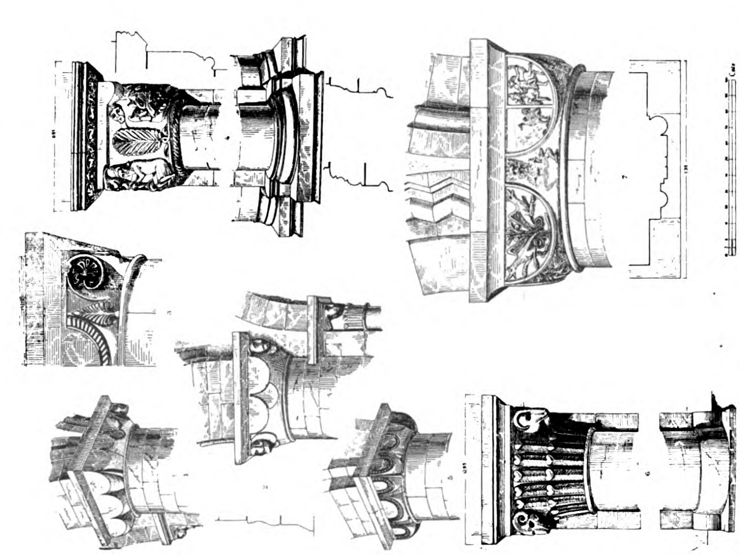


Fig. 144 *a*.

Cathedral of Stavanger. Details.

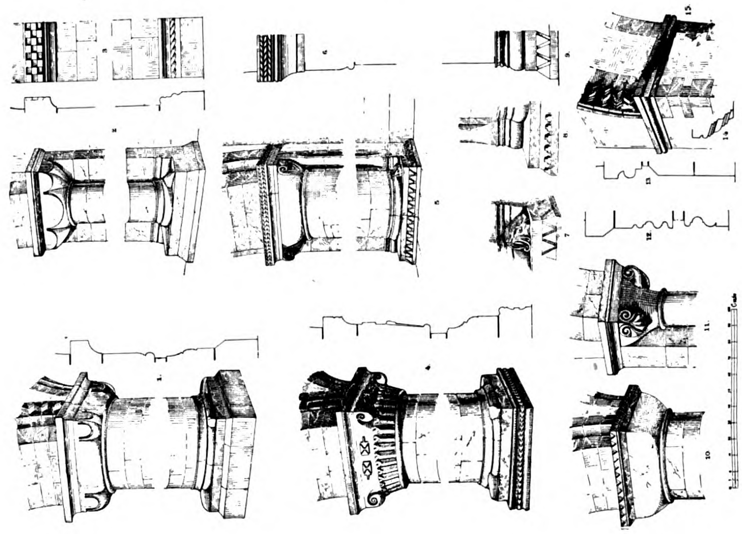


Fig. 144 *b*.

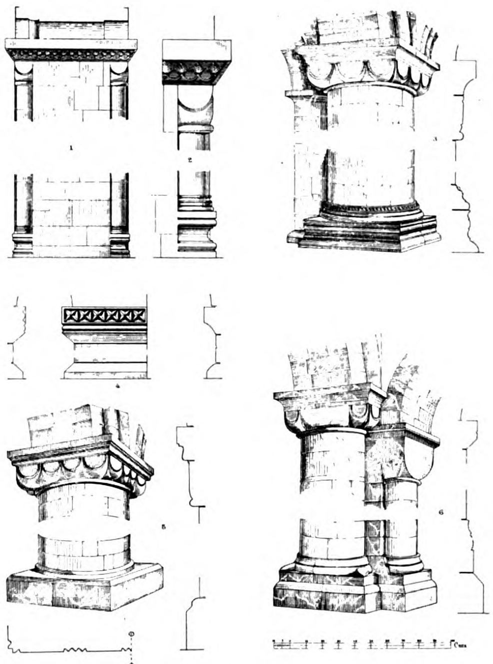


Fig. 144c.—Cathedral of Stavanger. Details.

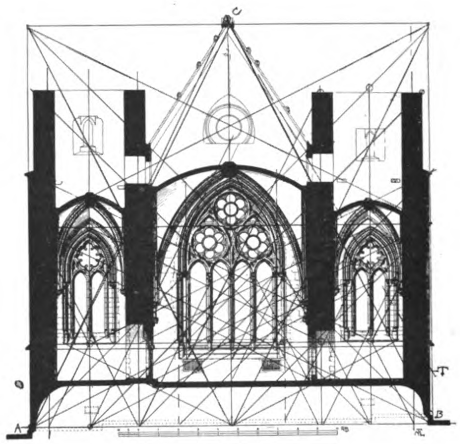


Fig. 145.—The same. Interior of the east gable wall, analysis.

Fig. 146.—The same, seen from S.E., analysis.

The Gothic gable-wall of the chancel, however, is designed *ad quadratum*, as shown by the diagonals A—C—B (fig. 145). As shown by the longitudinal section (fig. 143), the ground sinks at the beginning of the chancel, and slopes towards east. Thanks to this, while retaining the original height of the ridge of the church, the architect has been able to realise the front *ad quadratum* by lowering the plinth line from level T to level B. Instead of an orthogonal drawing of the east front, a drawing of the inner side of it must serve as a demonstration of how the architectural distribution of the elevation has been determined *ad quadratum* on the lowered level. The drawings need no comment.

(3) The Cathedral of Kirkjuvaag

The Cathedral of Kirkjuvaag (corrupted in English to "Kirkwall") on Orkney Islands,* was built by a Norwegian chief, vassal of the King. It was founded in 1137, and inspired chiefly by the cathedral of Nidaros. The lines of the system drawn in the plan (fig. 147) show the extensions carried out, in the thirteenth century, when it was also provided with a vault (transverse section fig. 148). Both the plan and the transverse section, as well as the longitudinal section through the transept (fig. 149), show the use of the system *ad quadratum*, although, as in all provincial imitations, it has not been fully understood.

(4) The Cathedral of Kirkjubö, Faroe Islands

Fig. 150 gives the plan of the cathedral of Faroe Islands.† This was built in the last part of the thirteenth century, and is a small construction for a small and poor diocese. As it will be seen, the church takes two squares and the chancel one. It has no roof at present, and shows various provincialisms. Judging from its whole character, it is not built by a master, but probably by a head workman or foreman sent from Stavanger, or from Bjorgvin, where the chapter elected the bishop of Faroe Islands. The cathedral was not a collegiate church.

(5) The Cathedral of Gordum, Greenland

Of this small cathedral, dating from about 1120, only scanty remains of the foundations are extant (fig. 151). As it will be seen, the plan shows a completely regular construction with nave *ad quadratum* on the length of the transept. Both nave and transept contain the obligatory two squares.

* We find it necessary in these times to attract attention to the fact that Orkney Islands as well as Shetland were mortgaged in 1468 by King Christian I to the King of Scotland. They are still, according to the deed which cannot be repudiated, the property *de facto* of the Kingdom of Norway. Each one of the five times that the King of Norway offered to pay the mortgage, which carried no interest, and demanded his property back, the mortgagee only made evasions, little to the honour of Scotland and of England.

† The Faroe Islands, Iceland, and Greenland, which had always belonged to Norway, were separated, with their Norwegian-speaking populations, from the mother-country at the Peace Conference of Kiel in 1814, through the deceitful informatio nof the Danes—to the great damage of the national raising of Norway and of these countries themselves, nationally and economically. The representative of the Dano-Norwegian King, the diplomat Bourke, went to Kiel with the draft of a peace treaty. According to article 4, the Kingdom of Norway and all its possessions, without any reservation, were to be ceded to the King of Sweden. The Swedish representative, Baron Wetterstedt, appeared to be quite ignorant of these old Norwegian possessions, and the Danish representative made him believe that "l'Islande, le Groënland et les îles Faroé n'avaient jamais appartenu à la Norvège"; he allowed, therefore, without protesting, article 4 to mean a cession of "the Kingdom of Norway, with all its possessions, excepting Island, Greenland, Faroe Islands," while the Kingdom of Norway with its possessions is defined by an enumeration of the four "bishoprics" of Norway itself. After the national assembly of Norway had elected the Swedish King as King of Norway, it was decided to demand the Norwegian colonies from Denmark. But, as the demand of Denmark that Norway should contribute to the payment of the common debt was agreed to by the congress of princes at Aix-la-Chapelle, a settlement between the two formally united countries was arrived at, in the Treaty of Stockholm, September 1, 1819, and the Norwegian colonies did not come back to Norway. The Danish deceit, at Kiel, was sanctioned under pressure of the Holy Alliance.

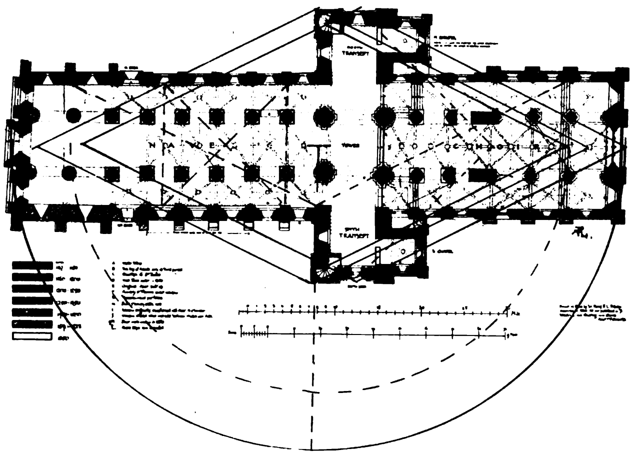


Fig. 147.—Cathedral of Kirkjuvaag. Plan, analysis.

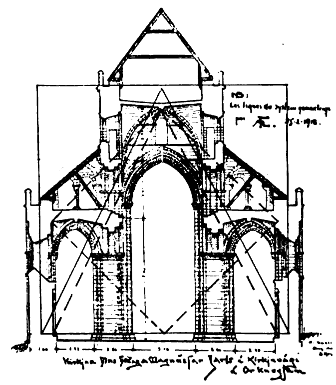


Fig. 148.—The same. Transverse section, analysis.

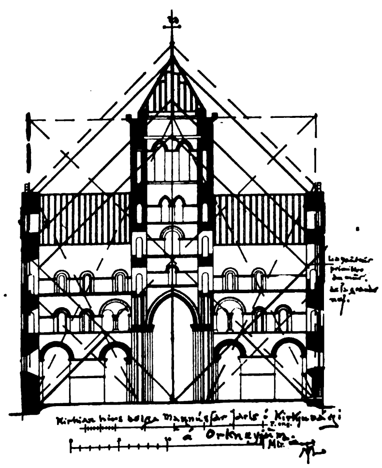


Fig. 149.—The same. Longitudinal section of the transept, analysis.

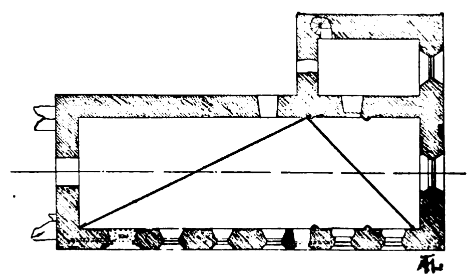


Fig. 150.—Cathedral of Faroe Islands. Plan, analysis.

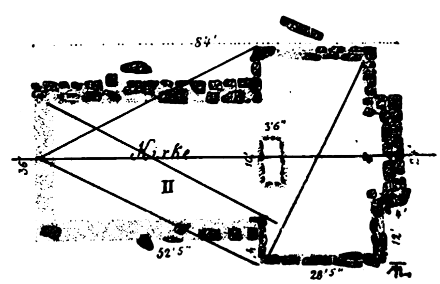


Fig. 151.—Cathedral of Greenland. Plan, analysis.

Of the two cathedrals of Iceland, not even the foundations remain. They were both built of wood. A drawing from the eighteenth century shows that the cathedral of Holum had been a three-aisled church with transept.*

We regret to say that we have no drawings of the cathedrals of Jona, Sudreyjarne (Hebrides) and of Peel (Isle of Man), which were ceded to Scotland in 1266; but, judging from photographs, these churches were built according to the same principle as other Norwegian cathedrals. After having gone through the stone churches, we shall now consider:

The "Stav Churches" †

These churches are famous, and justly so, because of their technically perfect construction. No other country can show their equal in that manner of building. Some old wooden churches are preserved in England, of which Greenstead church is the oldest. It is supposed

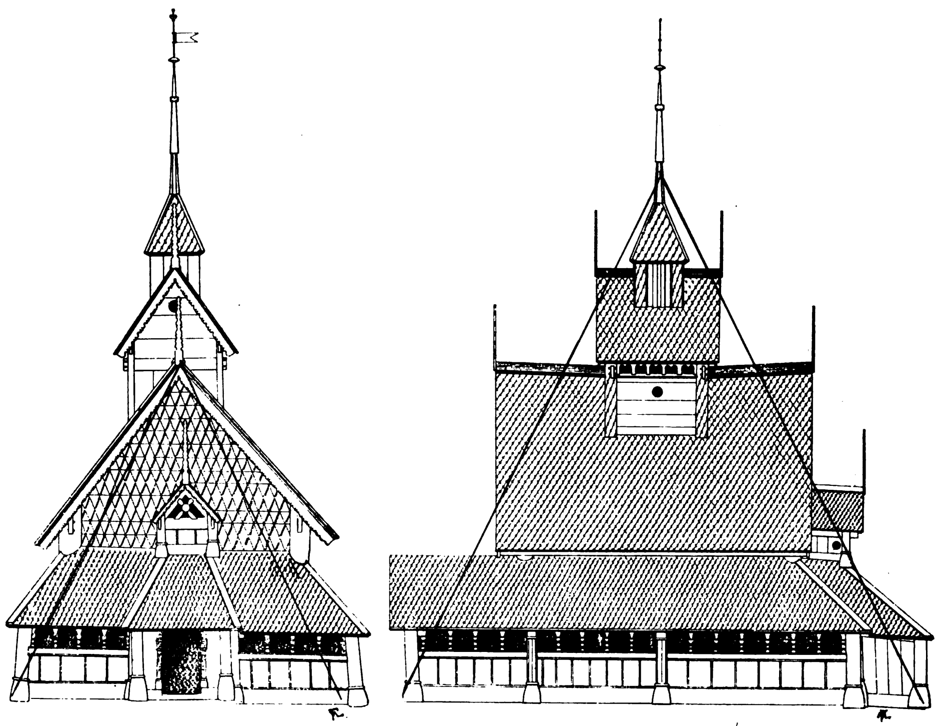


Fig. 152 *a* and *b*.—Church of Eidsborg. West front and longitudinal section, analysis.

to have been erected before 1066, and shows a manner of building having a certain resemblance to the Norwegian. The walls consist of half trunks standing up between the corner posts, which are tied together by horizontal beams; the halved trunks were then inserted in a groove in those beams; but the similarity does not go any further. Moreover, the manner in which the parts of the building are combined does not stand comparison with the ingenuity peculiar

* Annual report of 1908. "Foreningen til norske Fortidsmindes-mærkers Bevaring."

† *Stav* means a post.

to Norwegian churches, which show a strong evidence of a very ancient art of building in wood. The remote civilisation of Norwegians is evinced by the highly developed art of ship-building of this people, an art which was on a higher level than in any other country in the Middle Ages.

Some of these beautiful buildings date certainly from the eleventh century, indeed even further back, and are probably under their form of churches a transformation of the heathen —"Hov," or place of worship. As in Italy and in Greece, several of these "hov" have been probably transformed into churches; or their building material has been used to raise churches,

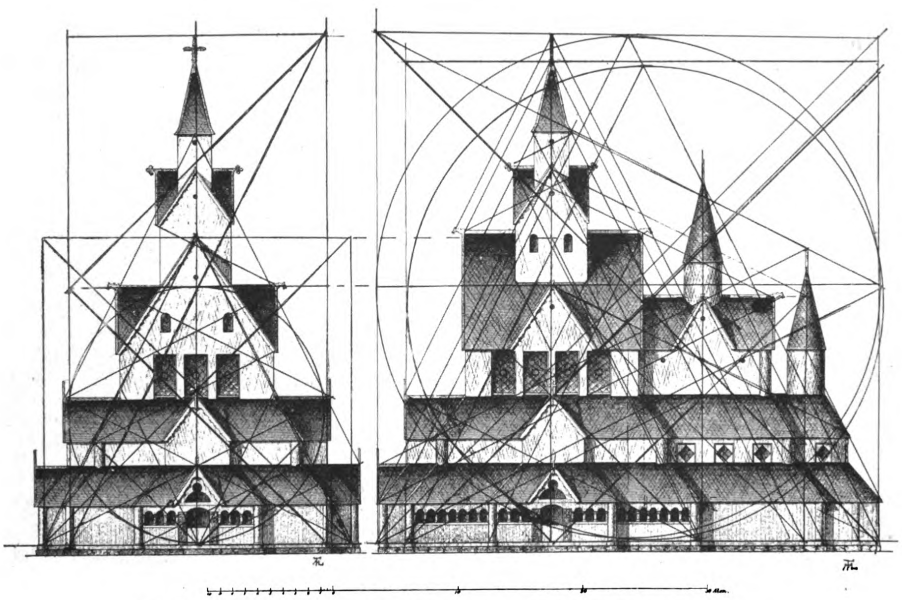


Fig. 153.—Church of Hitterdal. West front and longitudinal section, analysis.

as it was the case probably with the church of Ornæs in Sogn—which the Norwegian archæologist and architect, *Herman M. Schirmer*, has explained and made evident.*

These churches might be one-aisled and three-aisled. As three-aisled they have the character of the Christian basilica, with arcade, triforium, and clerestory.

Similar to those of stone, these wonderful wooden structures seem to be proportioned after the method *ad quadratum*.

Figs. 152*a* and 152*b* give the west front and the side elevation of *the church of Eidsborg*, Thelamork. Any comment is superfluous. The oldest existing document where it is mentioned is from 1354; but it dates probably from the thirteenth century.

Fig. 153 gives the front and side elevation of the *church of Hitterdal*, Thelamork. This church is older than the previous one. This drawing also needs no explanation.

* Annual report of 1910. "Society of Norwegian Antiquarians." The church of Ornæs is entirely built *ad quadratum*. This explains why the richly ornamented beams in the panelling of the wall have been brutally cut off; the old heathen part of the building had to fit into the system of proportionality of a Christian church as fixed by its symbolism.

Finally, as a last example, we give the *church of Gol*, Hallingdal, from the beginning of the twelfth century. King Oscar II restored it, and transported it to the royal estate at Bygdö.

Fig. 154*a* gives the transverse section and fig. 154*b*, the longitudinal section of this church. We leave it to the reader to study the analysis. As it will be seen, these seemingly capricious buildings are not the result of chance or accident, but of an ancient artistic civilisation, governed by a scientific principle, having nothing to do with the slangy and arty expression of "jolly."

It will be understood that the men who built these wonderful churches were not ordinary

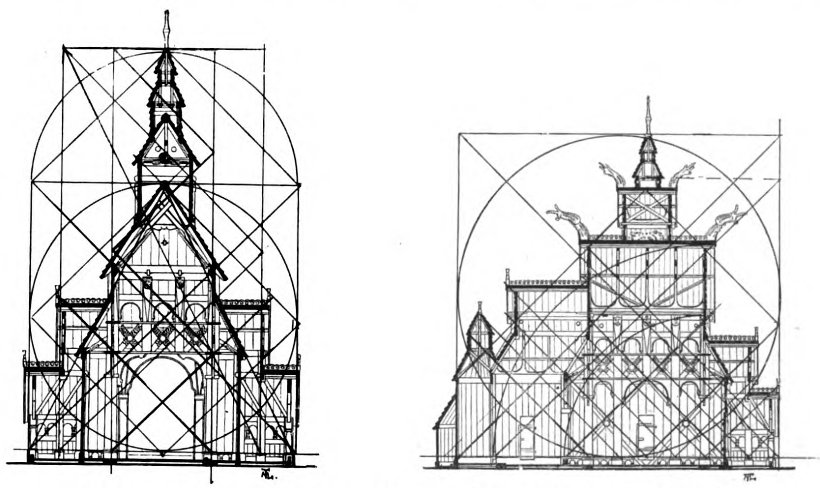


Fig. 154 *a* and *b*.—Church of Gol. Transverse and longitudinal sections, analysis.

rural carpenters, but trained master builders. These buildings belong absolutely to the same sphere of development as the cathedrals of the Middle Ages and the temples of the ancients. From the temple of Solomon to the cathedral of Greenland and the Stav churches of the valleys of Norway, we have been able to demonstrate the same fact—the square as the harmonic medium.

* * *

We have seen here how the architects of the Middle Ages succeeded in obtaining unity in the design of their enormous and complicated cathedrals—a unity which has made Gothic art the greatest, the most perfect in its power to carry out the idea which it embodies: to be in honour of God and to reflect His unity and His perfection.

This severe demand of unity in the geometrical raising of the church *ad quadratum* was not only an inheritance which the Middle Ages preserved, because it recognised its value from an æsthetic and philosophic point of view; it did more than preserve it, it increased and developed this inheritance to perfection. From the simple use of the principle of the square in the temple of Solomon, and the simple antique temple, there is a long and continuous development, to its use in the thirteenth century, when we find it developed into a descriptive geometrical and stereometric authority, whereby unity is realised in the distribution of the masses of these enormous cathedrals.

This strong requirement of unity was the outcome of medieval development. It is not the requirement of our time, where development is on other lines. It has been the interest of Prussia and North Germany, in their official writing of history, to praise Luther and the Reformation at the expense of the Middle Ages. Voltaire and the Encyclopædists supported this view. From that time we have been taught at school to call the Middle Ages, the age of darkness in opposition to classic times.

Nothing can be more foolish. There is a greater relationship between the Middle Ages and classic times than between ours and the latter. In both epochs society was founded on religion, and, with religion, followed the study of the metaphysics of antique civilisation. Aristotle and Plato, who, in their cosmologic philosophy, had nearly reached the monotheistic idea, were embodied in Christian teaching. But what the classics lacked the Middle Ages possessed—the belief in *one* God. The attempt of classic philosophy to find unity and reason in existence was solved by this belief. The logic and dialectic of classic metaphysics became thereby independent.

The belief of the Middle Ages was not the lay preacher's fear of hell. It had evidently its origin in the need of refuge felt by every human being in the cruel solitude of life.

It was also an expression of the need of the more complicated mind to find reason and unity in the world. This is expressed precisely by the words *Credo ut intelligam*—I believe, to understand. This belief which seeks reason was therefore a matter of knowledge, and it was supported in scholasticism by a perfect and unsurpassed work of dialectic, a masterpiece of unity itself.

It seems as if the principle of unity *ad quadratum* followed each step of the development of scholastic philosophy. It cannot be a mere accident that the time which is characterised by the cathedrals of Amiens, Reims, and Cologne, where we saw the principle brought to its perfection, should coincide with the time of Albertus Magnus and Thomas of Aquino. Albertus Magnus was named, as we know, Doctor Universalis, and, like his pupil, taught partly in Paris and partly in Cologne, where students from all countries of Europe fought to find a place under the chair of those universal teachers, who had succeeded in collecting the learning of centuries on the mysteries of the universe, on God, on the Trinity, the Reincarnation, in short, in accumulating all the philosophy and conceptions of the universe of the Christian ages in one sum: *From God to God* through Christ, crucified, risen from the dead, ascended to heaven, Mediator and Saviour, continuing his Redemption through His Church to the end of the world.

All was from God and in honour of God, and man was the highest creature. In this grave and magnificent conception of the world, the Middle Ages had one advantage over any other age, both before and after: it attained a universality which reached as far as a church was built, from Antioch and Byzantium to the Norwegian colonies in Iceland and Greenland, from Africa (St. Augustin), and Sicily to Trondenæs and Trums "apud paganos" in the north of Norway.

Later development has destroyed this universality. Modern civilisation is without a centre, and loses itself in details and specialisation without any comprehensive view. The servants of science are all occupied with their special branch, each counting the bones of his particular fossil. Philosophy is no longer philosophy, but it has come down to be an anti-speculative, experimental psychology; laws are promulgated from the absolutely materialistic theory, that everybody has a right to live. Life is no more the reward of the one who does his duty; there is no more room for the moral factor, mercy to the weak—by which all forces are equalised.

With our artificial lighting, our insurances, our police, our board schools and our newspapers, we have chased away solitude, ordinary danger, the fear of the dark and of life, the dreaded presence of God, and thereby the ability to admire; Pan is stuffed and put in the

museum. The faith of the Middle Ages was *universal*. God in all and all in God. The parts were only separate; they differed in their functions, but they were one only, because they aimed at the same goal.

The legend of St. Francis of Assisi preaching to the birds is a beautiful example of this feeling. The simple and great words of the creed were enough to give the simple-minded a feeling of human dignity: " Credo in unum Deum, Patrem omnipotentem, Creatorem cœli et terræ." Not only the great, but even the thrall could be a link of that chain if he did his duty, according to his ability and qualifications; he became their equal at the court of the great King who sits in His heavenly castle on His " throne of stars," as it is said so beautifully in an old Norse homily.

It is pure malice against the Catholic Church and Christianity in general, or sometimes ignorance, or perhaps to serve political interests, when history depicts the people of the Middle Ages tormented and persecuted by the fear of hell.

In the Middle Ages life was not at all the dark valley of tears of protestant puritanism, nor the hopeless desert of modern materialism, with its rather peculiar requirements; but it was a merry life, all the more beautiful and full of colour because it stood against a background of earnestness and belief. It was an earnestness full of joy, a religious feeling of relationship between man and the whole of nature as the creation of God, with a radiant belief in an eternal life as the reward of good. The Danish romantic poet Ingemann has found a most poetical and happy expression in five lines, as fitting as it is descriptive on the piousness and charm of the medieval view of life:

" Lovely is the earth,
Splendid is the heaven of God,
Beautiful is the march of the pilgrimage of the souls.
Through the fair kingdoms of earth
We enter singing into Paradise."

Medieval literature shows frequently a more highly developed sense of nature than the classics. In the profane literature of the Middle Ages we find evidence of the joy of living, and a highly developed sense of the beauty of nature.

In the old songs of Norwegian literature from Norway and Iceland there are descriptive renderings of nature in a form so artistic, so brief, and so concentrated that only an intense feeling can demand and achieve it. Not only in profane, but in religious literature also we can notice the same joy of beauty. Some homilies are entirely built upon meditations on nature or on observations of nature put in parables. Like a ray of light on a melancholy autumn day, this feeling of nature can suddenly shine and give a merry lustre to these severe thoughts, or in religious hymns it can break out in jubilant song to the praise of God—the splendour of life.

In the famous " Hymn to the Sun," by St. Francis, we find:

" Laudato si ' Misignore ' per sora nostra
matre terra
la quale ne sustenta et governa
et produce diversi fructi con coloroti fiori et herba."

The fact that the medieval church favoured divine service in the open air, and religious feasts having processions with banners and song, agrees completely with this joy of life.

Several medieval hymns are undoubtedly written in view of these open-air celebrations. Thus we find a hymn preserved in *Missale Nidrosiense*, which seems to be composed for a

harvest festival. It is probably of Norwegian origin, as it is known only from the mentioned missal of Nidaros. We quote therefore most of its verse *:

" Jubilemus cordis voce
nostro salutari,
jubilemus et psallamus
(nunc) omnipotenti.

" Qui ornavit aërem
volucrum catervis,
qui vestivit aridam
frondibus et herbis

" Qui ditavit Thetidem
piscibus marinis,
locupletem fecit terram
quadrupedum formis.

" Qui deducit super terram
rivulos aquarum,
qui suspendet super astra
nimbos pluviarum.

" Qui vernali facit flatu
flores redolere
autumnali sole coctos
fructus colorati.

" Albicare late campos
hiemali nocte,
cum flavescunt late messes
aestivale luce."

Another beautiful example is preserved in an Easter hymn by an unknown writer from the end of the Middle Ages. We give two characteristic verses:

" Surgite verni
Surgite flores,
germina pictis
Surgite campis!
Teneris mixtæ
Violis rosæ
candida sparsis
lilia calthis.

" Plaudite montes,
ludite fontes,
resonent valles,
reflectant colles:
J'o, revixit,
sicuti dixit,
pius illæsus
funere Jesus."

As we shall see later, the Greek temple represented a thought—the wonderful harmony of nature and of creation. There is, therefore, something abstract, a kind of geometrical coldness over it. It is the same with Greek ornamentation, which, by conventionalising, has reduced flowers and leaves to their geometrical and fundamental forms, their essence. The classic temple was for philosophers; it was built individually, each separately for a single idea within a cold rationalism.

* Both the hymns given are taken from J. H. H. Brochmann, 40 Latinske Salmer, Christiania, 1901

The Middle Ages had not only inherited, but it developed the classic temple. It added its temperament to it, a somewhat mysterious feeling of nature belonging to a northern clime. A merely intellectual joy of constructing could not have produced the heaven-reaching, fantastic cathedral, which technically surpasses the classic temple by a long way. By subjecting itself to the artistic and philosophic discipline of an older civilisation, this temperament became a source of richness and of further development.

We know that Tacitus thought it peculiar that the Germanic races used forests and dark groves as places of worship.

The Middle Age transferred the forest to the temple. It is worth noticing concerning this that it was just in the part of France subjected to Germanic or Norwegian influences that cathedrals grew higher and higher on their columns, branching out like trees in a grove under the high vaulting. They obtained the darkness of the grove, while the light from stained glass windows produced a wonderful play of colour in the dusk of the stone forest.

The medieval builders infused in the stone of their cathedrals the whole love of nature of the time. Leaves and flowers sprung from the capitals in the folds of the arches, not stuck on haphazard nor lumpy, coarsely modelled indications of any kind of blossom, but a bloom of distinct flowers, grew organically from the stone, with all the charm and expression of nature, as only the individualised form can give and the humble respect for the thought in that form.

All that man had supposed and humbly wondered at, since antiquity—the miracle of life—was put into the cathedral by the medieval workers.

The temple stood open day and night, and had equal place for everyone, rich and poor, Sundays and week-days. It was from there that the King borrowed his crown when all the bells rang; here the child was carried in to baptism and from here that man was carried to be laid to rest under its walls. Feasts were held in it, meetings and profane gatherings, sermons were preached and instructive lectures were given—university and public academy all in one.

In the Middle Ages the church was "the people's house," and it was loved by the whole people. In medieval times wages were very high, but, nevertheless, we see that even small dioceses and small towns could afford to build these enormous works of splendour, which, like a microcosmos, mirrored the time's idea of universal unity: from God to God through Christ.

The plan of the church was an expression of this. The Chancel symbolised the sanctuary; the nave—the name is already a symbol—is the movement of the faithful towards the sanctuary. The transept, like the arms of the cross, symbolises Christ as the Redeemer. The obliqueness of the intersection of the arm of the chancel with the transept is too common an occurrence not to signify the death of Christ on the cross, when the Saviour turns in agony and bends His head before He dies; this theory, which was put forward in France as early as 1820, was denied later. From the threshold and down to the floor of the church the step is a reminder and a symbol that man must lower himself humbly in order to be raised: "Stig nidr thú at upp stigir thú; ver thú litill, at thú sér mikill fyrir augum Guds" (Lower thyself that thou mayest rise; be little in thine own eyes, that thou mayest be great in the eyes of God), as it is written in an old Norwegian homily.

And similarly every vital part of the edifice had its symbol according to the importance of its function, even to the spires which rise to heaven as a symbol of the prayer and hope of the Church. We see this in Cologne, where the spire was designed within the same square as the chancel.

And this material image of the unity of the world is realised by the image of antiquity, unity in imperfect matter: "the broken circle," the square.

In the actual use of this principle—from God to God through Christ, from the square

to the square through the square—can we not see an analogy in these two sentences? Unity in the world finds its counterpart in the realisation of its symbol—the house of God.

Therefore it is not only a poetical metaphor, but an image taken directly from reality, when the great writer of psalms, *Adam of St. Victor*, in the twelfth century, compares the four evangelists to a square foundation wall, on which the Church is firmly built.

"Non est domus ruitura
Hac subnixa quadratura,
Hæc est domus Domini;
Glorietur in hac domo,
Qua beatus homo,
Deus junctus homini."

We have found proof of the employment of this principle *ad quadratum* in the Middle Ages and in classic times, and, as regards antiquity, we have also retained a linguistic evidence of it. The Latin verb *quadrare*, besides meaning "to make something square," means also "to make something fit." The last signification must have been transferred from the use of the square into the building art, which through it has been able to project a drawing into other plans and create unity in these. The adjective *quadratus*, which Latin authors occasionally use when speaking of a statue or a person, means, therefore, in such a case, not formed in a square, but regular, well shaped, and harmonic.

But, in addition to the method *ad quadratum*, the Middle Age has inherited from antiquity another auxiliary, by which it created harmony in proportioning the different parts within the main results of the squaring, namely, the *sectio aurea*, or *proportio divina*, as we find it termed towards the end of the Middle Ages.

* * *

CHAPTER X

THE SECTIO AUREA

CONNECTION BETWEEN CLASSIC AND MEDIEVAL SCIENTIFIC GEOMETRY

THE manner in which a discovery or a new observation is made is always of importance in judging its value. The history of the discovery is often the thing itself.

Here we must remark at once that our discovery of the geometrical system, as well as our observations on the use of the sectio aurea in the proportioning of the cathedral of Nidaros, were made entirely without knowing the work of previous investigators concerning it and its use in Gothic building art.

During our study of the history of economics we often met problems which could only be solved by mathematics; we had the surprise to find that the knowledge of it acquired at school was lying dormant and could be called back when needed. Such was the case now with the sectio aurea, the construction of the pentagon and the geometrical constructions combined with it. It was so even with our studies on ethics; we had forgotten that these had played a certain part in Greek and Egyptian architecture, but all came back when it could be of use. Using the floor of our study as a drawing-board, we had traced in chalk what remains of the large canopy from the cathedral of Nidaros which we took, like everyone else, to belong to the large central window of the west front. For the purpose of enlarging it we had drawn it in a network of squares. After looking at it for some time, it dawned upon us one day that this was indeed similar to the angle we had drawn so often at school when we constructed the sectio aurea and the pentagon. In order to test our memory, we did not refer to any book, but we endeavoured to trace our way back in our studies; we succeeded finally in constructing the pentagon just as in the good old days at school.

Following the pentagon there appeared the pentagram, the five pointed star, which in the latter part of the Middle Ages, was used in a prophylactic way as a talisman against evil powers, and in ancient times by the Pythagoreans as a sign of recognition and a symbol of the harmony of the universe. It was not only a foolish superstition, but a hidden knowledge which was the foundation of this popular prophylaxy, with its many weird names, such as the seal of Solomon, the ring of Solomon, Salus Pythagorea, the last one most of all pointing to a real connection with ancient philosophy.

The frequent occurrence of the five-pointed star as a stonemason's mark, and certainly also as the mark of a particular guild of stonemasons did not seem to us less remarkable. These five-pointed stars are found abundantly in the cathedral of Nidaros, cut in the stones belonging to the oldest part, dating from the eleventh century (fig. 155) and from the thirteenth.

As we know, the pentagon, or the sectio aurea on which the construction of the pentagon is based, can be constructed in many ways.

We shall first give the construction of the pentagon as it is most usually done in Scandi-

navian and German school-books, where it is known under the probably German name of "Cirkelens Höideling," which in our opinion means nothing. This is the division in extreme and mean ratio.

We have a circumference with radius A B (fig. 156). This circumference is divided into four parts, and the radius B H is divided in half at point D. The line D C (diagonal of 63° 26′) continues to D on the diameter and reaches E; the line E C is then the side of the equilateral and equiangular pentagon when it is used as a chord to divide the circle. At the same time point E divides radius A B according to the sectio aurea because A E is to E B as E B is to A B—or expressed mathematically, A E : E B = E B : A B, or briefly A E : E B : A B. Therefore E B is the mean ratio between the smaller extreme A E, and the larger extreme A B; according to the classic way of designation the line A B is divided into a mean ratio E B and two extremes: the part A E and the whole line itself. The larger part E B is called *major* (M) and the smaller A E *minor* (*m*). The line E B is also the side of the regular inscribed decagon.

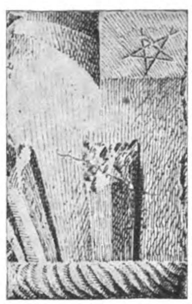


Fig. 155.—Pentagram on a stone from the ruins of the oldest part of the cathedral of Nidaros.

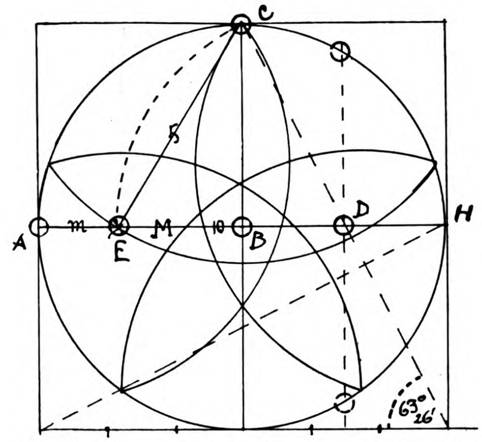


Fig. 156.—Dividing the radius in extreme and mean ratio, construction of the regular pentagon.

On the left of fig. 157 is given the usual *construction of the sectio aurea*. The line A B is to be divided; at right angle with it we draw the line A C of equal length and divide it at D. The line D B is drawn from here, on this the distance D A is marked and reaches to point H. The distance B H is fixed from B on the original line A B, which in this manner at E is divided into B E or major (M), and E A or minor (*m*). In a circle with radius A B we find that B E is the side of the inscribed regular decagon.

To the right in the same figure is given the construction of Euclid from II, 11. The line A^1 B^1 is to be divided and on a line forming an angle of 90° with A^1 B^1, as for example on the left front point A^1, half the distance from A^1 B^1 to point A is marked. The line in strokes A B^1 is marked upon the horizontal line through A^1 and reaches to F. The distance A^1 F is fixed from A^1 on the dividing line A^1 B^1 which in this way at point E^1 is divided into A^1 E^1 or major (M) and E^1 B^1 or minor (*m*). In a circle with A^1 B^1 (= A B) as radius we find A^1 E^1 to be the side of the decagon, and B^1 T the side of the pentagon.

Both the methods of construction given above are very ancient heirlooms from antiquity.

The construction in fig. 156 is used by Claudius Ptolemæus * in the first half of the second century A.D. It is undoubtedly older. The construction of Euclid (fig. 157) on the right) proves to be identical with it. But Euclid is not satisfied with demonstrating the construction of the sectio aurea, he uses it to show the marvellous function of this division. Similar to all other geometrical axioms and problems, this construction of Euclid is not his own discovery. It is significant that, the very first time he uses it, he does not give any definition of it, but it is to lead the attention of the reader into a way of thinking, so to speak, geometrically. This occurs in Book II, proposition 11, on addition and subtraction of squares, rectangles, and the various divisions of a given line and its combinations, where he puts the problem thus: "To divide a given straight line in such a way that the rectangle enclosed by the line and one of the parts is equal to the square of the other part." He does not give the definition of the sectio aurea itself before Book VI, on the equivalence and proportionality of figures, while it is first in the thirtieth proposition that he gives the problem "to divide a given limited line into extreme and mean ratio," employing the method of Book II, prop. 11. It is evident that Euclid did not so much want to point out the algebraic qualities of the sectio aurea, than, as remarked, to draw attention to its wonderful geometrical function, or to the part it plays in geometrical phenomena. In this manner he reaches to the very centre of ancient geometry, the object of which is precisely to make clear the connection between the various figures and the various bodies.

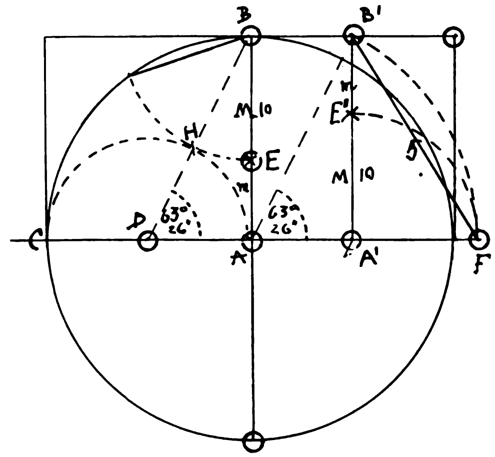


Fig. 157.—The usual construction of the sectio aurea, after Euclid.

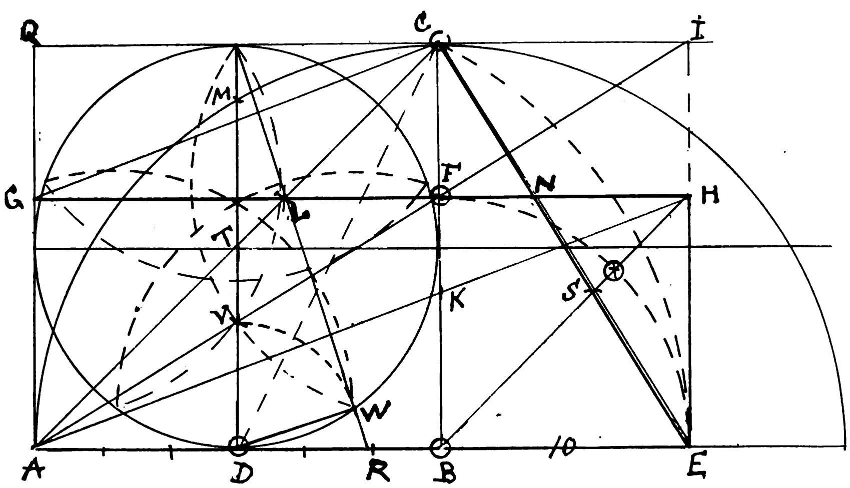
The use Euclid made of the construction in the two mentioned problems is given in fig. 158. We take B C as the line to be divided. Its half is fixed from B to D; the distance D—C is fixed from D—E and B E is marked on B C; the required dividing point is then at F, so that B F becomes M, F C = *m*. Through the intersecting point F, we draw the line G H, and from E the line E H. The rectangle between *m* (F C) and F G (= B C), therefore G, F, C, Q is then = to the square on M (B F), therefore B, E, H, F and the large rectangle A, E, H, G = the square on the whole line B C (= A B).

But besides the astonishing result of the function of the sectio aurea, we find already in this figure the geometrical qualities of the pentagon. With a glance at fig. 156, we see that the side E C of the pentagon is the hypotenuse of the rectangular triangle on the major (E B), and the radius (B C). According to the theorem of Pythagoras the square on the side of the pentagon E C is then = the sum of the squares on major E B and on radius B C. Now, as in fig. 158, B F is major of B C, and B C = A B, the hypotenuse A F is consequently the side of a pentagon in a circle with B C as radius. The same applies to hypotenuse C E

* Cantor, *Gesch. d. Mathematik*, 2nd edition, vol. ii, p. 462, when mentioning Albrecht Dürer (1471–1528) writes: "Für das Fünfeck ist ausser der Zeichnung mit unveränderter Zirkelöffnung auch eine genaue Zeichnung gelehrt, welche bereits im 9 Kapitel des I. Buches des Almagestes vorkommt." (As regards the pentagon, besides the drawing with altered radius, there is also given a distinct drawing which is already shown in chapter 9, of the first volume of the *Almagestes*.)

in the rectangular triangle B C E. The square upon these hypotenuses, A F and C E, is then = the sum of the squares on B C, and the square on B F (as B F^2 = the rectangle C F G) = the sum of the square on A B and the rectangle C F G. The diagonal of the rectangle on the major and the whole line A, B, F, G is thus the side of a pentagon in a circle having as radius the whole line B C. As A B = B C, and B F = B E, A F becomes also a side of the pentagon in the circle with B C as radius, because the diagonal of the rectangle on the major and the whole line A, B, F, G is also the hypotenuse of the rectangular triangle on F B (M) A B.

If this line of the hypotenuse is continued farther until it cuts the perpendicular from E in point I, there appears to the right of C F (= *m*), the small rectangle C, F, H, I, which develops the whole figure of the construction into a large rectangle, A, E, I, Q, the sides of which A E and E I stand in proportion to each other as M : *m*. The diagonals of all these figures produced by the sectio aurea form a number of proportions according to the same conditions.



(1) AE : EL = M : m	(13) GL : LF = M : m
(2) AB : BF = „ „	(14) LF : FN = „ „
(3) AK : KH = „ „	(15) HN : FN = „ „
(4) GC : KH = „ „	(16) GL : HN = „ „
(5) AF : FI = „ „	(17) BE : HN = „ „
(6) AL : LC = „ „	(18) BH : BS = „ „
(7) AF : AT = „ „	(19) BH : LC = „ „
(8) AF : TC = „ „	(20) EC : CS = „ „
(9) BF : FC = „ „	(21) BS : SH = „ „
(10) BK : KF = „ „	(22) EN : ES = „ „
(11) CF : KF = „ „	(23) ES : SN = „ „
(12) DV : VT = „ „	(24) FC : EN = „ „
	OSV.

Fig. 158. Euclid's use of the construction of the sectio aurea.

The drawing shows altogether twenty-four proportions and their number is not yet exhausted. If a circle is inscribed in a square on the original line B C, we find the large circle with B C itself as a radius and in the new small circle respectively that:

(1) the side of the triangle = A H and A L.
(2) the side of the square = A C and A T.
(3) the side of the pentagon = A F or C E, and A V or V F.
(4) the side of the hexagon = A B or B C, and A D.
(5) the side of the decagon = B F and D V.

And finally, what is outside our interest, that the diagonal B H (= D M) will be = the side of the heptagon inscribed in the large circle.

Moreover, it must be observed that a line from the zenith of the small circle through the intersecting point L, between the lines A C and G F H, and farther down to the line A B, cuts the small circle W and gives the side of the decagon inscribed in this circle from D, or D W = D V = the major of the radius D T of the circle.

All this abundant play of constant irrational proportions between lines and figures divided according to the sectio aurea, is developed logically from the constructions of Euclid. The name itself "sectio aurea" is used for the first time in writings towards the end of the Middle Ages by Luca Paciuolo, the Italian translator of Campanus, in his *De Divina Proportione* (1509). But Euclid himself had already accepted a known and recognised description of this linear division, which forms the central element of his whole geometrical method. In the translation of Campanus, we read in Book XIII, prop. 6, of Euclid, as follows: "Directa linea rationalis extrema et media ratione secta fuerit: utrumque segmentorum irrationale est appellaturque apotome." In the Danish translation (1912) by Thyra Eibes from the Greek text revised by the publisher, Professor J. L. Heiberg, and the historian of mathematics, Professor Zeuthen, we read: "When a rational straight line is divided into an extreme and a mean ratio, then each of the parts is irrational and is called section." Here we have Euclid's own admission for using the said ἀποτουή, "section," or only τομή. Taken in a narrower sense, the excellent German author, Dr. J. Xavier Pfeifer, is right when he says, in his book *Der Goldene Schnitt*, concerning the terminology, "One does not find in Euclid a special name for the proportion of the sectio aurea in the abstract, but only for the line derived from the proportion—that is to say, for the concrete base of it" ("Bei Euklides für die Proportion selbst in abstracto noch keine besondere Benennung sich findet, sondern blos für die nach solcher Proportion geteilte Linie, also für das concrete Substrat der Proportion" *). But from this there is a leap into the darkness of ignorance when the compiler of Trondhjem repeats parrot-like in his Extracts, the following self-sufficient sentence: "In Euclid the sectio aurea has no special name, and is given no preference over the other mathematical proportions." † The coolness evinced in such a rash statement proves that the "author" has never read and still less understood Euclid. Of all the geometrical proportions, the sectio aurea is the only one on which Euclid bestows a definition: "A straight line is said to be divided in an extreme and a mean ratio when the whole length of the line is to the larger part as the larger is to the smaller." Within the frame of his work, Euclid could hardly have said more than this. Of all imaginable proportions this is the only one which contains its divisions in two parts on one length, it becomes the proportion *par excellence*. For the thinker, the definition of Euclid contains the explanation of the æsthetic superiority of this proportion over all others, *because the section and the totality are joined here in an indissoluble union.*

All the "elements" of Euclid are, as we know, a single united system, where each

* *Der Goldene Schnitt* (Augsburg 1885), p. 42.

† Dedekam, *Gotik og. Geometr. Systemer*, p. 42.

recognised fact is unavoidably and logically built upon the previous one to form a single chain, until it culminates into the stereometric proof of the harmonic relation of the five Platonic bodies. The uniting link in this chain of recognised facts is the sectio aurea, *the only one of all geometrical proportions which enters constantly into a multitude of proofs.* One meets it altogether seventeen times: in Book II, prop. 11; in Book IV, props. 10 and 11; in Book VI, definition 2 and props. 17 and 30; also in the first eleven of the eighteen propositions of Book XIII. Book XIII treats of the inscription of regular polygons in the circle, viz. the equilateral triangle, the square, the pentagon, the hexagon, the decagon, which again are used as side surfaces for the five Platonic bodies inscribed in one and the same sphere, the tetrahedron, the cube, the octahedron, the dodecahedron and the icosahedron. The sectio aurea forms the connecting element between all these bodies. At the same time, fig. 158 shows us the five regular polygons constructed inside the same circle by means of the sectio aurea.

Geometry, in antique science, was not studied for its own sake only, but principally as a means of attaining to the knowledge of the various phases of human existence.

The Neo-Platonician Proklos (A.D. 412–485) who, besides his dialectic, possessed also an extensive knowledge of ancient philosophy, expresses himself entirely in the spirit of the classics when he says of Euclid that, in accordance with his scientific position as Platonician, he put forward the theory of the construction of the platonic bodies, as a goal in his *Elements*. We cannot but look upon the reservation of Cantor as due to a lack of comprehension caused by modern indifference regarding religious aspirations, which were everywhere the motive of the efforts of ancient times towards human wisdom and the understanding of beauty—the cosmos. In consequence geometry was the centre-point of ancient philosophy, this philosophy of which the talented French writer, Étienne Vacherot, has rightly said, when mentioning Proklos, that it could understand everything, express everything, and find the connection in everything.* The means of reaching this understanding and this explanation of the world, which was conceived as an ordered work of art (cosmos) was geometry, in which (see fig. 158 and the introduced diagonals) this very sectio aurea and the pentagon produced by it, are the combining elements.

If one cared to examine all the combinations of the figure 158, it would easily be found that the entire system of Euclid is condensed in the manner in which he applied the proportion of the sectio aurea.

We shall now examine the pentagon itself. We have just remarked that a line from the zenith of the small circle in the square on A B, in fig. 158, through point L, sections out the arc D W of one-tenth of the circle. In addition to this line being the diagonal of a decagon, it is also the diagonal of the pentagon, the side of which we found to be = the section A V and V F. To mark the pentagon produced by the mentioned diagonal, we have drawn a curved pentagram. In connection with this the pentagon is produced with all its five diagonals (fig. 159). It can be seen that the pentagon, like the previous figure (158) must be said to consist only of proportions of the sectio aurea.

The diagonal EB is to the side of the pentagon $ED = M : m$; $ED : EG = M : m$; $EF : FC : FG$, $FG : FH : HI—M : m$, as it can be seen: a harmonious geometrical progression according to the sectio aurea; it can be continued endlessly in increasing or decreasing progression

The given construction in fig. 158, taken from Book IV, tenth and eleventh propositions, for the construction of the pentagon, is attributed to Euclid because the *Elements* are the oldest written document in which it occurs. As remarked above, it is probably older. Euclid used it in a theoretical and scientific manner with a theoretical object, and it has its theoretic

* "Proclus fut plus qu'aucun autre philosophe de cette époque pénétré de l'esprit alexandrine, de cet esprit qui aspire à tout comprendre, tout expliquer, tout concilier. Proclus s'appelait le pontife de toutes les religions ; il aurait pu ajouter : et le philosophe de toutes les écoles" (Étienne Vacherot, *Hist. critique de l'école d'Alexandrie*).

condition: the acknowledgment of the Pythagorean theorem of the square constructed on the hypotenuse of the rectangular triangle.

There exists, however, a still more ancient construction of a perfect pentagon which can be produced without this condition.

It will be remembered that, in connection with the practical geometry of the medieval artisan, we have already somewhat touched upon the subject, viz. how the "Fünfeck mit einspringendem 'Winkel'" (the pentagon with inverted angles) mentioned by Cantor, and which is attributed to Hippocrates of Chios, is a formula of remembrance for the construction of the pentagon; at the same time, we pointed out that this "Fünfeck" is identical with the pentagon constructed with a stretch of the compasses such as it is found in *Geometria Deutch* of 1487, and in Albert Dürer's work.

We will now consider the "formula of Hippocrates" in connection with the construction of Euclid. In fig. 160*a* we see the "pentagon" of Hippocrates with "inverted angles" in darker shading, and in fig. 160*b* we see the corresponding figure from *Geometria Deutsch* emphasised also in darker shading. As fig. 160*a* shows, this figure of Hippocrates is produced between the diagonals of the primary and secondary pentagons, and it shows unmistakably that Hippocrates has made theoretical speculations on the angles produced by the diagonals of the pentagon. The line $a\ b = a\ d = b\ e$. The angles $d\ a\ b$ and $a\ b\ e$ = the angle $d\ c^1\ e = 144°$. In the corresponding "popular" figure A B = A D = B E, and the angles at A and B = the angle at c^1 = 150°. The "Hippocratic" figure cannot be drawn unless the equilateral and equiangular pentagons with their diagonals are drawn first. We shall now demonstrate, further, that this pentagon is to be found by the corresponding "popular" figure.

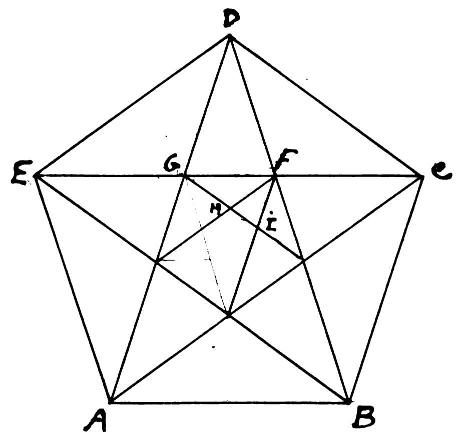


Fig. 159.—The pentagon.

In fig. 160*a* the given radius A B, and its three circles, are drawn in strokes. We trace the vertical $F^1\ H^1$. By drawing lines from points A and B, for example a line from B, through C^1 and similarly by drawing a line from the intersecting points of the circle (for example point *a* lower to the right) through A, the two lines will intersect the circle drawn in strokes, at point D higher on the left (see the similar points B, C^1, D, also *a*, A, D, in fig. 160*b*, where the introductory construction is given entirely). With $C^1\ D = C^1\ a$ as radius the fully drawn circle is produced, in which $H^1\ F^1$ becomes the vertical diameter. By drawing lines from zenith F^1 through the extreme points A and B of the radius, marked by small rings, until these lines intersect the circle at points X and Y, and by drawing the line X Y between these points, finally by halving the angles F^1 X Y and F^1 Y X, we get the circle divided into five equal parts, $F^1\ e$, $e\ y$, $y\ x$, $x\ d$ and $d\ F^1$. By drawing lines from the two corners *d* and *e* of the inserted pentagon, through C^1, the angles of *d* and *e* are halved and the "hippocratic" "pentagon with inverted angles" is produced.

It can also be interesting to point out that a line from the corners of the base of the square circumscribing the circle, for example from point a^1 on the right, through the extreme point A on the radius, together with the vertical diameter, sections out the opening of the arc $d\ F^1$ = one-fifth of the circle.

Thus, with the help of the "Hippocratic commemorative formula," we have in the meantime found two ways of constructing the regular pentagon and the sectio aurea according to a harmonic geometrical progression.

The following analysis will give us two ways more, however.

For the sake of continuity, the triangle between the diagonals X F' Y of the pentagon is transferred to fig. 160*b*. The two equally large radii D A and B E are lengthened, and with their intersecting point C as centre and C D (= C E) as radius, a new circle is drawn. This is circumscribed by a square, marked by the red diagonal L V. Through the centre *c*, we introduce the horizontal diameter O P. This diameter O P is intersected at points *l* and *m*, by the circle round centre C' with C' E as radius. This section *l m* = F' X, and thus equals the diagonal of the pentagon in this circle having C' E as radius. Similarly, it will be seen that the same circle intersects the higher side of the square through zenith F at points *p* and *g*. This section P *g* is the side of the same pentagon, where the chord *l m* forms the diagonal.

The "Hippocratic commemorative formula" has thus given us four different solutions for the construction of the regular pentagon—that is, two in fig. 160*a*, where we get the summit of the pentagon to coincide with zenith F', and two in fig. 160*b*, where it coincides with nadir H'.

In addition to the already indicated development of the proportion of the sectio aurea which is included in the pentagon, we shall now find, in the interplay of equilateral triangles, squares, and regular polygons which are produced by the "formula" a characteristic harmony similar to the construction of Euclid, because, like the latter, it develops constantly after the proportion of the sectio aurea. By adding several lines between the various intersecting points, a wonderful new play of proportions will be developed.

The list under fig. 160*a* and 160*b* show fifteen of these proportions of the sectio aurea, which could be multiplied indefinitely.

It can also be seen that in these circles the sides of the five regular polygons are produced:

(1) The equilateral triangle: by line D E in the large circle with C as centre and C E as radius (later called circle No. 1).
By D β in the circle with C' as centre, and C' E as radius (later called circle No. 2).
By N G in the three original circles with A B as radius.

(2) The square:
By H P in circle No. 1.
By α β in circle No. 2.
By D N in circle No. 3.

(3) The pentagon:
By H T in circle No. 1.
(By diagonal H N' in circle No. 1).
By *p q* in circle No. 2.
By F X and F Y } Diagonals in circle No. 2.
By *l m* }
By H' O (fig. 160*a*) diagonal in circle No. 3.

(4) The hexagon:
By C E in circle No. 1.
By C' E in circle No. 2.
By A B in circle No. 3.

(5) The decagon:
By H' C in circle No. 1.
By T N and X H' in circle No. 2.

In addition we find by H R, t u and H^1 R the side of a heptagon in circles No 1 and 2 respectively, besides octagons and dodecagons.

We find also trapezes with three equal sides, such as D, A, B, E, D, and *f*, *g*, *h*, *i*, *f*, the side of which $f\ g = C^1\ E$, and finally :

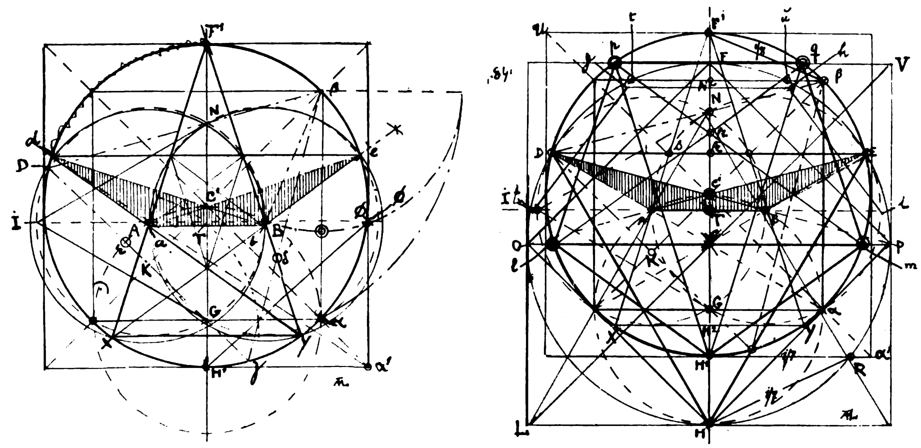
Segments of circles and their angles.

Although these phenomena are outside the limit of our subject, we could not leave them out because they complete the impression made by the marvellous geometrical play which the figure of Hippocrates sets in motion.

We know of Hippocrates that he wrote the first book on geometry, and, as we remarked earlier, he was expelled from the sect of the Pythagoreans for having taught their doctrine publicly.

We saw that, by drawing the diagonals in Euclid's construction of the pentagon, we practically produced a commemorative formula for his whole geometrical system.

The case is the same with the formula of Hippocrates, although it is not evolved with so much subtlety. This cannot surprise us, as it is probably older than the discovery of the Pythagorean theory. In Hippocrates we do not only find again the problems which are contained in the "formula" of Euclid, but—what we consider a most valuable criterion of its genuineness—that, in addition to showing the five regular polygons as bases of the five regular bodies, it also leads to the very cosmic problems which we know have been treated by



(1) $H^1F : NI = M : m$
(2) $D\beta : IT$ = ,, ,,
(3) $\beta\delta : S\gamma$ = ,, ,,
(4) $\gamma\epsilon : \epsilon D$ = ,, ,,
(5) $a\beta : \beta B$ = ,, ,,
(6) $NI : \frac{K\text{-}\beta}{2}$ = ,, ,,

See Fig. 157a

(7) $LV : DE$ = ,, ,,

(8) $Ua : D\beta = M : m$
(9) $HF^1 : K\beta$ = ,. ,,
(10) $HP : HR$ = ,, ,,
(11) $a\beta : H^1R$ = ,, ,,
(12) $\dot{a}\beta : m^{a1}HF$ = ,, ,,
(13) $D\beta : H^1Æ$ = ,, ,,
(14) $HF : H^1r$ = ,, ,,
(15) $HC : H^1C$ = ,, ,,
etc.

Fig. 160 *a* and *b*.—The "Hippocratic formula" and the sectio aurea.

Hippocrates, viz. parallel trapezes inscribed in the circle which were to be used in his attempt to square it. We also find half-circles and various segments with their angles in the circle. Lastly, the following problem will be found illustrated: the surfaces of the circle are to one another as are the squares on their diameters.

But the spirit—to speak in the words of the ancients—of this marvellous geometrical play of the figures of Euclid and Hippocrates is the sectio aurea, or, if one prefers, the pentagon. No wonder, therefore, that "the sectio aurea, as it was called, had a mysterious meaning among the Pythagoreans." *

We know, moreover, that the Pythagoreans saw in the pentagon and the pentagram the symbol of the harmony of the universe. Having mentioned that the pentagram was the sign of recognition and of greeting among them, in their letters, Cantor remarks: "In many cases, the mysterious meaning which this equally strange figure of the pentagram had for the Pythagoreans confirms the supposition, hardly doubted, that the regular pentagon was discovered by the Pythagoreans themselves." †

He attributes to the Pythagoreans the construction of the five cosmic bodies. The ancient authors are also of this opinion, but they think that these constructions could not be made without the Pythagorean theory. Nevertheless the formula of Hippocrates shows that this condition is not necessary, and it seems to us very probable that the construction of the pentagon according to this formula is very ancient and the admiration of the people of antiquity for it and for the constructive power of the sectio aurea can be just as ancient. Be it as it may; *this* at least is certain: the sectio aurea plays such a dominant part in classic geometry that we can see in this fact an expression of the Greeks' admiration for its wonderful characteristics. As the "Hippocratic formula" shows, these characteristics must have been known to the Greeks long before Euclid, if they were not already known to the Egyptians, to whom several writers attribute the use of the proportion of the sectio aurea in their buildings. The figure of Hippocrates shows in any case that the Pythagoreans knew this proportion. Cantor is of the same opinion, as shown by his explanation of Proclus's list of ancient mathematicians.

"Eudoxus, says the register, explains moreover that Plato had begun to treat of the section, using analytic methods. We are told in an unclear sentence which means, if we understand it rightly, that the section, ἡ τομή, which Plato had begun to examine, was considered as a quite special section, held to be very important at that time. This seems to be the proportion called later the "sectio aurea." This sectio aurea is now found in the analytic method of the first five axioms of Book XIII of the *Elements* of Euclid, after being already taught in Book II, axiom 11. The supposition that these five axioms were the discovery of Eudoxus, and were carefully kept by Euclid in their proper order, is very probable. It may be added that Eudoxus, in dealing with proportions, must necessarily have come to the one in question, which cannot be dealt with arithmetically, but can only be arrived at by geometry. We think he must have been led to it, as we said before, because the Greeks used numbers in preference to figures and symbols in space, and, if Eudoxus has done the same, then we understand why in the register of mathematicians his works on the proportions and on the sectio aurea are mentioned together." ‡

* *Der Goldene Schnitt* wie ihn eine spätere Zeit genannt hat bei den Pythagoräern eine mystische Bedeutung hatte" (H. Weber, *Encyclopædie der Elementarmathematic*, i, Leipzig 1903, p. 31).

† "Unter allen Umständen ist diese seltsame Bedeutung, welche die freilich auch seltsame Figur des Sternfünfecks bei den Pythagoräern besass, eine Unterstützung der kaum mehr bestrittenen Vermuthung, dass das regelmässige Fünfeck von den Pythagoräern selbst entdeckt worden sei" (Cantor, *Gesch. d. Math.*, 2nd edition, vol. i, p. 166).

‡ "Eudoxus, sagt uns das Verzeichniss noch, führte weiter aus, was von Platon über den Schnitt begonnen worden war, wobei er sich der analytischen Methode bediente. Der Schnitt, ἡ τομή, über welchen Untersuchungen von Platon begonnen waren, muss wie in richtigem Verständniss dieses lange für unerklärbar dunkel gehaltenen

If the following chronology is correct, Pythagoras 569–470 B.C., Hippocrates about 450, Eudoxus 408–355, Euclid about 300, then we have fully 200 years for the development of Greek mathematics, in which we find that what we could call the geometrical and dynamic qualities of the sectio aurea were fully admitted and looked upon with astonishment and admiration.

With this acknowledgment, there must necessarily follow the acknowledgment of the æsthetic qualities of the sectio aurea, and its superiority over all other proportions. It is only natural, therefore, that we should find it used in architectural works from those times.

The sectio aurea played no small part in Greek architecture at the time of Pericles. This mighty and beautiful proportion, that is, the "stetige," is at its best in the Athenian buildings of the years 450–430. It occurs with such regularity that we cannot believe it to be mere accident, when we remember the connection between the sectio aurea, the regular pentagon, and the Pythagorean theory.*

The view of the compiler from Trondhjem, as regards the history of the sectio aurea is taken entirely from Pfeifer, whose production he embellishes with quotations from other writers. As an example of the theoretical knowledge of the Middle Ages concerning this proportion, Pfeifer quotes a German work on geometry of 1599 and its rendering by Michel Chasles, the French historian of mathematics, on a very important point historically, in the work of Johannes Campanus from Novare, who wrote in the second half of the thirteenth century. The compiler gives the quotation as a result of his own reading, but, thanks to his misunderstanding of Pfeifer's rendering of Euclid, he represents Johannes Campanus as "the first who introduced the sectio aurea as a specially important mathematical proportion." † In his ignorance of these subjects, the compiler puts history upside down. Johannes Campanus, like nearly all mathematicians in the Middle Ages, was no great scientist, inasmuch as he did not make any new discoveries within his own science; but he knew his classics and understood their meaning. As regards the sectio aurea it is these very classics he refers to in the quotations which the compiler has used third hand. This third-hand quotation of the latter is taken by Michel Chasles from what is called Book XIV of Euclid. This Book XIV is now admitted by all to have been written by Hypsikles in the second century B.C., and is really only a continuation of Book XIII of Euclid on the five regular bodies. A comparative analysis of the reciprocal proportions of these bodies is given in the former. Naturally the sectio aurea is used here constantly. Johannes Campanus made a few annotations to Hypsikles; he also gives in Book XIV, proposition 10, a résumé of the speculations made by the

Ausspruches erkannt worden ist, ein ganz bestimmter gewesen sein, ein solcher, dem die damalige Zeit die grösste Bedeutung beilegte. Das aber war der Fall mit dem Schnitt der Geraden nach stetiger Proportion mit dem sogenannten *goldenen Schnitt*, wie die spätere Zeit ihn genannt hat. Der goldene Schnitt tritt nun grade in Verbindung mit Anwendung der analytischen Methode in den fünf ersten Satzen des XIII Buches der euklidischen Elemente auf, nachdem er schon im II Buche als Satz 11 gelehrt worden war. Die Annahme, jene fünf Sätze seien Eigenthum des Eudoxus und von Euklid in ihrem Zusammenhange pietätsvoll erhalten, hat sonach eine grosse Wahrscheinlichkeit für sich. Es sei ergänzend nur hinzugefügt, dass Eudoxus bei Untersuchungen über die Proportionenlehre fast mit Nothwendigkeit auch zu solchen Verhältnissen geführt werden musste, für welche Zahlenbeispiele nicht möglich waren, und deren Behandlung nur geometrisch gelang. Wir sagen, er musste dahin gefühhürt werden, weil wie wir im Vorbeigehen bemerkt haben, der Grieche die Zahl vorzugsweise in räumlicher Versinnlichung zu betrachten pflegte, und hat Eudoxus sie ebenso betrachtet, dann verstehen wir, warum das Mathematikerverzeichniss die Leistungen des Eudoxus in der Proportionenlehre und um den goldenen Schnitt in einem Athemzuge ausspricht" (Cantor, *Gesch. der Math.*, 2nd edition, vol. i, p. 228).

* "Der goldene Schnitt spielte in der griechischen Baukunst der perikleischen Zeit eine nicht zu verkennende Rolle. Das ästhetisch wirksamste Verhältniss, und das ist das stetige, ist in den athenischen Bauten aus den Jahren 450–430 aufs Schönste verwerthet. Wir können bei solcher Regelmässigkeit des Auftretens nicht an ein instictives Zutreffen glauben, am wenigsten, wenn wir des eben berührten geistigen Zusammenhangs zwischen goldenem Schnitte regelmässigen Fünfecke und pythagoräischen Lehrsatze gedenken" (Cantor, *Gesch. d. Math.*, 2nd edition, vol. i, pp. 166 f.

† Dedekam, *Gotik og Geom. Syst.*, p. 44.

Greek geometrician Aristæus (about 320 B.C.), and by Apollonius (about 350 B.C.) on the inner relation between the five bodies inscribed in one and the same sphere or in one another. Johannes Campanus adds a conclusive remark to this résumé, which can probably be attributed to Hypsikles: "Mirabilis itaque est potentia lineæ secundum proportionem habentem medium duoque extrema diuisæ. Cui cum plurima philosophantium admiratione digna conveniant: hoc principium vel præcipuum ex superiorum principiorum inuariabili procedit natura, ut tam diuersa solida tum magnitudine, tum basium numero, tum etiam figura, irrationali quadam symphonia rationabiliter conciliet." * (There is a marvellous power, therefore, in a line divided according to the sectio aurea. Most of what attracts the attention of people with a scientific mind falls under the reach of this proportion, because out of the unchangeable character of the laws mentioned above, there stands forth as a fundamental or main law the fact that the sectio aurea, in a calculable way, causes to agree, in an unreckonable harmony, bodies mutually so different regarding size, number of surfaces and shape, as the five regular bodies treated above by Hypsikles").

It is in keeping with his ignorance of Euclid and Greek geometry, that the compiler, misunderstanding a careless phrase of Cantor (Book II, 1st edition, p. 93, 2nd edition, p. 105, etc.) explains to the world that Campanus, "in his thirteen additions to Euclid, Book IX, proposition 12, has also demonstrated, among other things, the irrationality of the sectio aurea."†

Campanus has made thirteen additions to Euclid Book, IX, prop. 16. The last of these is to prove that it is impossible to divide a number in such a manner that the product of the whole and of one of its smaller parts should be equal to the square on the larger part, or to use the very words of Campanus: "Quod 11 secundi proponit faciendum in lineis: demonstrat hoc impossibile esse in numeris" that, what in Book II, prop. 11, Euclid proposes to do geometrically cannot be done in numbers—or, in other words, that the sectio aurea is unreckonable. Cantor finishes his account of this by saying: "Die Irrationalität des goldenen Schnittes ist somit festgestellt" ("The irrationality of the sectio aurea is thus proved").

This rather careless and vague statement of Cantor has led the inexperienced Norwegian compiler to credit Campanus with having "demonstrated the irrationality of the sectio aurea"! As if this had not been proved ages before, not only by Euclid in Book XIII, prop. 6, among other places, but recognised also much earlier by the Pythagoreans. Without this knowledge the *Elements* of Euclid, particularly Book XIII, and the whole of antique geometry, could never have been produced.

In this geometry we find the irrational, the unexplainable, the unnamed, to be the creative principle of life, the very unexplainable idea of God. Among the Jews, it was said that no one had ever seen God, and that he who did would certainly die. Proclus relates the following Greek legend: "The Pythagorean Hippasos, who first brought the idea of the irrational, from the hidden world into open knowledge, perished through this ungodliness, in a shipwreck at sea; because that which cannot be expressed by numbers, and is without image, should always be kept hidden: therefore Hippasos, who accidentally happened to touch and discover this representation of life, was transferred to the origin of times and kept isolated there, surrounded by the eternal currents. Such a respect had these men (the Pythagoreans) for the teaching of the Irrational." ‡

This beautiful legend reflects the conception of ancient times, that geometry was a divine science, the mystery of which should be reserved for those possessing intellectual capacity; this explains the secrecy with which the Pythagoreans surrounded themselves and their science.

* XIV, 10, in Paris edition of 1516, folio 250*a*; Basle edition of 1546, p. 462.
† Dedekam, *Gotik og Geom. Syst.*, p. 44.
‡ Cantor, *Gesch. d. Math.*, 2nd edition, vol. i, pp. 164 and 171.

That is why they used the sectio aurea as a sign of recognition and the pentagram as the key to their geometry. Even from the time of Pythagoras, the irrationality of the sectio aurea was known, and *must* have been. When Cantor says of Johannes Campanus's thirteenth addition to Book IX, prop. 16, of Euclid, that "the irrationality of the sectio aurea is thus proved," he only means that this was the case in arithmetic, because, as Cantor himself says: "The Pythagoreans taught the irrational. For the Greeks, the irrational could not be expressed in number, therefore we have to call it the irrational, and not the 'irrational number.' For the Pythagoreans the unit was not a number, but a multiple of it.' *

The compiler has not only lacked the capacity to understand the meaning of Cantor, but he has also lacked the capacity necessary to understand the statement of Johannes Campanus on the wonderful and admirable qualities of the sectio aurea.

The historical interest of this statement, as shown by our examination, is not at all that Campanus is "the first who put forward the sectio aurea as a specially important mathematical proportion," nor his demonstration of the arithmetical irrationality of it, which follows naturally the geometrical one; but that he renders the ancient conception of the sectio aurea and *proves precisely, through it, the connection between the scientific attainments of ancient and medieval times*.

An example, no less amusing, of the lack of geometrical understanding of the compiler is when he relates of Johannes Campanus that he "is the first who, in his edition of Euclid, has calculated the sum of the angles in an irregular pentagram, viz. 2 R"; and, referring to Cantor, he adds: "a calculation which is not found in Euclid." He continues naively: "In this connection one must also add that in the previously mentioned French *Traité de Géométrie*, from about 1275, one finds the pentagram inscribed into the pentagon." Cantor says on this point: "The figure of the external convex pentagram with the many summits is inscribed by means of the pentagon, which seems remarkable." From this we must conclude that there is no evidence of the mathematical figure which consists of a combination of the pentagon and the pentagram having occurred in mathematical works before 1275. We can say, further. *that the first who used the regularity of regular polygons* as a means of testing the correctness of the sum of their angles was Charles de Bouvelles (1503). ***He was also the first who united the pentagon and the pentagram into the figure which is produced by lengthening the sides of the convex pentagon and by drawing its diagonals***."†

In explanation of the science of the compiler and of his ability to understand what he reads, we will first quote what Cantor really says of de Bouvelles:

"In the French edition [it was published in Latin in 1503, and then in French in 1547] de Bouvelles speaks of *the regular polygons and other figures which are derived from them*. He begins with the pentagon A, B, C, D, E [fig. 161]; through extending all the diagonals he forms another inner pentagon, having its summit turned down, and which forms a pentagram by extending all its sides [fig. 162]. By using both these methods together, the sum of the angles of the pentagram can be found. All the angles of the pentagon together = 6 R, and each angle = 108°. The diagonals divide again each angle into three equal angles of 36°, having the total of 2 R. *Whether the limitation of the regular polygon, which de Bouvelles calculates is new, can be questioned*. Bradwardinus and others, who treated of pentagons, scarcely mention this limitation; we do not mention it in our account, as we do not tie ourselves to regularity in our figures, *but it is a fact that the figures of this ancient* writer are all

* "Pythagoräisch war die Lehre vom Irrationalen. Vom *Irrationalen* sagen wir und müssen wir sagen, nicht von der Irrationnalzahl; denn das Irrationale var den Griechen keine Zahl. War den Pythagoräern doch sogar die Einheit noch keine Zahl, sondern erst eine Vielheit von Einheiten" (Cantor, *Gesch. d. Math.*, 2nd edition, vol. i, p. 175).

† Dedekam, *Gothik og Geom. Syst.*, p. 45.

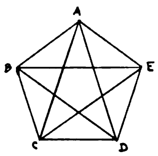


Fig. 161.—Pentagrams inscribed in the pentagon. From de Bouvelles.

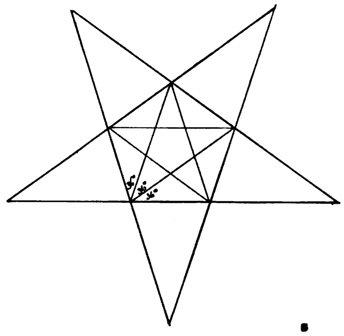


Fig. 162.—Pentagram developed from the pentagon. From de Bouvelles.

regular. The only novelty in de Bouvelles is that he uses the regularity as a test and that he emphasises it." *

The compiler's quotation of Cantor is wrongly interpreted, as usual, from beginning to end. Even Cantor's remark of the use made by de Bouvelles of the regularity as a test is not quite correct; what is new is only that de Bouvelles *emphasises* the regularity—while all previous writers take it for granted,—for the sole reason that the test becomes in this way simpler and therefore easier. Even from the time of the simple test of the Pythagorean theorem by means of the square, which diagonal becomes the hypotenuse, all the ancient geometricians used regular polygons, except in one or two cases in Euclid. This is the case without exception in the Middle Ages, as claimed by Cantor. Johannes Campanus has not, in consequence, calculated the sum of the angles of an irregular pentagram. He made five annotations to Euclid, Book I, prop. 32, which show that the external angle, by the extension of one side of the triangle is equal to the two opposite inner angles added together, and that the three angles together are equal to two right angles. The last annotation calculated the sum of the angles of the pentagram.

As external angle of $\triangle\ fdb$, $\angle\ afg = \angle\ b + \angle\ d$, just as $\angle\ fga$ as external angle of $\triangle\ gce$ is equal to the two angles c and e; but, as the two angles afg and fga with the angle $a = 2$ R, so it follows that the four angles b, d, and c, e, with the angle $a = 2$ R.

As the sum of the angles of any triangle is = 2 R, the sum of the angles of any—that is, irregular pentagram—must also be = 2 R. Cantor,† who takes for granted that the reader possesses some reasoning power and some knowledge of the subject, has accompanied his free rendering of the proof taken from Campanus with a freely drawn irregular pentagram.

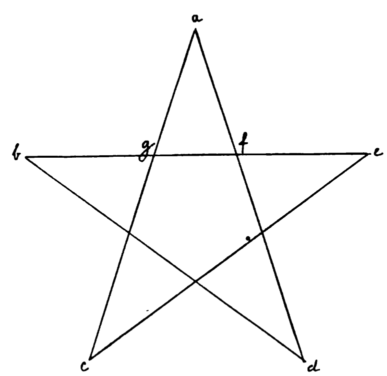


Fig. 163.—Pentagram.

* " In der französischen Ausgabe [there came out first a Latin edition in 1503 and one afterwards in French in 1547] spricht *Bouvelles von den regelmässigen vielecken und anderen, welche sich daraus ableiten.* Er beginnt mit dem Fünfeck A, B, C, D [fig. 161]; und leitet durch Ziehung aller Diagonalen ein ähnliches inneres aber mit der Spitze nach unten gekehrtes Fünfeck ab, welches selbst wieder durch Verlängerung sämmtlicher Seiten zum Sternfünfecke wird. Diese beidem Entste hungsweisen vereint betrachtet lassen aber die Winkelsumme des Sternfünfecks erkennen. Alle Fünfeckswinkel zusammen betragten 6 Rechte, der einzelne 108°. Die gezogenen Diagonalen zerfällen jeden Winkel in 3 gleiche Winkel von je 36° mit der Gesammtsumme von 2 Rechten [and this is to be noticed:] " *Ob die Einschränkung auf regelmässige Vielecke, welche Bouvelles sich auferlegt, neu ist, dürfte fraglich sein.* Bradwardinus (1290–1349) und die Anderen, welche Sternvielecken ihre Aufmerksamkeit zuwandten, sagen zwar nirgends etwas von dieser Einschränkung, und desshalb haben wir geglaubt, in unseren Berichten gleichfalls zschweigen, in unseren Figuren uns nicht an die Regelmässigkeit binden zu dürfen, *aber die Figuren jener älteren Schriftsteller sind thatsächlich alle regelmässig gezeichnet.* Neu ist nur bei Bouvelles, dass er die Regelmässigkeit als Beweismittel sich bedient und sie desshalb betont " (Cantor, *Gesch. d. Math.*, 2nd edition, vol. ii, p. 380).

† *Gesch. d. Math.*, 2nd edition, vol. ii, p. 104 (fig. 20).

Campanus, whose work dates from the Middle Ages, knew also that regularity makes no difference to the sum of the angles, therefore he says logically: "Patet etiam quod omnis pentagonus cuius unumquodque latus duo secat ex reliquis habet 5 angulos duobus rectis æquales." * This sentence is not quoted by Cantor, that is why the compiler does not know it, but he accepts wholly the irregular figure of Cantor and announces to the world that Johannes Campanus is the first who has calculated the angles of the irregular pentagram = 2 R, a calculation not found in Euclid.†

Nor is it found. Euclid does not repeat himself. He wrote for savants and not for dilletantes. He knew the regular pentagram as well as did the Pythagoreans, whose teachings he combines, through his genius, into a united system, and we take the liberty to tax Cantor of inconsistency when he says of the Roman writer Boëtius, that he did not find any "Angerung" (encouragement) in Euclid to bring the pentagram into relation with the regular pentagon of chords.‡ This thoughtless statement is also naturally accepted by the compiler.

Euclid constructed the regular pentagon (Book IV, props. 10 and 11) by starting from the isosceles triangle with the angles of the base double as large as the angle of the summit.

We take the triangle $a\,b\,d$ (fig. 164). The angles $d\,a\,b$ and $d\,b\,a$ are each equal to 2 d, and all three angles $d + b + a = 5\,d = 2$ R. By halving we get the new angles $d\,a\,c$ and $d\,b\,e = d$. It is known that we do not find in Euclid any pentagon having the fifth diagonal $e\,c$ drawn in: he did not need it in his system, but Euclid would not be Euclid if he did not know that by drawing the fifth diagonal, two new angles would be produced, e and $c = d$, therefore, in all $5\,d = d + a + b = 2$ R. Since the creation of the world there has not been any other triangle than this one possessing the qualities required by Euclid to construct the pentagon, which is the reason that it is called the triangle of the pentagon. It is Euclid himself who proves, by means of this triangle, not only the regularity of the pentagon, but also that the sides of the triangles, as diagonals, cut one another according to the proportion of the sectio aurea and in irrational "sections"; it is unthinkable, then, that he should not know and still less give the "encouragement," spoken of by Boëtius,—to draw the fifth diagonal which any child, when drawing, will trace quite instinctively, following the human instinct of connecting points with lines—and to speculate wonderingly on the laws governing the relation between the various values such as Boëtius does in vague and half mysterious sentences:

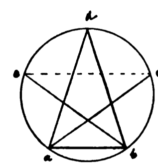


Fig. 164.
Pentagram inscribed in the circle.

"Circum datum circulum quinquangulum æquilaterum et æquiangulum designare geometres præcipiunt. Intra datum circulum quinquangulum, quod est æquilaterum atque æquiangulum, designare non disconvenit. Nam omnia, quæcunque erint, numerorum ratione sua constant et proportionabiliter alii ex aliis constituuntur, circumferentiæ æqualitate multiplicationibus suis quidem excedentes atque alternatim portionibus suis terminum facientes." §

The end of the quotation cannot be translated in any other way than the following. Everything referring to numbers exists by its relation; they are produced by one another proportionally; arcs are multiplied in even sizes alternately producing the periphery by means of their chords.

The advice cannot be misconstructed: we draw a regular pentagon, a, b, c, d, e, the arcs $a\ b$, $b\ c$ are doubled, and the chord $c\ a$ is drawn; the chords $c\ e$ and the chords $b\ d$ and $b\ e$ are carried across the doubling of the arc $b\ d$, and if these are taken in pairs, $c\ e$ and $c\ a$, $b\ e$ and $d\ b$, then the outline of the pentagon a, b, c, d, e is produced.

* Paris edition, fa 16*b* Basle edition, p. 28.

† Dedekam, *Gotik og Geom. Syst.*, p. 45.

‡ Dedekam, *Gotik og Geom. Syst.* p. 45.

§ Cantor, *Gesch. d. Math.*, 2nd edition, vol. i, p. 547. Note 2, quoted from Boëtius (edition Friedlein), p. 389.

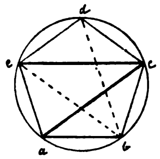


Fig. 165.
Diagonals of the pentagon : Euclid, XIII, 8.

One thinks involuntarily of the 8th proposition in Book XIII of Euclid: "In a pentagon having equal sides and equal angles, when the diagonals are situated facing two angles on a level, they divide each other in an extreme and a mean ratio, and the larger parts are equal to the side of the pentagon."

The oldest existing manuscript of the *Geometry* of Boëtius dates from the eleventh century, and it has been claimed that it does not give the original text, but that it is only an extract or a later version. The mysterious and poetical expression in the quotation on proportionality speaks for the correctness of this opinion; this mode of expression is thoroughly medieval, and has not much similarity with the comparatively far clearer way in which Boëtius expresses his thoughts on proportionality in his treatises on music. There is no reason, consequently, to do like Cantor, who often teaches history like a pedagogue when he seems unwilling to allow Boëtius so much geometrical knowledge or to admit that he arrived by himself to the idea of uniting the pentagram and the pentagon.*

* It was only after we had written what precedes concerning Boëtius and the pentagram that we thought it incumbent on us to read the work of Chasles on the *History of Geometry*. We reproduce lower a lengthy extract of his statement. It coincides entirely with our own view, as it can be seen, and it proves precisely the unbroken connection between the science of ancient and medieval times.

Michel Chasles: *Aperçu historique sur l'origine et le développement des Méthodes en Géométrie*, 2me édition conforme à la première. Paris, 1875, 4to, p. 476:

On a part of the "Geometry" of Boëtius relating to the pentagon of second order.

ORIGIN AND DEVELOPMENT OF STAR POLYGONS

In the first book of his *Geometry*, which is a translation of the propositions taken from the first four books of Euclid, Boëtius gives with each theorem or problem his definition only, and the figure belonging to it.

His first proposition, taken from Euclid is the problem of inscribing a regular pentagon in a circle (11th proposition, Book IV). After the statement of this problem, there is only the accompanying figure as usual, but the remarkable thing about it is that it shows at the same time the ordinary pentagon and the star pentagon, or pentagon of *second order*. In addition to this figure there is afterwards an explanation which does not usually accompany any of the other propositions, and which seems to us, therefore, to have been put here in order to explain this double figure, or rather this new pentagon corresponding to the new problem.

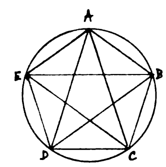


Fig. 166.
Star polygon.

As this passage of Boëtius is rather difficult to understand, and as it is easy to be deceived as to its meaning, we quote it from the manuscript which is more correct than the Basle edition (1570):

"Intra datum circulum, quinquangulum quod est æquilaterum atque æquiangulum designare non disconvenit."

Here we find the figure which corresponds to the subject in question, and the writer continues:

"Nam omnia quæcumque sunt numerorum ratione suâ constant; et proportionaliter alii ex aliis constituuntur. Circumferentiæ æqualitate multiplicationibus suis quidem excedentes; atque alternatim portionibus suis terminum facientes."

Translated: To inscribe in a circle an equilateral and equiangular pentagon.

The figure answering to this gives two pentagrams, one having a new shape and consequently differing from the usual pentagon. Boëtius explains it as follows:

All that can be expressed in numbers exists by the peculiar proportion of the numbers, and these are produced proportionally from one another.

The arcs[1] get larger (grow) of a length equal to themselves, through being doubled, and their chords[2] taken in pairs form the periphery[3] of the figure.

[1] In several parts of Boëtius "circumferentia" means the arc of the circle.

[2] We translate "portionibus" by the word "chords"; "portio" means the segment of the circle, which on the whole has no other name in Latin. ("Portio circuli est figura quæ sub recta et circuli circumferentia continetur.") Here we suppose that Boëtius has taken the whole for the part, that is, the segment for the chord, because the word "chord" had then no other name; one had to say "linea inscripta."

[3] The Latins called "terminus" the extreme point of a line and the periphery of a polygon or of any other figure. ("Figura est quod subaliquo vel aliquibus Terminis continetur"—definition of Boëtius).

Boëtius was considered by his contemporaries to have reached the summit of every science, and a modern writer, Gibbon, says of his elegant and firm writing in *De Consolatione*, that it was worthy a Plato or a Tullius (Cicero). Boëtius translated the great Greek philosophers; his books on arithmetic, music, geometry, and astronomy were the "quadrivium" of the medieval schools; we also know that his translations of the Greek philosophers, in the tenth century, became the text-books of philosophy in the grammar schools.*

In connection with this it may be of historical interest to point out that, among the friends of Boëtius, we find the mathematician Cassiodorus, who, following the example of his friend St. Benedict, founded a wealthy monastery in Calabria, where the main occupation of the monks was to copy ancient writings.

The Benedictine monks came to Ireland already in the fifth century and from there they went to England and Scotland, and they also settled at St. Gallen in Switzerland. Some verses have been preserved on the kind of teaching in the monastery of St. Gallen in the tenth century†; these verses, which date from about 970, give a poetic

If this translation of the text of Boëtius is admissible, it seems to answer to the construction of the star pentagon. Let us take, for instance, A, B, C, D, E, as the five points of the ordinary regular pentagon. The arcs on its sides are A B, B C, C D, D E, E A. If we take them two and two together, they will be A B C, B C D, C D E, D E A, E A B, and the chords of these arcs are A C, B D, C E, D A, E B. If we take these chords two and two together, we get A C, C E, E B, B D, D A; treated in this way, we obtain the star pentagons.

However, we ought not perhaps to be astonished at finding this figure in Boëtius, because, as it will be shown below, it seems as if it had been known in ancient times, especially by Pythagoras. We find it again in the thirteenth century in the *Comments on Euclid* by Campanus; during three or four centuries the theory of the star polygons, then called reversed polygons, was adopted, even developed. But since then this theory has been lost sight of and remained unknown, because, even apart from the novelty of the algebraic analysis, it was thought to have only an interest of curiosity, without being of any real advantage to geometry. But the famous geometrician who created it anew at the beginning of the nineteenth century, and whose name it now bears, has given it a significance which it will lose no more, by showing its true scientific character and the tie which binds it necessarily in an indissoluble manner to the ancient polygons.

Nevertheless, this theory speaks in honour of the Middle Ages, where there is so rarely an opportunity of coming on traces of genius and of fertile new shoots. That is why we now wish to give what we have found on this subject in the much too rare documents from this historical period.

But let us first name the source from which we get the knowledge that the star polygon was an object of study in ancient times, especially by Pythagoras.

In Alstedius's *Encyclopædia* (1620, 1630, 1649), Book XV, which treats of geometry, immediately after the construction of the ordinary regular pentagon we find the following statement: "Pentagonum etiam ita scributur, et a superstitiosis notatur hoc nomine *i, e, s, u, s*." ("The pentagon is drawn here, marked with the letters *i, e, s, u, s*, on its five summits.")

"Si pentagono ita constructo addas lineam ex superiori angulo in oppositum angulum ductam, fiet illa figura, quam vocant sanitatem Pythagoræ; quia Pythagoras, hac figura delectus, adscribebat singulis prominentibus angulis has quinque literas υ, γ, ι, η, α, Germani vocante in Truedenfus: quia sacerdotes veteres Germanorum et Gallorum vocabantur Druidæ, qui dicuntur calacos ['perhaps calceos'—Chasles] hujus figuræ gestasse."

In his *Arithmologia* (1665) in the fifth part of "De Magicis Amuletis," Kircher pronounces himself in the same manner on the star polygon, which he calls "Pentalpha" because two connected sides, having another side intersecting them, form the letter A. He marks the points with the letters υ, γ, ι, η, α. The author says:

"In quibus (sigillis magicis) nil frequentius occurrit, quam pentalpha et hexalpha; est autem pentalpha nil aliud quam linearis figura in quinque A diductum, quibus Græci υ, γ, ι, η, α, id est salutem et sanitatem exprimebant, quo Antiochum vexillo imposito, iussu Alexandri in somno apparentis, mox admirabilem a Galatis victoriam reportasse Magi fingunt, eoque tanquam summæ felicitatis symbolo in suis nugamentis utuntur."

Kircher gives further on several mysterious examples where the Pentalpha is used.

In the sixteenth century the renowned alchemist Paracelsus considered the star pentagon as the emblem of health.

We see in Murhard's *Mathematiske Bibliothek* that the learned professor Kästner has treated of the Pentalpha and of the Hexalpha in his *Geometrische Abhandlungen*, 1st edition, Göttingen, 1790, 8°.

We now turn to the theory of the star polygon in its more limited aspect. We find the first signs of it in the Commentaries which Campanus, a geometrician of the thirteenth century, added to his translation of the *Elements* of Euclid, etc.

* Compare Cantor, *Gesch. d. Mach.*, 2nd edition, vol. i, pp. 535 and 799.

† Cantor, *Gesch. d. Math.*, 2nd edition, vol. i, p. 802.

description of what they knew of geometry, while bringing the thought back to Book XIII of Euclid :

> Inde superficies ponens ex ordine plures
> Trigona tetragonis coniunxit pentagonisque,
> Strenua Pyramidum speciem ductura sub altum.
> Tum laterum miras erexit ut ipsa figuras.

Translated :

> Then she (Geometry) placed five surfaces according to their order,
> Putting square, pentagon and triangle together,
> Constructing diligently the forms of pyramids.
> The figure of the sides she built up marvellously like herself.*

It is not displeasing to know that heathen science had some excellent exponents among the Church Fathers, such as St. Augustine, nor that the Primate of the Church, Pope Sylvester II,

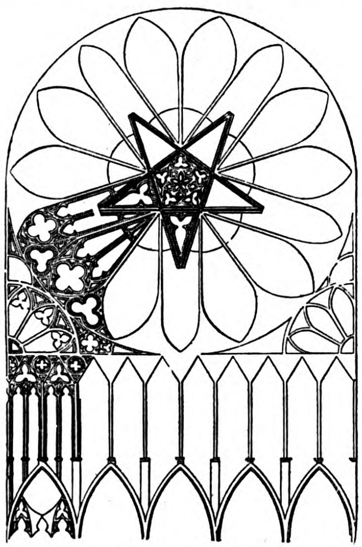


Fig. 167.—Window from the cathedral of Amiens.

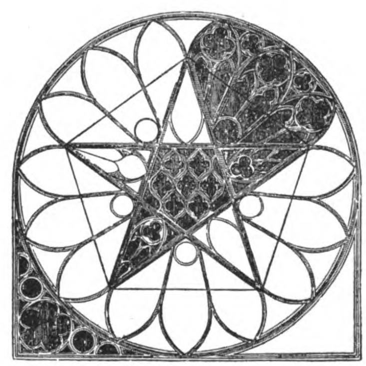


Fig. 168.—Window from the cathedral of St. Ouen in Rouen.

towards the year 1000, was an enthusiastic scholar of Greek philosophy, when, as a writer on geometry, he gave as his authorities Plato, Timæus, Chalcidius, Eratosthenes, Aristotle and Boëtius.

It is significant that these are the very same classic writers who are quoted by Johannes Campanus. Through these quotations it is easy to trace back the geometry of the Middle Ages directly to classic times.

All these names were known in Western Europe long before the interest in mathematics obtained a new stimulus due to Latin translations of Arabic translations of Greek science.

* The author of these verses, Walther von Speier, is probably the Meisteri Gualterus, quoted in the old Norwegian computative work *Rimbegla*, of the twelfth and thirteenth centuries, written partly in Norway, partly in Iceland.

Therefore, it is not through the Arabs that the Middle Ages first obtained the knowledge of mathematics, as the compiler repeats it blindly after others.

Finally, it may be of interest to put forward some examples of the combination of the pentagon and the pentagram, taken from practical geometry in medieval architecture.

The first illustration is from a window of the fourteenth century in the cathedral of Amiens; the second from a window dating towards 1440 of St. Ouen in Rouen. We have the pleasure of taking them both from the compiler's own work (pp. 16 and 17).

As will be seen, these windows from the thirteenth and fourteenth centuries contain precisely the combination of a pentagon and a pentagram, of which the compiler says, on page 45, that Charles de Bouvelles was the first to bring it into use, although his work first appeared in the year 1503!

What remains now of the whole following statement*: "It is first at the time of the Renaissance that the sectio aurea played any part as basis of theories on proportions and in nature" (sic!).

It was on the recommendation of two professors at the Technical High School of Norway, and with the consent of the Royal Ministry of Cults and Education, that the work of compilation was printed at the expense of the State, and according to the compiler's introductory preface, "as a guide to the public, as a support for those who have the responsibility of judging and of settling this question!"

* Dedekam, *Gotik og Geom. Syst.*, p. 45.

CHAPTER XI

THE SECTIO AUREA

ITS ÆSTHETIC SUPERIORITY OVER ALL OTHER PROPORTIONS

WE have shown, in the previous chapter, that the sectio aurea, as a geometrical phenomenon, played the principal part in ancient geometry, and that it was continued in the Middle Ages. It will now be our task to make clear why this proportion is superior to all others from an æsthetic point of view.

It has been claimed that it was not until the end of the Middle Ages, or, as expressed by the compiler, it is only since the time of the Renaissance that the sectio aurea is seen to have played any part as basis of theories on proportions and in nature.* The æsthetic estimation of the sectio aurea is said to be due to Luca Paciuolo, who in 1509 published his famous work, *De Divina Proportione*, in collaboration with Leonardo da Vinci. As this is the oldest existing writing which uses that name, it was concluded, without any further investigation, that the term originated with Luca Paciuolo.

It will be remembered that the ancient world looked upon the Irrational as the imageless expression of the unfathomability of God. We find this thought also in Paciuolo, who compares the unity of God in the Trinity with the sectio aurea: the latter is composed in its entirety of three parts, which even separately contain each the possibility of the same proportion, of the same nature, in the smaller as in the greater part. Like the Greeks, Paciuolo sees in the irrationability of the sectio aurea an analogy with the incomprehensibility of God. Therefore he calls it the "divine proportion" ("proportio divina"), which he praises moreover in ridiculous and enthusiastic terms such as: "Essentiale," "singulare," "ineffabile," "mirabile," "innominabile," "inestimabile," "sopra gli altri excessivo," "supremo," "excellentissimo," "quasi incomprehensibile."

When it is remembered that Paciuolo was a great admirer of Johannes Campanus, whose scientific reputation he courageously defended against the attacks of the Venetian Zamberti,† and also how the enthusiasm of the classic writers for the sectio aurea reappears in Johannes Campanus, we can hear in the exalted adjectives of Luca Paciuolo the ever-rolling and reverberating echo of the admiration of antiquity and of the Middle Ages. Consequently, it was not a discovery of Paciuolo; it was commonly known in medieval science, because, if not, whence does originate the use of the sectio aurea in the architecture of the Middle Ages, centuries before the time of Paciuolo? Can anything be more medieval than the very adjective "divine"? As sure as the Bible is older than the oldest existing Codex (from the fourth century), and that the "Norröne," or Norse pagan Bible, and the ***Eddakvadene***, are older than the oldest existing manuscript (about 1280), so is the expression "proportio divina" undoubtedly much older than the oldest existing written mention of it. The case must be the same with

* Dedekam, *Gotik og Geom. Syst.*, p. 45.

† Cantor, *Gesch. d. Math.*, 2nd edition, p. 340.

the expression "sectio aurea," which is thought to date from the sixteenth century. This term also is quite medieval. The adjectives "divina" and "aurea" were in favour in the Middle Ages, because they are emphasising and exalting epithets to qualify some especially remarkable phenomena. There is a similar example from the early Middle Ages, such as "divisio aurea," which is the technical term for a division of arithmetic. "Proportio divina" and "sectio aurea" must have been common expressions in the language of educated people, especially among builders who used it as a recognised term for the æsthetic valuing of this proportion.

Early in the Middle Ages the science of architecture was a monastic science, and later, in the twelfth century, a guild science, which was kept secret and preserved solely perhaps by verbal tradition within the guild. It seems evident that the master builders must have had a suitable term for this wonderful proportion, which had been continually employed in practical work from ancient time. We have seen that the Pythagoreans considered the teaching of *ἡ τομή* as divine; nothing is more likely than that the building guilds inherited the very ancient terms used for the æsthetic qualifications of this geometrical proportion which was always employed in the art of building. Before verifying this we will first point out why the proportion of the sectio aurea is superior to all others æsthetically.

In planning a monumental building the principal rule is that the distribution of the plan and of the horizontal lines should be symmetrical; the plan is divided with the same measure, and on the right is repeated what has been done on the left of the central axis. It is quite different when dealing with the height and the vertical lines; here the words of Viollet-le-Duc can be considered as law: "In fact, one of the conditions of harmony in architecture is to avoid obviously equal divisions, but to take care, however, that there should exist a correlation between them." * In other words, the important thing, in architecture, as in music, is not to use hard, decided parts divided alike, but to take care that notes glide over one another to avoid the ordinary march rhythm. It is supposed that the oldest song was rendered by a single voice or chorus of several voices singing the same tune at different octaves, according to the depth or height of the voices. Northern people, who are supposed to have invented the chorus, have, however, introduced an intermediate stage, which they called *fábord*, in this primitive mode of singing. In Anglo-Saxon there is still the word *faburdan*, to signify the kind of tune which was introduced among the other voices, having a sort of contrepoint function. English historians of music have thought that the word signified "to carry the burden," it was, therefore, translated thus "to bear the burden was to sing the base below, either a single part, or fuller harmony." In popular etymology the word has also gone into French as *faux-bourdon*, and in Italian as *falso-bordone*.† The Norse substantive from which derives the verb *fábyrda*, is composed of the verb *at fá*, which signifies to adorn by breaking the uniformity of the surface of an object, and of the verb *at byrda*, which means to make stripes or to weave patterns in the cloth. The Anglo-Saxon *faburdan* signifies, therefore, to interlace into the singing of the chorus a voice which is inserted like an ornamental musical pattern among the other voices to break the monotony. It is this very musical function which later was called in English *round*, or *catch*; that is to say, the voice which effects the transition in the changes of measures and of tunes.

The same is the case in architecture. Equally large, or, more correctly, equally high divisions appear monotonous and hard, and therefore tedious. The eye soon tires of such an elevation because it lacks mystery; the various divisions can be too easily measured upon one another and divided into each other. It is not transformed into a mystery of life, it lacks

* "En effet, une des conditions d'harmonie en fait d'architecture, c'est d'éviter, en apparence directe, les divisions égales mais cependant de faire que des rapports s'établissent entre elles" (Viollet-le-Duc, *Dictionnaire*, vol. vii, p. 552).

† *Encycl. Brit.*, 9th edition, vol. xvii, p. 82.

variety in opposing contracts, because it is only built on the monotony of uniformity. It is not without reason that the Pythagoreans' philosophy had based on the famous sentence "The unity of variety and the agreement of opposites"—as a condition of harmony—the fundamental opposites, *tretragonon* and *heteromekeia*, square and "rectangle" of the table of categories; and to translate simply heteromekeia by "rectangle" as do lexicologists, in no way conveys the real meaning of the word. To grasp it, it is necessary first to have understood completely what is meant in the table of categories by the square as the opposite of heteromekeia.

The square (tetragonon) is similar in Greek geometry to *dynamis* in Greek arithmetic, a rational quantity, because it is the sum of equal factors, which can always be measured by each other. A rectangle, in geometry as well as in arithmetic, is the sum of unequal factors, but these factors can nevertheless measure each other and give a rational sum, as, for example, 2×4. A "rectangle" of such factors does not answer, then, to the opposite which the table of categories requires. The rational tetragonon (*dynamis*) can only have, as opposite, an irrational heteromekeia in order to fulfil that condition. The German historian of mathematics, Hankel, appears to have held this opinion, which is also supported by Cantor.* It is only by the notion of the rational and the irrational that tetragonon and heteromekeia can get their place and meaning in the table of categories. By the rational, as the opposite of the tetragonon, a rectangle is meant, the factors of which consist of the side of a square and an irrational "section" (τομή), by which they both fall into an unreckonable harmony—to use the words of Johannes Campanus—such as it is demonstrated in Book XIII of Euclid, and as we have illustrated it in the figs. 158, 160*a*, and 160*b* given above. There we saw vastly different geometrical figures having a common uniting element, the sectio aurea.

In all that has been preserved of Greek, and particularly of Doric architecture, the tetragonon and the heteromekeia (viz. a rectangle having the side of a square and a sectio aurea from the latter as factors) are used as harmonising mediums, both in plan and elevation.

Probably on account of the Greek religion being polytheistic, the planning of the various temples is individual—that is to say, they vary according to the god to which they are dedicated. It is beyond the scope of our subject to go through the various temples to prove this. We will content ourselves to remark here generally that Doric temples, as a rule, are proportioned according to the sectio aurea in such a way that the length is fixed by either the major or minor of the width being multiplied a certain number of times. Various manners of using the pentagram and the square are the instruments of this proportioning.

Of all the Doric temples we have found the *Temple of Concordia* at *Girgenti*, in Sicily, the ancient Greek colony of Akragas, to be the most striking example of how the principle of opposites of the Pythagorean table of categories, tetragonon and heteromekeia, rational and irrational, has been carried out in Greek architecture, and also how the regular square and the seemingly accidental appearance of the sectio aurea as opposite, is just the secret of the beauty of Doric temples, and of their great silence, as Winckelmann expresses it:

"The Temple of Concordia is the one (Doric temple) which most of all gives the impression of a geometrical intention. The length is equal to accurately four times the side of a decagon inscribed in a circle, the radius of which is equal to the width of the front." †

This important information is due to the French mathematician Jules Tannery, to whom

* Cantor, *Gesch. d. Math*, 2nd edition, vol. i, pp. 150, 171, 223.

† "Dans le parallèle du tableau A (Tableau comparatif des proportions générales du temple dorique), le temple de la Concorde est celui qui laisse le mieux soupçonner une préoccupation de géomètre. La longueur en est rigoureusement égale a quatre fois le côté du décagone régulier inscrit dans un circle dont le rayon serait égal a la largeur de la façade. Il n'est pas impossible que ce résultat exact géometriquement, soit aussi historiquement vrai. C'est l'impression qui s'est dégagée pour un mathematicien distingué, M. Jules Tannery, des calculs qu'il a bien voulu faire sur les données que nous lui avions fournies."– Perrot et Chipiez, *Histoire de l'Art dans l'Antiquite*, vol. vii, p. 557.

the well-known archæologists, the Professor Perrot and the architect Chipiez had submitted their measurings. But the enormous importance of such an epoch-making observation does not seem to have been realised by any of these scholars; at any rate, it did not make them persevere further in their researches. In the course of such examinations, we find that the Temple of Concordia of ancient Akragas not only confirms the opinion we have given above, but that it provides also material for a rich chapter on the history of Greek geometry, and even of Greek philosophy—a chapter which will finally make it perfectly clear that it is this philosophy which is the foundation on which are built the Greek temples and *all* later religious architecture, including medieval architecture.

It is clear that this temple of Akragas is entirely constructed according to the Pythagorean theory of harmony, exactly like a demonstration of the interplay between tetragonon and heteromekeia,—between the square and the sectio aurea. We must, therefore, give the plan of this building the full consideration it deserves.

The general scheme of the plan of the Temple of Concordia, according to the calculation of Tannery, is reproduced in fig. 169 (Pl. XII). We draw a circle having the width of the front as radius (AB), and we divide this radius (Ak—AB) according to the sectio aurea at point F.

The distance A F is, then, the side of a decagon inscribed in the circle. This side of the decagon, which is the major of the radius, is carried on the vertical diameter, and is repeated four times on its extension ($Ah + hd$ and $Al + lb$); this gives the length of the temple ($b\ d$) according to Tannery. If the width of the front, with its major, is multiplied four times, the plan of the temple forms an irrational rectangle, or heteromekeia. For convenience we can use the expression, "temple-heteromekeia."

On the sides of the centre A of this heteromekeia, two squares or tetragons are produced ($\square$ mAk and $\square$ Aig). The sides of the square Am and Ai (= the radius of the circle) are divided according to the sectio aurea at points h and l, and the major of the radius $AF = Ah = Al$, and the minor of the radius $Fk = hm = li$.

Through the construction carried out on the right, the major ($dh = Ah$) of the radius is furthermore divided according to the sectio aurea; the major of the radius' major is therefore $h\ m$ (= the minor of the radius); the minor of the radius' major is then $d\ m$ (= $d^1 . m^1$).

If expressed in arithmetical values, as, for instance, radius = 13, its major ($AF = Ah = dh$) appears to be approximately = 8, and its minor ($hm - Fk$) $h = 5$, and the minor of the radius' major ($d\ m$) = 3; the diameter is therefore = 26.

The length of the temple consists, then, in the following arithmetical values in a harmonic ascending and descending progression of the sectio aurea 3 : 5 : 8 and 8 : 5 : 3. The plan of the temple consists, then, of the heteromekeias of the sectio aurea $3 \times 13 : 5 \times 13 : 8 \times 13$ and $8 \times 13 : 5 \times 13 : 3 \times 13$.

Divided by the width of the front or the major of the radius, the temple-heteromekeia (the plan) consists of four equally large heteromekias whose factors are = 8×13. The diagonals of these heteromekeias, such as ch = the side of the pentagon inscribed in the circle having AB as radius. Likewise it will be noticed that the diagonal mv of this pentagon intersects the diagonal ge in the square circumscribing the circle at the same point R as the two dividing lines $t\ u$ and $q\ s$ produced by the sectio aurea. If we draw now a vertical line parallel with the longitudinal centre line of the temple through the intersecting point of the sectio aurea on radius (F), there appears a new set of tetragons and heteromekeias as the products of the major and the minor of the radius and of the minor of the radius' major. Through the chosen arithmetical values the products of the major of the radius on the right of the dividing line give the proportions $3 \times 8 : 5 \times 8 : 8 \times 8$, and the products of the minor of the radius on the left of the dividing line give the proportions $3 \times 5 : 5 \times 5 : 8 \times 5$. The

diagonal of this last heteromekeia 8·5 = the side of the pentagon in the smallest circle with AF as radius.

In this manner of dividing with the sectio aurea, the tetragon of the radius (□ A i g) we recognise the same problems of Euclid as in fig. 159. We see clearly in the same way the Euclidian interplay between the temple-heteromekeia and the large tetragon inscribed in the circle with side 2 R (= AB + Ak) = the double width of the front.

The temple-heteromekeia *abcd* = 4 heteromekeias of 13 × 8, giving a surface of 13 × 32 = 416, while the large tetragon on the double width of the front gives $(2 \times 13)^2 = 676$. If we multiply now the introduced arithmetical values in harmonic geometrical progression, according to the proportion of the sectio aurea, 1 : 2 : 3 : 5 : 8 : 15, etc., by 2, we obtain a new progression with values 2 : 4 : 6 : 10 : 16 : 26, etc. So that when the radius consists of the parts 5 + 8, the side of the tetragon on the diameter will = 2 (5 + 8) = 10 + 16. The large side of the tetragon *gp* or *pe* is then divided in the parts 10 and 16, which are thus the factors of the heteromekeias *g, u, t,* or *q, p, g*. The values of the tetragon ÷ 676 (10 × 26) = 416 = the temple-heteromekeia. The geometrical proof of this is glaring.

The side *g p* of the large tetragon is divided by the sectio aurea at point *u*; the line *u t* cuts then in that tetragon the irrational heteromekeia having the major *g u* of the side of the square as base. The diagonal *g b* of heteromekeia *g i b* and the diagonal *c m* in heteromekeia *c d m* = the diagonals A *t* and A *u* of the two heteromekeias A *m t* and A *i u*. Consequently the surface of the major-heteromekeia of the large tetragon *t u g* = the temple-heteromekeia (*a, b, c, d*) itself.

The statement of Jules Tannery, that the width of the Temple of Concordia is the radius of a circle and its length four times the side of a decagon in that circle, has thus led to the important acknowledgment that the thought lying at the base of the conception of the plan is: tetragon and heteromekeia. The statement does not really give the meaning or the aim of the plan, or even its formula, but it only shows one of the many modes of construction, in which the plan of the temple can realise the following claim: that it is the heteromekeia of a tetragon, and that its surface measurement stands in proportion to this, according to the sectio aurea, as *m* : M. In other terms, we get an authentic testimony for the fact that a square forms the basis of the horizontal plan of the Temple of Concordia. We shall also come to acknowledge, later, that this applies equally to the vertical plan—the conception of the elevation of the front—not only this one, but all Greek temples—in the Middle Ages equally, because the proportioning is governed by the tetragon and the heteromekeia of the table of categories, although in a different manner.

In order to examine further the working of these two opposite conditions of harmony in the plan, we shall continue the analysis.

In the centre of the plan *a b c d* (fig. 170, Pl. XII) we trace the circle having as radius *a b*, the width of the front of the temple. We call this circle ○ III, and the circumscribed tetragon □ III.

Furthermore, we draw inside this circle having as radius the major of radius AF in Fig. 169 the ○ IV and its tetragon, □ IV.

Again we draw in ○ IV having as radius half the width of the front of the temple, ○ V and its circumscribed tetragon □ V, its side being then = the width of the front.

If we draw ○ II having as radius half the length of the temple (Ab = Ad) round ○ III, we get the circumscribed tetragon □ II.

Lastly we draw ○ I having half the diagonal (Ad) of the plan of the temple as radius, round ○ II and the circumscribed tetragon □ I.

In fig. 170, ○ III and ○ IV and their tetragons □ III and □ IV will be recognised as circle III of fig. 169, having the width of the front as radius and circle IV having the

major of the width of the front as radius, and also their circumscribed tetragons. Later on—in the following chapter—we shall see that these various circles agree unanimously with the interior organic distribution of the temple, and that this is due to the mutual conditions of these circles as well as to their dimensions and proportions.

We saw, with the rough arithmetical values, that $\square$ III $= 676$ and the temple-heteromekeia $= 416{\cdot}676 \div 416 = 260$. If we take as major the more accurate value of $0{\cdot}618$ (unit $= 1$) we get $416 \times 0{\cdot}618 = 257$; this value is also the dimension of surface of $\square$ IV $= (2 \times 8)^2$, and of the four tetragons on the major of the width of the front in the temple-heteromekeia $= 4 \times 64$ or $24 + 40 + 64 + 64 + 40 + 24$. If we had set the value of the base higher in the progression of the sectio aurea—for instance, if we had made the radius $= 55$ (*m*) $+ 89$ (M) $= 144$, or even higher—we would have obtained an arithmetical value, by which the difference 260-256 would have almost disappeared. The condition, consequently, is practically that $\square$ III $=$ $\square$ IV $+$ the temple-heteromekeia $= 676 = 416 + 256$ (260). In other words, we have a harmonic progression according to the sectio aurea $676 : 416 : 256$ (260).

Moreover $\square$ V $= 13^2 = 169$ and the heteromekeia having as factors the radius and the major of the radius $= 13 \times 8 = 104$.

Taking as starting-point the rough calculations in fig. 169, it will be seen that we have obtained a harmonic progression, beginning with the surface dimension of 4 as value of the smallest tetragon. We get then $4 : 6 : 9$ (10) $: 15 :$ (16) $: 24$ (25) $: 39$ (40) $: 64 : 104$ (105) $: 169 : 256$ (260) $: 416 : 676$.

The surface value of $\square$ I is derived from the equation $ad^2 = ab^2 + bd^2$, where $ab =$ the width of the front and $bd = 4 \times$ the major of the width of the front, and $ab =$ the diameter of $\bigcirc$ I $=$ the side $\square$ I; $13^2 + 32^2 = 169 + 1024 = 1193 = ad^2 =$ the surface value of $\square$ I. The surface value of $\square$ II $= bd^2 = 1024$. Likewise $\square$ I $\div$ $\square$ II $= 169 = \square$ V. We shall also find, later on, that there exists a proportion of the sectio aurea between the temple-heteromekeia and $\square$ I.

The corresponding interplay among the various circles is obvious. Therefore as $\square$ I $\div$ $\square$ II $= \square$ V or $\left(\frac{13}{2}\right)^2 \pi$, it must also be similar to $\bigcirc$ I $\div$ $\bigcirc$ II. But the interplay between circles and tetragons is also surprisingly evident. As example of this we can state this astonishing fact, that the difference between $\square$ I and $\bigcirc$ I $= \square$ IV $= 256$. The surface of $\bigcirc$ I $= 17{\cdot}27^2$. $\pi = (\pi$ reckoned to $3{\cdot}14159) = 936{\cdot}988328111$. This value subtracted from the surface of $\square$ I $= 1193$ gives 256.011671889!

The equations that follow are not naturally the product of an abstract mathematical analysis; it is evident that, for example, the functions of the equilateral triangle are not the same in abstract mathematics as the functions of the sectio aurea. But these equations, which partly agree up to $0{\cdot}00005$ (width of the front $= 2$), show that the mentioned functions of the sectio aurea in the plan of the temple contain such an incredible wealth of geometrical proportions and equations between figures wholly different that it must have caused the greatest astonishment to the Greeks, just because they were unable to measure such small differences, and they must have been struck by the overwhelming abundance of coincidences between geometrical values; therefore it is quite unnecessary to explain that, in this examination, we have purposely used a method quite empirical with ruler and compasses, as the Greeks did; because it is only in this way that we can hope to understand the motion of their thoughts and their astonishment over their observations.

We saw above that the surface value of the temple-heteromekeia stands in proportion to $\square$ III as minor to major, or that the surface of the temple was the major of the surface of this tetragon III with the double width of the front as side.

Let us extend, in fig. 170, the longitudinal sides of the temple to points *a1 b1* and *c1, d1*,

by which the side of □ II = the distance AI — CI becomes = the full length of the temple including the krepis or step like foundation, on which it rises.

As shown in the construction of the sectio aurea marked NB higher in fig. 171 (Pl. XII), the width of the temple *cd* is the minor of the side of □ I. This result is carried to fig. 170, where point T divides the side of □ I in *m* (minor), and M (major). If the width of the front is multiplied by this side the product = the minor of the surface of □ I, which is the reverse of the proportion between the temple-heteromekeia and □ III.

We see, from fig. 169, that major A*y* of diagonal A*g* = the side of a regular heptagon in ○ III; furthermore that major (*g*R) of the whole diagonal (*ge*) in □ III = the side of a tetragon inscribed in ○ II, and that minor (R*e*) = the side of the heptagon in the same ○ II.

Moreover, as mentioned before, the diagonals in the four equally large heteromekeias of the temple-heteromekeia (*al, lk, kh, hc*) = line F*m* (in fig. 169) = the side of the pentagon in ○ III. The major of such a diagonal (*hf*) = the side of the pentagon in ○ IV.

The minor of the side of □ III (*qe* = *pu*) is at once = the major of the side of □ IV = the side of the octagon in ○ III = the side of the decagon in ○ II. Furthermore, the major of the side of □ III = the side of a hexagon in ○ II.

The diagonals (*uq* and *st*) of the heteromekeia, whose factors are the major and the minor of the side of □ III = the side of the pentagon in ○ II (fig. 169).

The diagonals *gq*, for example, of the large heteromekeias, the factors of which are the side of □ III, and its major = the diagonal of the same pentagon.

The diagonals *pt* = *se* of the heteromekeias, the factors of which are the side of □ III, and its minor = the side of the regular triangle in ○ II.

The interplay continues between tetragon and heteromekeia. We see that the pentagon in ○ II (fig. 170) circumscribes ○ III and that the triangle in ○ II circumscribes ○ IV and that the side of the same triangle coincides with the side which is the base of □ IV. We see, similarly, that the extremities of this base coincide with the intersecting points (E and F) of the side of the triangle with the diagonals of the pentagon. □ IV lies, then, between the parallel diagonals of the hexagon inscribed in ○ II. Furthermore, the base of □ III and of the pentagon in ○ II coincide (G and H are the intersecting points of ○ II and □ III). Moreover, the minor of the side of □ I (the width of the front) = the radius and the side of the hexagon in ○ III.

To test what we have stated above, we will measure, in fig. 171, the width of the front in relation to the side of □ I by means of a construction marked NB, where the two punctuated oblique parallels from the corner of the temple-heteromekeia show the width of the front to be = to the minor of the side of □ I.

Moreover, in fig. 171 we have inscribed the equilateral triangles △ I, △ II, △ III[1], △ IV, △ V, in the five circles, and to the right we have marked the height of the triangles and the length of their sides.

The minor of the side of △ I = the major of the side of the tetragon inscribed in ○ III = the side of the tetragon inscribed in ○ IV.

The major of the side of △ II = the radius and the side of the hexagon in ○ I = ½ the side of □ I and = ½ diagonal of the temple-heteromekeia (A*d* in fig. 170). In the side of the triangle marked △ II divided according to the sectio aurea, introduced in the above fig. 171, the major is PR, whilst the minor is PS (Q R). After having marked the angles of □ I of the main figure with α, β, γ δ, we can observe that the minor of the side of △ II (PS) = the distance γC = ½ the major (TU) the side of □ I (UV); also that the minor of the major of the side of the triangle, the part RS = ½ the width of the front and thereby = also ½ the minor, part of the side of □ I.

In △ III the major of the side is approximately = ½ the side of △ II = the side of △ IV. Half the side of △ III = the side of the tetragon inscribed in ○ IV, fig. 172 (Pl. XII).

In △ IV the major of the side = the minor of the side of △ III, and the minor of the side of the tetragon inscribed in ○ II.

In △ V the side = the minor of the side of △ I.

* * *

The height of △ I = the diameter of ○ III = double the width of the front.

In △ II the minor of the height = the side of the pentagon in ○ IV = ½ the side of the pentagon in ○ II.

In △ III the major of the height = the height of △ IV.

In △ IV the height = ½ the height of △ II.

In △ V the height = the minor of the height of △ II.

It will be interesting now to examine the relative proportions between the tetragons ◇ I, ◇ II, ◇ III, ◇ IV, ◇ V, inscribed in the circles, and which we have drawn in fig. 172, accompanied by a scale of the length of their side with divisions according to the sectio aurea.

In ◇ I the side = the diagonal of the pentagon in ○ III, consequently the major = the side of the same pentagon = the diagonal of the pentagon in ○ IV.

In ◇ II, the side = the side of △ III.

In ◇ III, the side = the major of the side of △ I.

In ◇ IV, the side = the minor of the side of △ I.

In ◇ V, the side = the side of the pentagon in ○ IV.

We cannot spend more time in treating of inscribed tetragons, but we will make an analysis of the relation of the pentagon to the various figures. As a preliminary we have introduced, in fig. 172, the pentagon inscribed in ○ I. The internal connection is already visible; because, as it can be noticed, the diagonals of the pentagon intersect one another and form a pentagon of second order, inscribed in ○ V, the diameter of which is the width of the temple-heteromekeia and the minor of the side of □ I.

We carry over this construction to fig. 173 (Pl. XII), which will lead us back finally to our starting-point. Pentagons are introduced in all the circles.

In pentagon I the minor of its side, or of the major of the diagonal—as we have already noticed in the previous figure—is the side of pentagon V.

In pentagon III the major of the diagonal = the side of the pentagon = the diagonal of pentagon IV. The minor of the diagonal = the side of pentagon IV.

In pentagon IV the diagonal = the major of the side of ◇ I.

In pentagon V the diagonal = the minor of the diagonal of pentagon I.

We see that pentagon IV circumscribes ○ V. Through the intersecting points of the diagonals of pentagon IV a sixth circle, ○ VI, is drawn, and a pentagon of second order is inscribed into it.

In this pentagon VI, the diagonal = the minor of the diagonal of pentagon IV = ½ the minor of the diagonal of pentagon II = one-third of the radius of ○ I.

The radius of ○ VI = the minor of the radius of circle IV. But the radius of circle IV is A F, which is ¼ of the length of the temple and the major of its width.

Here we come back to the beginning. If we now employ the previously chosen arithmetical value for the width of the front (R = 13), for its major (= 8) and its minor (= 5) we could, as an approximate value of the minor of the major, introduce the number 3, the value of which we find beneath on the left in fig. 169, and as side of the small tetragon, the surface value 9. The diagonals *g y* and *g q* divide both this tetragon and tetragon *g f y* in other tetragons and heteromekeias. The minor of the small tetragon with surface value 9, produced by the intersection of the diagonal, is equal to half the side of the very smallest tetragon with

surface value 4. Thus we have found the arithmetical unit, but, in a drawing on a larger scale and by carrying the division further, we could have found the geometrical unit, which is the base of the construction of the temple of Akragas.

We find again the tetragon with surface value 9 in the tetragon having A R as diagonals in the same fig. 169. Its side is marked by the letters (A) R.

This value (A) R appears again in fig. 170 as radius (marked AR) in the circle taken from fig. 173, where it circumscribes the pentagon of second order in circle IV. The diameter of circle VI is then the side of the smallest (VII) tetragon in fig. 170. The dotted line indicates the tetragon standing on its point, which is inscribed in ○ IV and ○ V. As it will be noticed, the diagonal of tetragon □ VII = the side of this tetragon ◇ V standing on its point.

Just as the radius of the circle introduced from fig. 173 is the minor of the radius of ○ IV, so the side of tetragon VII, which, in fig. 170, circumscribes this introduced circle, will be obviously the minor of the side of □ IV.

Furthermore, it will be noticed, that the portion *s* R in fig. 169 = *s* R in fig. 170. Consequently, as the parallels running from these points will show, the portion *p q* in fig. 169 has its corresponding value in fig. 170.

Thus, by means of squares and pentagons we have come back to the values which we obtained in the original division according to the sectio aurea in fig. 169.

This interplay between the tetragons and the circles of the above figures is the result of the geometrical entelechie, produced by the introduction of the sectio aurea.

We see now in fig. 173, that ○ I circumscribes an octagon, which again circumscribes ○ II, which circumscribes a pentagon, which circumscribes ○ III; this circumscribes a hexagon, the side of which is the width of the front. A tetragon on the side of this hexagon = tetragon V, which circumscribes ○ V. Furthermore, we see in figs. 170 and 171 that ○ V is circumscribed by a triangle which is inscribed in ○ III, whilst ○ IV is circumscribed by a triangle inscribed in ○ II.

By these various roads we come to the same interplay between the different values or their irrational sections, produced by the sectio aurea, or *τομή*.

The Greek conception of the world was based upon geometry; they used expressions and pictures from it to symbolise their idea of life, which was stimulated and made poetic by their astonishment at the inner relationship of geometrical phenomena. The philosophy of the Greeks became a teaching of the beautiful; its fundamental elements were numbers and the "elements" of numbers: even and uneven, tetragon and heteromekeia, which by their unreckonable and mutual interplay shape the harmony of the world, the building up of the universe. They found, in the construction of the five platonic bodies in one sphere, the picture of the union of opposites, the work of the Demiurge, the creative artist of the world.

We have just seen that the whole geometrical foundation which underlies this view of the mysticism of the world, of life and of beauty is brought to its fullness in the Temple of Concordia in Akragas, without the help of any written document, just as it is standing there at the present day, exactly as it was conceived and erected in stone in the time of the Pythagoreans' school—a wholly authentic document, older than Aristotle, and than Plato, unimpeachable to criticism as to faulty copying or fanciful interpretation.

In the geometrical analysis of the proportions of the plan of the temple in fig. 169, we found a united movement, a principle of activity, an *ἐντελεχεία*, existing within its own conception, produced by an interplay between tetragons and heteromekeias, between the square and the sectio aurea as means of division, creating everywhere harmonious proportion of numerals deriving from one another, from the unit itself to the extreme limit, while they partook as factors, in the condition of harmony.

We saw the same state of harmony in figs. 170 and 171, as a development of the connection between the largest surface and its smallest unit. We saw also the development of the connection between the various regular figures of surface, triangles, squares, pentagons, hexagons, and octagons—not as in the examples taken from Euclid, nor as in the commemorative formula of Hippocrates—but developing themselves as a connected *living* picture.

We noticed the harmonic proportion between the various circles of fig. 173, in an interplay where they are in turn influenced and being influenced without monotonous arithmetical cadence, unrolling in a quiet, unreckonable rhythm, like the rings formed by a stone thrown into the water; playing in a scale of intervals, unequally but not accidentally uneven, on the contrary, immovably determined from the uttermost limits to the centre, or vice versa. A thing which is rationally finished, which is kept within an insuperable barrier, is restful, because movement itself is rest where it is occasioned by the constant seeking of the level—there is ποίησις, and παθησις, active and passive. What seems accidental is no accident, nor sprung from the depth of subconsciousness; it exists fatally. The play of contrasts is not unlimited, but it is a continual movement within a given limit, a "marvellous" result of the conditions of opposite, as limited by the laws of the table of categories; such as tetragon and heteromekeia.

Only the square and the sectio aurea could produce a play of harmony with so many strings and a concert of so many various configurations.

What we claimed, at the beginning, to be a pure logical necessity, must now have been proved to be a geometrical and arithmetical fact, namely, that "heteromekeia," as the opposite of "tetragonon" in the table of categories, signifies only the proportion between the rational square and the irrational product, one factor of which is determined by ἡ τομή of the square.

All the wealth of undulating harmony explained above, which is in itself a complete interplay of the various figures, can be produced by no other proportion than the sectio aurea.

The unique æsthetic value of this proportion, superior to all others, must now also have been proved by this abstract examination, showing that the sectio aurea and its functions contain, just as in a great sphere, the harmonies of all the regular figures. We will underline the proof by some concrete examples.

We quoted above a statement of Cantor *: The proportion which is æsthetically the most powerful is the one which is absolutely continuous. This is true, of course, but there can only be one perfect proportion—at least in architecture. The sectio aurea owes its perfection, not only to its irrationality and thereby its power to bring together the different figures, but also to its essential difference from all other constant continuous geometrical proportions, that is, while all these are synthetic, the sectio aurea, as the very word "sectio" or "section" indicates, is analytic: it acts within a given limit, and it cannot add or deduct arbitrarily anything from the whole which is itself a term of the proportion—a thing which is not and cannot be the case with the other constant proportions. We see at once, by means of the figures used as examples, $5:8 = 8:13$, that the whole is the sum of the smaller terms ($13 = 8 + 5$). If we take another constant geometrical proportion, for instance $2:4 = 4:8$, the total gives a surplus ($2 + 4 = 8 \div 2$). A front with the main divisions $2:4:8$ will have 14 as total value, a sum which stands in no proportion to any of its terms; wherefore the elevation will lack the uniting harmony required of an ideal work of art which ought to give the impression of something complete in itself, and not receiving its expression from its surrounding or something outside itself. In classic temple architecture, created by thinkers and not improvised from the "feelings" of "half educated" individual artists, the totality is the given factor from which the character of the building, or εὐρυθμία is determined. By making the frame of the building a term of the division the Greek temple-builder created an

* Cantor, *Gesch. d. Math.*, 2nd edition, vol. i, p. 167.

independent vibrating life *within* that frame, and this division is produced only by the sectio aurea, while all other constant proportions are continuously connected terms only without limit within themselves.

Moreover, the other continuous proportions 1 : 1 — 1 : 1·5 — 1 : 2 — 1: 2·5 — 1 : 4 are rational; they measure each other and form an obvious ordinary rhythm. From the Greek point of view, such a division appears formless and imperfect, and therefore, undesirable. To use the words of Aristotle, it would fall in the category of endlessness, to which belong things evil and ugly; while goodness and beauty belong to the category of the limited: *the right thing can be done in one way only, but the wrong one can be done in many.*

It is through the analytical sectio aurea that the entire building gets its potency. It has to all appearance a visible vibrating life in itself; its *restful* form, which is movement —gives it emotion. It moves us because it is itself moved—from its inner self.

It is very significant of the half education which is prevalent to-day in architecture, as in other branches of art, as a result of present democratisation, that people have talked with unction of an "epic in stone," when the question has risen to restore the disappeared part of the front of the cathedral of Nidaros. Such an expression discloses indeed the helpless ignorance of its originator and his friends, when referring to an ancient product of civilisation! The sound of the word has been a snare. An epic can belong only to literature; true architecture, like higher music, can only have emotion. Architecture, like music, does not seek its goal outside itself or outside the limits of its material. Within this, it has to transform weight into play, time into chords, to form a cosmos for itself, with its own independent life.

And this life, this living form, obtains its potency within its own frame by means of the unique proportion called sectio aurea, or to use the terms of the table of categories, by obtaining the keynote from tetragonon and heteromekeia.

CHAPTER XII

THE SECTIO AUREA

ITS USE IN ANTIQUE GREEK ARCHITECTURE

THE Temple of Concordia has helped us to establish, in the last chapter, the meaning of the words "heteromekeia" and "tetragonon," and to see that they represent the play between the irrational sectio aurea and the square as condition of harmony. We have been able to demonstrate quite diagrammatically the Greek teaching of harmony and beauty, and we have convinced ourselves that, in contrast to the modern art talk, it was quite scientific and based upon a knowledge of geometry.

We have accepted the theory and given our reasons for it; we shall now convince ourselves of its practice in antique Greek architecture, and of its transmission to the architecture of the Middle Ages.

Starting from the Temple of Concordia, we shall first consider the geometrical constructions which were developed by the main proportion of its plan. A searching examination of these constructions where the various figures grow out of one another, through the entelechie of geometry, into one harmonic connection, can be compared to a glimpse into the workshop of creation.

The astonishment created in our own time by physic and chemistry, geometry created also for the Greeks. Therefore it was natural that this entelechie and this harmony-creating power, which they found in the sectio aurea and in the connection between tetragon and heteromekeia, should rouse their astonishment and be considered as a holy, divine mystery. It was no less natural that temples in honour of the divinity and of the various personifications of divine power should be planned according to this divine geometrical mystery.

The Temple of Concordia fulfils this idea; the whole edifice, in plan as well as in elevation, and in details, is the result of the play between ἡ τομή and the tetragon.

As it may have been seen already in fig. 173, we have introduced the main lines in the plan of the Temple of Concordia, which is emphasised in hatchings.

We have ascertained that the width of the temple is the minor of tetragon I or the square which circumscribes circle I.

We see, furthermore, that the front of the temple lies between the parallel sides of an octagon and of a hexagon inscribed in circle I. The sides of the hexagon are marked by the letters *x y*. Moreover, the inner sides of the front coincide with the sides of a pentagon in the same circle; see, for instance, the base *e f*. It would be obviously the same with the opposite inner side of the front, if a pentagon was inscribed with its base in this part of the circle. The inner width between the longitudinal sides of the colonnades = the side of a decagon in circle I.

Moreover, it will be noticed that diagonals *e g* and *f k* of pentagon I intersect circle II at points *l* and *m* respectively. A chord through these points is the side of a hexagon in

circle III, which is inscribed in the pentagon in circle II. The total outer length of the cella including pronaos and corresponding opisthodomos, or back room, treasury or vestry, lies thus between the parallel sides of the hexagon in circle III, as shown by the dotted lines in fig. 173; it will be remembered that the sides of this hexagon = the exterior width of the temple.

It will also be noticed that the base of the pentagon in circle III falls just in the middle of the foundations under the columns, facing the pronaos or vestibule before the cella or enclosed room of the temple proper.

We notice that the external width of the cella = the major of the side of the triangle in circle V.

The inner width of the cella = the side of the octagon in circle IV marked *o n*. The diagonals of the pentagon in circle III, which are parallel with the diagonals *e g* and *f k* in circle I, intersect also circle IV at these points *o* and *n*. The side of the octagon in circle IV = the diameter of circle VI,* which is inscribed between the longitudinal walls of the cella. We also see that the pentagon inscribed in this same circle VI, is, as mentioned already, the pentagon of second order in circle IV.

The base of this pentagon in circle IV corresponds with the inner transverse wall of the cella, absolutely as we found it to be the case with the inner side of the front and the base of the pentagon in circle I. The base of the latter corresponds also, as we have observed above, with the base of the equilateral triangle in circle III, which circumscribes circle V, the diameter of which = the front width of the temple.

This very front width (= the diameter of circle V) corresponds to the inner length of the cella, which is determined by the distance between the two opposite sides of the tetragon which circumscribes circle V and correspond firstly with the bases of the two triangles inscribed in circle III, and which stand above each other, having their summits respectively up and down, and secondly with the bases of the two pentagons inscribed in circle IV.

The total inner length of the cella with pronaos and opisthodomos included (fig. 173 G H) = two sides of a decagon in circle II and the major of the side of tetragon II.

We could continue indefinitely to show the interplay between the proportions of the cella and the elements of the special configurations of the construction. But we shall turn instead to fig. 174*a* (Pl. XII) where the complete plan of the Temple of Concordia is given. Already at first glance it seems to correspond with the geometrical construction and to be entirely regulated by it.

The length of the stylobate, or the distance from front to front, from edge to edge of the platform on which the columns stand = the diagonal of the regular pentagon in circle II = the sum of two diagonals of the pentagon inscribed in circle IV.

We see from the dotted diagonals of the octagon in circle I, which intersect one another at an angle of 45°, how the two columns at the entrance of the pronaos and of the opisthodomos—the front hall and the vestry of the cella—are regularly placed according to the construction.

On the right of the drawing the numbers 1, 2, and 3 show the different measurings. No. 1 is the measure of half the total length of the temple. It shows that the inner side of the open row of columns of the opisthodomos coincides with the proportion of the sectio aurea.

As an example of how the different figures stand in relation to the temple, we have shown in No. 2 that the side of the tetragon in circle II = the side of the triangle in circle III. It can be seen that the distribution of the plan corresponds with the division of the line

* In mathematics there is a small difference which the Greek could not calculate (0·964 : 0 944).

according to the proportion of the sectio aurea. Line No. 3 is the side of the triangle in circle II. Here also divisions according to the same proportion correspond with the distribution of the plan.

The preceding examination of the relation between the various figures of the geometrical construction has shown the harmonic mutual relation.

The plan is produced by the same geometrical construction or its configurations. It is, therefore evident that, in its entirety and its main features, it must stand in relation to all the figures, their height, sides, and diagonals, or sections of these. It is unnecessary to point out all these proportions.

* * *

We had no other material at our disposal for our previous analysis than the little plans taken from the above-mentioned work of Perrot and Chipiez (Pl. XV). The scale is so small that the front has a width of only about 21.mm.5. We have doubled the size of this drawing. It goes without saying, however, that it is easy, with such a small drawing and with the empirical method used, to make mistake and to deceive oneself. In order to prevent this, we have tested the results of the analysis by mathematical calculations.

After this was written we had the good fortune to receive drawings of the Temple of Concordia on a larger scale—of the plan as well as of the front; the latter is not found in Perrot and Chipiez. These drawings are traced from authentic measured drawings in *Méthodes des Proportions dans l'Architecture*, by Endric Henzelmann, Paris, 1860.

The scale of the plan is twice as large and the drawing of the front is six times larger than our drawings (figs. 169–174, Pl. XII).

The drawing of the front, of Henzelmann, is reproduced in fig. 176, Pl. XIV, and his plan in fig. 179, Pl. XV.

This plan is copied in fig. 175, Pl. XIII, where we have collected the most important of our previous observations, which here in the larger drawing appear with greater clearness and reliability.

The temple-heteromekeia, taken from the data of Tannery, is transferred and marked with the letters *a*, *b*, *c*, *d*. We understood Tannery to mean that, having used the width of the stylobate as radius, he had calculated the length of it as equal only four times the side of the decagon in the circle. We increased this length by introducing the krepis before each front. We got thus the total length of the temple to be = the side of tetragon I, which circumscribes circle I. As shown by the drawing of Henzelmann, Tannery has reckoned the krepis in the length of the temple. It appears, however, from the analysis transferred that circle I with tetragon I belong entirely to the construction, as the octagon, the hexagon, and the pentagon in this circle determine the division of the plan. Thus the drawing of Henzelmann does not shake the slightest part of our previous theoretical suppositions and their results; on the contrary, it adds a new interesting fact, as it shows, by calculation, that the surface value of the stylobate—that is, the platform of the colonnade, or, if one prefers, of the temple itself under the krepis—is the minor of the surface of the square on the length of the krepidom, or the square on the side of tetragon II.

It will be remembered that the surface value of tetragon II = $32^2 = 1024$. $1024 \times 0{\cdot}618 = 632{\cdot}832$ = major. $1024 \div 632{\cdot}832 = 391{\cdot}168$ = minor of surface of □ II.

If we now take the value 13, used before, for the width of the front we get by calculation the krepis to project from the temple, somewhat less than 1/13, that is, 0·96962 of the unit. For both fronts ($2 \times 0{\cdot}96962$) we get 1·93927. $32 \div 1.9324 = 30{\cdot}06745$ = the length of

the stylobate. This value multiplied by 13 gives 390·8788 as the value of the surface of the stylobate. This figure, subtracted from the result 391·168 obtained before, gives the slight difference of 0·2892.

To test further we shall give a calculation according the measurements stated in Badeker, *Unter Italien*, 15th edition, Leipzig, 1911, p. 354. The length of the Temple of Concordia, including the krepis (that is, tetragon II) is stated as 42·12, the width as 19·68, and the diameter of the columns as 1.m.27. If we measure on the drawing of the front by Henzelmann (fig. 176, Pl. XIV), we find that the krepis projects outside the stylobate one diameter of a column, that is, 1. m. 27. 42·12 ÷ (2 × 1·27) = 39. m. 58 = the length of the stylobate, while 19·68 ÷ (2 × 1·27) = 17·14 = the width of the stylobate or of the front. $42{\cdot}12^2$ = 1774·0944 M^2 = the surface of tetragon II. 1774·0944 × 0·618 = 1096·3900952 M^2 = the value of major of □ II. 1774.0944 ÷ 1096·39009 = 677·70431 M^2 = minor of □ II. On the other hand, if we reckon quite simply with the length and width of the stylobate, we get for its surface: 39·58 × 17·14 = 678·4012. 678.4012 ÷ 677·70431, giving a difference of 0·99689 M^2.

For the length, Henzelmann has 39·2304, and for the width 17·22 M; these factors give 567·547488 M^2 as the surface of the stylobate. These differences, which show various ways of measuring, cannot, however, be taken into consideration. We could thus add to our earlier results the interesting statement that the surface of the stylobate is the minor of the square on the whole length of the temple—that is, tetragon II.

This fact goes still more to prove with what geometrical care the temple has been planned.

Except in a single instance, of which we shall treat in its right place, we have thus underlined, with the help of Henzelmann's drawing, the correctness of all our observations made from the small ones. It would be a tiresome repetition to go through the larger drawing to prove it, therefore we shall only show clearly from the latter that the organic division of the temple is brought about entirely by the geometrical entelechie.

We find that the whole temple, with its krepis, marked *a*, *b*, *c*, *d*, lies between the parallel sides of an octagon in circle I.

The stylobate, or the temple without the krepis—in other words, the lines of the front of the temple—lie between the parallel sides, marked *x* and *y*, of a hexagon in the same circle. It is, then, geometrically obvious that the length of the stylobate = the side of the equilateral triangle in circle I.

The total outer length of the cella—including pronaos and opisthodomos—lies between the parallel sides of a hexagon in circle III, the radius of which = the width of the front. The cella stands, therefore, in the same relation to the equilateral triangle in this circle.

The cella proper lies similarly in a hexagon in circle IV (see the small plan on Pl. XIV).

The inner length of the cella lies between the parallel sides of a hexagon, which circumscribes circle V, the diameter of which = the width of the front. This hexagon is equally inscribed in a circle whose diameter = the total width of the front; that is, including the krepis, or distance *n o* (marked in the drawing). At the same time the side of the hexagon in circle V is part of the side of the equilateral triangle in circle III.

We take now the pentagons. The inner surface line of the colonnade of the fronts coincides with the side of a pentagon in circle I. These sides are marked with α β and γ δ ("α" has dropped in the reproduction). The columns or their diameters lie thus between the sides of the hexagon and of the pentagon in circle I.

The total breadth of the temple, including the krepis = the side of the pentagon in circle III, the lines of base of the side of the pentagon lie between the centres of the two columns in front of the pronaos and of the opisthodomos.

The side of the pentagon in circle IV coincides with the side of the tetragon on the width of the front □ V, and with the side of the hexagon, which circumscribes circle V.

The outer width of the cella = the side of the pentagon in circle V. The lengthened diagonals of this pentagon correspond with the centres of the columns of the opisthodomos, as do also the lengthened sides of the hexagon which circumscribes ○ V.

The inner length of the cella proper = the minor of the diagonal of the pentagon in circle II = the sum of two diagonals in circle VI.

The lengthened diagonals of the pentagon in circle VII correspond furthermore with the corners of the cella and the opening between the side-walls of the cella—that is to say, the gable of the pronaos—and of the opisthodomos if the reversed pentagon had been drawn in circle VII.

The inner width of the cella = the minor of the diagonal of the pentagon in circle IV, or the diagonal of the pentagon in circle VI. Thus we have corrected with the help of the large plan of Henzelmann, the result obtained earlier, the width being equal to the side of the octagon in circle IV.

As we already remarked, the data of Tannery is not really quite accurate; the temple, as we saw, is the complete result of a geometrical order of thoughts; it is therefore natural that its conception should be expressed in some way, as soon as it comes in contact with this order of thoughts. It expressed the length of the temple as four times the side of the decagon in the circle with the width of the front as radius. Strictly speaking, this result is not correct, because Tannery mixes together two things of different nature, when he uses as radius in his calculations, the front, or width of the temple proper, getting as result the length of the krepidom. In the meantime, our observations have taken us into the geometrical conception of the temple, its sphere of harmony, but no further. We have obtained a number of grades of value in the scale of harmony; but this is not absolute. A glance at the scale of values in figs. 171 and 172 (Pl. XII) gives the impression of missing grades, or links, in the harmonic feeling which these grades of value must involuntarily arouse. It will be especially noticeable, for instance, that there is a blank—or, can we say, a gap—between grade III and grade IV.

We must reserve for a special study the full details of this progression of harmony; here we shall only state what is necessary to indicate the complete picture of the inner interplay, as it exists in the geometrical conception of the temple.

It will thus be seen that in the hexagon in circle I (fig. 175, Pl. XIII), we have inscribed a circle which we can call ○ II/III, because it lies between ○ II and ○ III. A hexagon in this new circle circumscribes circle III, which is equally circumscribed by a pentagon in circle II. Moreover, it will be noticed that an equilateral triangle in the new circle II/III circumscribes the equally newly introduced circle with diameter = the width of the krepidom—that is, *n o*. We call this circle ○ IV/V. If we inscribe now a circle in the hexagon in circle III, a tetragon in this circle will circumscribe circle IV, and so on. We notice a more lively interplay between the circles than before.

If we examine how the new figures stand in relation to the plan of the temple, we shall find that the diameter of the new circle II/III = the length of the stylobate or the temple proper, and also that the hexagon and the heptagon in the same circle = respectively the width of the krepidom and the width of the front or radius of circle III.

Moreover, the minor of the diagonal of the pentagon in circle II/III = the distance between the rows of columns of the side-walls.

We have now settled more definitely, or rather, we have justified the data of Tannery, as the total length of the temple—the krepidom—is determined by an octagon, the length of the stylobate by a hexagon, and the intercolumnation of the front by a pentagon in one and the

same circle, while the total width of the temple, of the krepidom, and of the front, the width of the stylobate, are determined respectively by a hexagon and a heptagon in a circle which is inscribed in the hexagon of the first circle.

We shall now study the connection between the plan and

The Front

It is obvious that, just as the case is with the plan, so it is with the front, that is to say, raised according to a geometrical construction in continuation of and in full unison with the construction of the plan. This accordance is already introduced by the rigid, law-bound manner in which the projection of the krepis is determined in several ways, which lead all to the same result.

We have seen that the total length of the temple is determined by tetragon II, or by the parallel sides of the octagon in circle I. It can be seen how the line of 45° passing through the centres of the corner columns and through the corner of the stylobate determines the width of the krepis by sectioning out the part *n o* of the side of □ II and the lengthened sides of the octagon in circle I. This section = the side of the hexagon in circle II/III = the side of the pentagon in ○ III = the diameter of ○ IV/V, which, besides being inscribed in the triangle in ○ II/III, circumscribes a hexagon which circumscribes circle V. It will be noticed that the circles IV/V and V have been made quite independent of one another—through the most different constructions, and yet play into each other, as if they were sprung from one and the same source, as if circle V had been purposely inscribed in the hexagon in circle IV/V, whereby the side of the inscribed triangle and the inscribed heptagon in this circle = each respectively the diameter and the radius of circle V (see fig. 176*a*, where the side of the heptagon is in dots and strokes and marked by two small rings and the fig. 7). There is accordance, then, between the several elements of the construction.

Within and through these two circles IV/V and V and their polygons, the cella, the most holy part of the temple, is proportioned in intimate accordance with the whole horizontal plan. It goes without saying that, when the front is proportioned inside the tetragons on the diameters of their circles and by their polygons, the proportioning can be only a continuation of the proportioning of the horizontal plan, starting from circle I itself, and its inscribed and circumscribed figures.

Before commencing the analysis of the front, it will be useful first to consider the interplay between circles IV/V and V, and the tetragons on their diameters. This is shown in figs. 176*a* and 176*b*. In the first we have used large letters and in the last small ones. We shall first consider the relation between the tetragons. The major of the side of the larger tetragon (□ IV/V) B^1 G^1 = half the diagonal, *a*, *r*, of the smaller tetragon (□ *v*) the half diagonal A^1 R of the larger tetragon = the section *a q* of the side (marked on the base) of the smaller tetragon, while minor E D^1 of diagonal A^1 D^1 = major *b g* of side *b d* of the tetragon. Diagonal A^1M = diagonal *a g*. Diagonal I L = diagonal *k l*. Minor B^1L of side $B^1$$D^1$ = diagonal *i m*, while diagonal O P (in the smallest tetragon on the base to the right) = sections *b m*. So as not to remain indefinitely on the subject, we shall only give, as an example of the harmonious interplay between the pentagons in the two circles, the fact that minor X T of the diagonal of the primary pentagon and the diagonal of the secondary pentagon T V = *i ö* * in the half tetragon on the section *i b*, while the side of decagon X K = the diagonal *tv* of the secondary pentagon in the small circle.

When we remember that the side of tetragon $A^1$$B^1$ = half the length of the stylobate,

* This letter, which comes last in the Norwegian alphabet, is pronounced almost like "ur" in "absurd."

and thus the radius of ○ II/III, and that diagonals C^1S and A^1G^1 = respectively the diagonal and the side of the pentagon of the same circle, and, furthermore, that the lines *ab*, *ag*, and *cs* stand in a similar relation to ○ III, it must be clear that, when the proportioning of the front is determined by the two squares on the width of the krepidom and of the stylobate, it must stand in an intimate and harmonious connection with the whole plan.

In fig. 176*a* the front and the principal features of its architectural distribution are drawn in red.

It can now be noticed that the front lies between the parallel sides of a hexagon in ○ IV/V—that is, in the same manner as the stylobate and the cella in the plan—as a whole, and the cella proper—are included between the sides of the hexagon. Moreover, we must notice that the distance between the axes of the corner columns = the minor of the diagonal of the pentagon in ○ II. The height of an equilateral triangle on this distance gives the height from the ground to the top of the tympanon, the triangle of the gable. The height from the stylobate to the top of the cyma, which is the gutter = the major of side B^1G^1 of the tetragon on the width of the krepidom = the half diagonal *a r* of the tetragon on the width of the stylobate. The front is, therefore, developed inside the major-heteromekeia of tetragon IV/V. This major = the diameter of the smallest circle, marked in the arab figure 1, and 2 sides of decagon XK = two diagonals *vt* of the secondary pentagon in circle V. Furthermore, the total height from the ground to the top of the cyma = the diameter of the circle marked in the arab fig. 2 = two diagonals VT of the secondary pentagon in circle IV/V. As shown by the construction of the sectio aurea marked in a red arab figure 3, beneath on the base, the major of this total height—the height from the ground to the lower edge of the architrave—gives the epistylion.

It will be noticed that the total height has been found by the tetragon on the width of the krepidom being lifted to the stylobate—or what we could call the top of the plinth. In the same way the level for the top of the cornice (geison) is determined, as the height from the lower edge of the architrave to the same top = the minor of the height from the mentioned lower edge to the side C^2D^2 of the lifted tetragon. The height from the stylobate to the top of the cornice = the side of the pentagon in circle IV/V.

If we now turn to the large drawing of the front, fig. 177, Pl. XV, we find that the proportioning of the smaller divisions in the architectural formation are determined logically in the same way by the sectio aurea.

The construction is transferred from fig. 176*a* and 176*b*, and the divisions of the sectio aurea already made are given their corresponding numbers.

We begin with the construction of the sectio aurea, marked with figure 6. This divides the height from the lower edge of the architrave to the top of the gutter; the part from the top to the lower edge of what is called the drops under the soffite of the cornice coincides with major, while the remainder of the lower edge of the architrave gives the minor.

The two sections marked 7 show that the minor of the tympanon is the part between the lower edge of the cornice over the tympanon to the lower edge of the architrave, and the whole entablature, that is, cornice, frieze and architrave—gives the major. It shows also how the profils marked NB^1 and NB^2 of the two cornices are of equal dimensions.

The construction marked with arrow No. 8 divides the whole entablature in its main parts, cornice and frieze as major, and architrave as minor. Finally, the division marked 9 shows that the frieze and the architrave are proportioned in the same way.

By comparing these results with the various values of the constructions of the pentagon it is shown that the minor of the diagonal of the pentagon of second order or the diagonal D of the pentagon of third order in circle IV/V = the height from the lower edge of the architrave to the part of the cornice marked NB^2 and = the height from this point to the top

of the cyma, that is the gutter. It follows that the major and the minor of the same diagonal (major = the side of the pentagon of third order in ○ IV/V) = respectively the cornice + frieze and architrave, or, in other words, it gives the same results as construction No. 8. Furthermore, the diagonal and its sectio aurea division in the pentagon of third order in circle V give the same result as construction No. 9. The diagonal of the very smallest pentagon of fourth order in circle V is then = to the minor in the last-named construction.

We take now the columns. In fig. 176*a* we found that the height from the stylobate to the top of the cyma or gutter was = the major of the side of tetragon IV/V (construction No. 1 in fig. 177), and again that this major = two sides of a decagon in ○ IV/V = twice the minor of the diagonal of the pentagon in circle V. This connection, which we emphasise in fig. 176 by a circle marked 1, having a radius = the minor of this diagonal, is also emphasised in fig. 177 by a corresponding circle, marked 12. As it can be seen, the height of the column to the lower edge of the top of the column, or, to be more precise, of the echinos = this radius. The construction of the sectio aurea, marked 5, to the right on the base, shows that the level of this part of the column, marked NB^1, lies at the point of division of the sectio aurea, on the line of height, from the ground to the top of the tympanon.

Lastly, the construction No. 10 shows that the height of the column to the covering plate, or abacus, = the minor of the side of the tetragon circumscribing circle V, or the width of the front. The radius of circle V is also = the height from the ground to the lower edge of the abacus. With this result we come back to our starting-point, the statement of Tannery.

It will be noticed that in fig. 175 (Pl. XIII) we have introduced in red the construction of Euclid from fig. 169. It is marked in small red letters. The small tetragon on the minor of the width of the front is marked as in fig. 169, with the letters *g l* (in fig. 176*b* with *b l* = major I N of diagonal I L in fig. 176*a*). If we continued to divide according to the sectio aurea, we could find in that tetragon the geometrical unit which is the basis of the temple of Akragas—as we pointed out previously. It will be seen that, within the small tetragon on section *g l* in fig. 175 (Pl. XIII) we have drawn two small circles, emphasised by shading. The diameters of these circles give the diameters of the columns. Owing to an oversight, the circle and consequently the plan of the columns introduced have become somewhat too large. The radius is a little smaller than half the side of the secondary pentagon in circle VII (fig. 175, Pl. XIII). In order to remedy this lack of clearness we have continued the analysis in fig. 178, on the scale of the drawing of the front. The height of the columns to the plinthos = the minor of the width of the front. For the sake of continuity in the development of the theory of the front, the same letters are used in this as in fig. 176*b*.

We have shown so exhaustively the concordance in the whole construction that we shall not tire the reader with repetitions, but we will indicate, however, that apart from the height from the krepis to the top of the gutter being = diagonal *f w*, we find here also the proportioning, not only of the columns, but of all the parts of the front down to the smallest. Thus the diagonal marked O N B = the entablature; it follows that its sections = the distribution, in the construction of the sectio aurea, marked 8, in fig. 177 (Pl. XV). We find the diameter of the columns in the section *y p* emphasised with the little circle. We find again the radius of this circle in the side of the pentagon of the fourth order in the pentagon in circle IV/V, fig. 177 (Pl. XV), as shown by the circles drawn here. We notice, in parenthesis, that the radius of the columns in front of the pronaos and of the opisthodomos seems to be = the radius of a circle circumscribing the pentagon of second order in circle VII, developed therefore from circle V (fig. 175, Pl. XIII).

Further, it will be seen that the small section, marked "la hauteur de l'abacus" (fig. 178) gives the height, or if preferred, the thickness of the covering plate of the column, and the

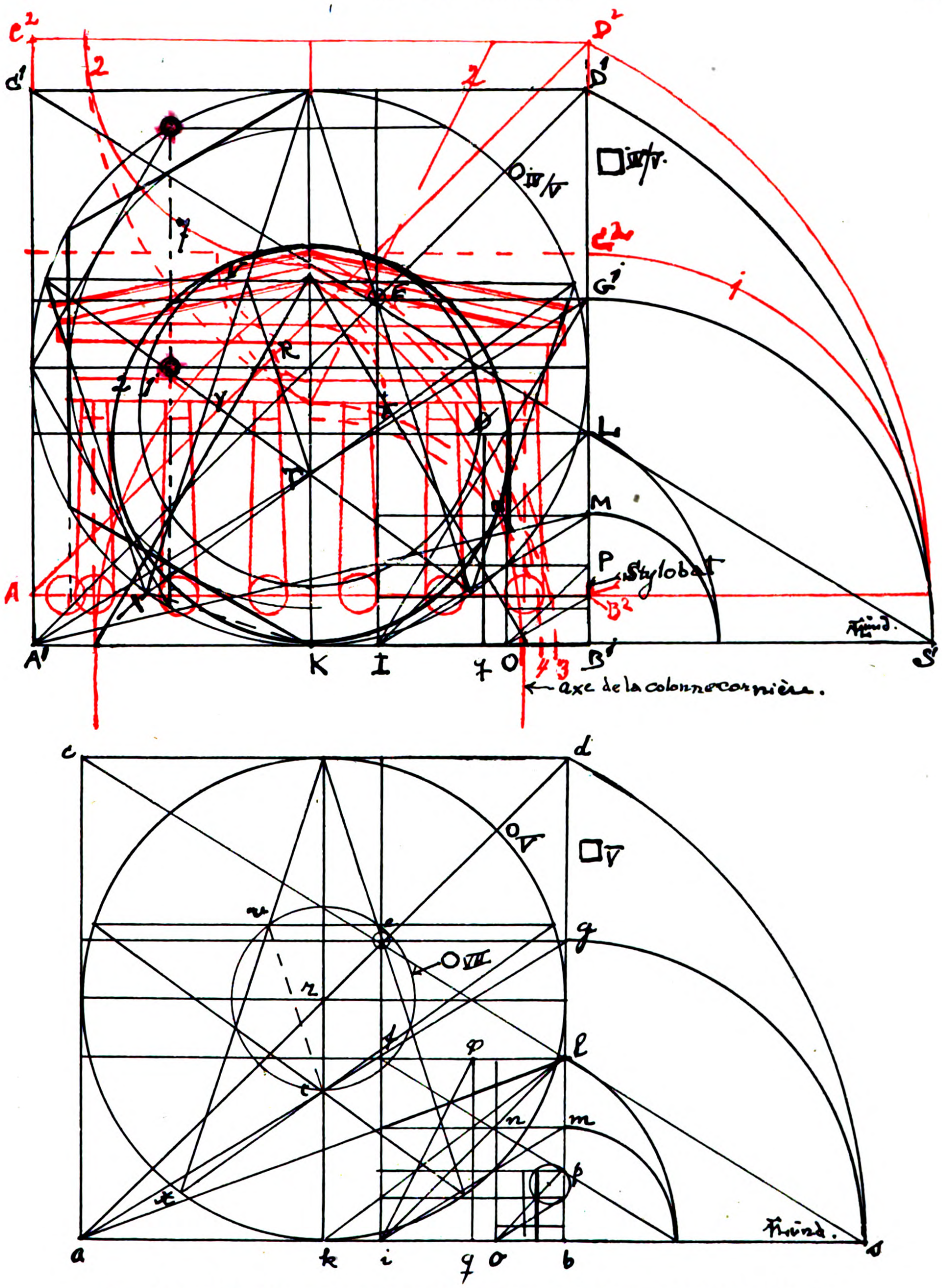


Fig. 176 *a* and *b*.—The temple of Concordia. Theory of the front developed from the plan.

section marked "1/2 longueur de l'abacus" gives half its length. There can be no doubt that the ell, or the foot, used at Akragas lies hidden in one of these smaller values.

As a last example of this remarkable harmonious play, we indicate the two small circles of fig. 175 (Pl. XIII), whose centres lie in circle V and whose radii = the distance between circles V and IV. In these circles there appear pentagons produced by the diagonals in various circles. The diagonal of the pentagon in these small circles is = the major of side $g\ l$ of the tetragon on the minor of the width of the front (fig. 175, Pl. XIII); see fig. 173 (Pl. XII).

Finally, we refer to fig. 174 (Pl. XIV), where we have drawn the front with the plan to make it clearer.

This analysis, now brought to an end, has convinced us that all the parts, from the smallest, stand in intimate proportion to one another, not only mutually and with the front, and with the plan in its totality, but in a mysterious connection with all the geometrical configurations, such as equilateral triangles, squares, pentagons, hexagons, heptagons, octagons, and decagons, produced by the remarkable creative interplay taking place in the geometrical construction which is the basis of the architectural conception, as it is disclosed to us in the statement of Tannery, and as we have seen it in the drawings, from fig. 169 to 178.

It is related of Plato that he used to receive candidates for admission to his Academy with the words *μηδεὶς ἀγεωμέτρητος εἰδίτω μοῦ τὴν στέγην*: No one ignorant of geometry shall cross my threshold! The contempt of the Greeks for unclearness and attempts has found in the word *Glossalgia* an expression full of meaning—it signifies speaking in tongues, rattling words out. In the Pythagorean table of categories, out of ten points as condition of harmony, nine are, it must be admitted, clear, but usual, limited and unlimited, odd and even, one and many, right and left, masculine and feminine, rest and movement, straight and crooked, light and dark, good and evil. It is obvious, then, that the tenth point of contrast *τετράγωνον* and *ἑτερομηκές*, rendered in the Latin translation of the fifteenth century, as *quadratum* and *longius altero latere*,* that is, *equilateral* against *unequilateral*, or *rectangular parallelogram*. "Tetragon and heteromekeia" is no "glossalgia" on the part of the Pythagoreans. On the one hand, through the obvious, the indubitable, they try to urge their students to independent thinking and precision, and, on the other hand, with true knowledge of man, to hold the timid layman spiritually inclined, or the fool, at respectful distance from the true and difficult divine science. Our analysis of the temples has proved irrefutably, in the meantime, that the tenth point means something quite real, and this reality is nothing else precisely than the construction of Euclid in Book II, prop. 11, which we developed in fig. 158. In this point, therefore, is gathered in one sum the meaning of the whole table of categories.

Through this construction we have discovered the secret of the proportioning of the Temple of Concordia. It dates from 480–400 B.C. Time has forgotten the name of its architect, but he must have been a deep and clear thinker; his temple is one great lesson on Greek geometry and on geometrical and æsthetic philosophy. We cannot help thinking of the Pythagorean Empedocles, who was from Akragas, who lived precisely at that time, and who was famous as orator, poet, philosopher, architect and engineer.

Our examination has settled the correctness of what we said previously, that the construction of Euclid, ii, 11, is not his own discovery. The manner in which he introduces it shows, moreover, that he supposed it known.

From this construction the temple of Akragas appears as a perfect illustration of the correctness of the conditions of harmony of the table of categories, as a perfect result of the

* Johannes Bessarione (1395–1472), Italian cardinal with honorary title of Patriarch of Constantinople. His translation of the *Metaphysic* of Aristotle was published in Paris in 1516. The table of categories are referred to in Lib. I. chap. v.

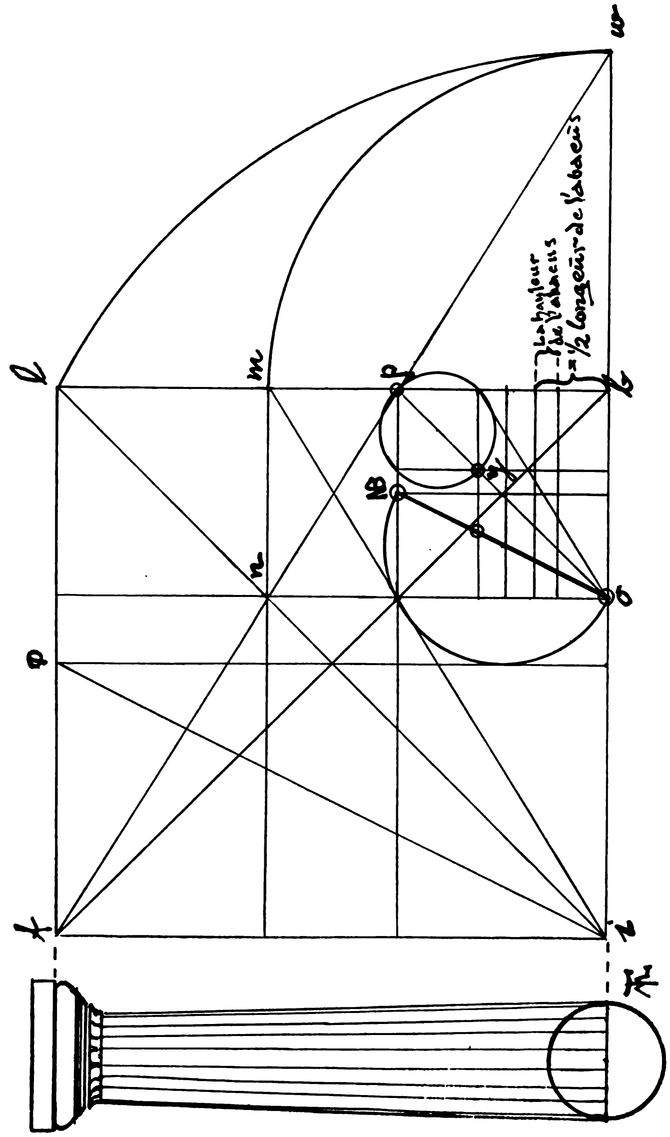


Fig. 178.—The temple of Concordia. Analysis of the proportioning of the columns.

play between the square and the sectio aurea—which, according to the compiler, was first discovered in A.D. 1509 by Luca di Paciuolo—that is to say, 2000 years after the Temple of Concordia was built!

We have seen that this temple of Akragas has been throughout conceived *κατὰ τετράγωνον καὶ ἑτερομηκείαν*, or, as it should be called after our examination, *ad quadratum et pentagonum*. We shall follow the same line here as for the Middle Ages, giving several examples to prove that the principle of Doric, and, for that matter, of all Greek temple architecture, was no dry formula—what the Hellenes, who understood beauty, would never have tolerated in their hatred of monotony, but that it was an ideal, living principle, a *method*, which presupposed sureness of thought and a full knowledge of the geometrical conception of a building.

As the next example, we now take the most famous of all Doric temples,

The Parthenon, Athens

We start with the front, which is reproduced in fig. 180.

It strikes us immediately that the principle is used here in a different manner; while the front of the latter in Akragas is conceived within two tetragons, one on the width of the krepidom, and one on the width of the front, here it is within a tetragon on the width of the front with its base on the lower edge of the krepis. It will be seen that the proportioning of the main part of the front is determined according to a harmonic progression within the tetragon because the height from the lower edge of the krepidom to the summit of the gable, cyma, or gutter deducted = the major of the height of the tetragon. This major through the construction of the section (marked 2 under the krepis) is again divided in such a way that the major of the height up to the cyma gives the height up to the lower edge of the architrave, while the minor of the same height gives the height from the lower edge of the architrave to the top of the cornice under the cyma. The section (3) of this part gives a minor which is the height from the lower edge of the architrave to the lower edge of the cornice, while the major and the minor of this part respectively give the height from the lower edge of the small moulding on the architrave to the lower edge of the cornice, and the height from the lower edge of the architrave to the lower edge of the same small moulding.

Therefore the distribution of the front is determined very simply according to a geometrical progression according to the proportion of the sectio aurea. It is thus shown by our previous demonstrations that the size of each part, from the smallest to the totality, all is brought into symphonic union, not only mutually but with all the regular polygons which can be inscribed in circles produced by the construction. The "instrumentalising" geometrical construction appears through the geometrical progression. We inscribe in the tetragon on the width of the front ABCD a circle, which on account of the following analysis of the plan (fig. 181, Pl. XVI), we shall call circle V. We divide the radius according to the proportion of the sectio aurea, and we take the major as radius of circle VI and the minor as radius of circle VII. The radius in this circle is divided in the same manner, whereby we get circles VIII and IX.

We cannot enter into a lengthy comparison of the values of harmony thus variously produced; we shall limit ourselves to pointing out some special results of the geometrical hegemonikon. The side of the pentagon in circle V = the height of the entire front from the stylobate to the top of the ridge, cyma included. Two sides of a decagon in the same circle = the height from the base of the tetragon of the front to the summit of the tympanon.

The diagonal of the pentagon in circle VIII = the height from the lower edge of the architrave to the summit of the tympanon.

The side of a bis-decagon in the same circle = the height from the lower edge of the cornice to the summit of the gable, cyma included.

The side of a decagon in circle VII = the height of the tympanon. Five sides of a decagon in the same circle = the height from the stylobate, the top of the krepis, to the summit of the gable, cyma deducted. Eight sides of a decagon = the distance between the axes of the corner columns.

The side of an octagon in circle VI = the height from the lower edge of the architrave to the lower edge of the cyma = section *a* in the sectio aurea construction marked 2.

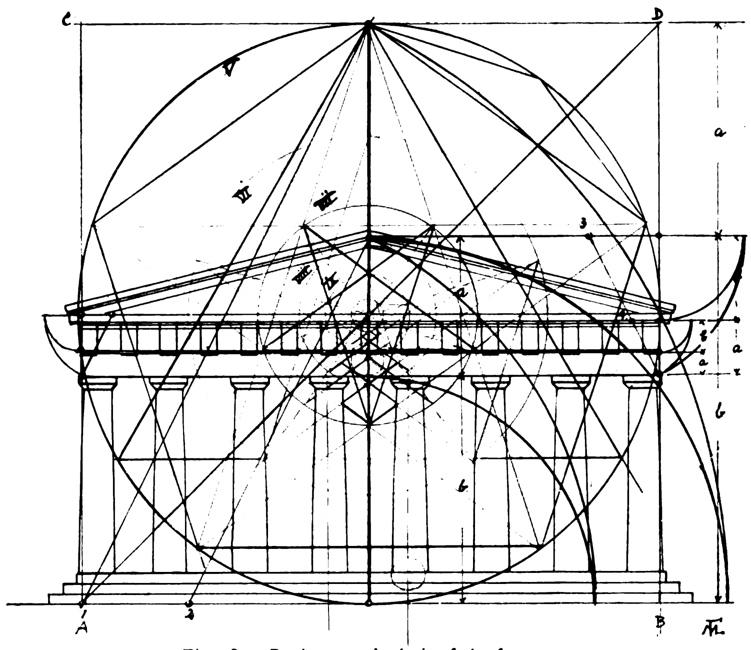


Fig. 180.—Parthenon. Analysis of the front.

The side of an octagon in circle V = the height from the lower edge of the krepis to the lower edge of the plinthos—the covering slab over the caps of the columns.

This construction of the front, to which we shall return later, is produced, as in the Temple of Concordia, by the geometrical hegemonikon of the plan, which is reproduced in fig. 181.

This connection will be proved immediately by transferring the auxiliary construction from the elevation of the front (fig. 180) to the horizontal plan. The sides of the pentagon in circle V are lengthened until they intersect one another. It will be seen that the whole plan, including the krepis, lies inscribed in circle I, which circumscribes the new pentagram. It can be said, in geometrical terms, that the Parthenon is conceived within a circle whose diameter is the diagonal of the temple, the heteromekeia of the krepidom, while the minor of this diagonal gives the diameter of circle V of the front. Half the width of the front is

thus the minor of the radius of circle I of the whole plan. Thereby it follows that the width of the front = the minor of the side of a tetragon which circumscribes circle I.

Four times the side of the pentagon in circle V gives the total length of the heteromekeia of the temple, krepis included. It can be mentioned, further, that ten times the minor of the side of the same pentagon or ten sides of the pentagon in circle VII = the length of the stylobate. (The side of an octagon in circle I is also practically = the width of the front, because the side of an octagon stands to the width as 2·0037 : 2·0000).

It will be seen by the hexagon *a, b, c, d, e, f,* inscribed in circle I, that the stylobate, in its length, lies between the parallel sides of this hexagon, such as is the case with the temple of Akragas—although, let it be noticed, the proportioning of the two temples does not thus become the same thereby.

By inscribing the plan of the stylobate in a circle called circle II, we find a new remarkable feature: the line from the extreme corner of the krepis coincides with the side of a decagon marked *g—h,* in circle II. Therefore the krepidom lies in its whole length, or, more accurately, the outer edges of the krepis of the west and of the east fronts are situated on the two opposite parallel sides of a decagon in circle II. It will be remembered that, in Akragas, the krepis of the fronts were lying between the parallel sides of an octagon in circle I.

Furthermore, one side of a bis-decagon in circle II = the diameter of circle VII, which is the minor of the diameter of circle V, which again is the minor of the diameter of circle I. The same accord is found everywhere.

We have obtained, moreover, a circle III, having half the length of the plan of the stylobate as radius. The circle is drawn in dots and divided in five parts; we have also dotted a pentagram in it. Here we meet a new interesting feature—the side of the pentagon of second order in circle III = the radius of circle V. Moreover, the side of a decagon, *i k,* in circle III = the external width of the cella. It will be equally noticed that the side of the hexagon, *l m,* in the same circle, goes through the centres or axes of the second columns, from the corner, in the row of columns of the longitudinal sides.

In circle V we see on the drawing the striking occurrence of the side of an inscribed equilateral triangle = the intercolumniation of the outer colonnades across the temple.

By comparing the mutual relations between the circles with their relation to the division of the plan, we can convince ourselves of the inner connection of its different parts. We remember that the minor of the radius of circle I = the radius of circle V.

The major of the radius of circle V is the radius of circle VI.
The minor of the radius of circle V is the radius of circle VII.
The major of the radius of circle VI is the radius of circle VII.
The minor of the radius of circle VI is the radius of circle VIII.
The major of the radius of circle VIII is the radius of circle IX.

It is through this accordance that the division of the plan is determined, and that the architecture of the temple is "instrumentalised."

We have mentioned before that the total length of the temple, krepis included = four times the side of the pentagon in circle V; therefore it is also = four times the minor of the side of the pentagon in circle I.

The total length of the stylobate = ten times the minor of the pentagon in circle V, and = also ten times the side of the pentagon in circle VII.

The total external length of the cella, including the six columns in every gable front = five times the diameter of circle VII, and therefore = also five times the minor of the diameter of circle IV.

The external width of the cella = (1) five times the minor of the side of the pentagon

in circle VI; (2) = five times the minor of the diagonal in circle VII; (3) = five times the side of the pentagon in circle VII; (4) = five times the diagonal of the pentagon in circle VIII; and finally (5) = the side of the decagon in circle III.

The inner width of the cella = the major of the diameter of circle IV = the diameter of circle V.

The inner length of the cella = seven times the minor of the side of the pentagon in circle VI, and = seven times the diagonal of the pentagon in circle IX.

The inner length of the cella to the six columns placed transversally = six times the minor of the side of the pentagon in circle VI.

The intercolumniation of the rows of columns in the cella = the diagonal of the pentagon in circle VII.

The inner width of the nave in the cella, that is, the width across between the two rows of columns = six times the minor of the diagonal of the pentagon in circle IX, or six times the diagonal of the pentagon in the very smallest red circle.

The width of the nave = also the side of an equilateral triangle in circle VII, just as the distance right across between the outer colonnades = the side of the triangle in circle V.

The width of the aisles of the cella = twice the diagonal of the secondary pentagon in circle IX. The diameter of the circle round this pentagon, *it should be noticed, is precisely = the diameter of the large columns,* except the corner column of the *outer colonnade* which carries the temple.

The radius of this small circle is determined here by the small pentagon which is inscribed in it. But it will be seen that we have drawn a similar circle above, the radius of which is a diagonal drawn in black in a small parallelogram. This small parallelogram is part of a larger one.* The reader will at once recognise Euclid's problem of the sectio aurea in II, 11, which we have already dealt with in several ways.

The major of the radius of the main columns is again the radius of the columns inside the cella.

Finally, it will be remarked that the axial distance between the columns of the external colonnade excepting the corner columns = the side of a pentagon in circle VIII, which in the drawing is demonstrated by the two dotted red parallel lines showing this axial distance, and seen clearly in the drawing of the front (fig. 180), as well as the axial distance of the columns of the cella which = the side of the pentagon in circle IX.

If we now analysed the front and its details, we would find the same accord; but we must limit ourselves to the main features given above and leave the Parthenon, which we keep for a detailed examination with other buildings belonging to the sacred architecture of the classic period.

It is said in the old Norwegian *Gulathingsret:* "Eftir Vitnum ok Gognum skal hvert Manna Mål döma, svá er ef einn beri Vitni med Manni som engi beri, enn tveir som tiu" —which means that a man's case must be judged according to witness and evidence; that the evidence of one man is worth nothing, but that the evidence of two men is as good as the evidence of ten.

We have produced evidence that, as conditions of harmony in the table of categories, the tetragon and the heteromekeia are meant, this is precisely the problem II, 11, of Euclid. We also proved that both the Temple of Concordia and the Parthenon of Athens are built according to this law of harmony.

* See that part of the plan (fig. 181).

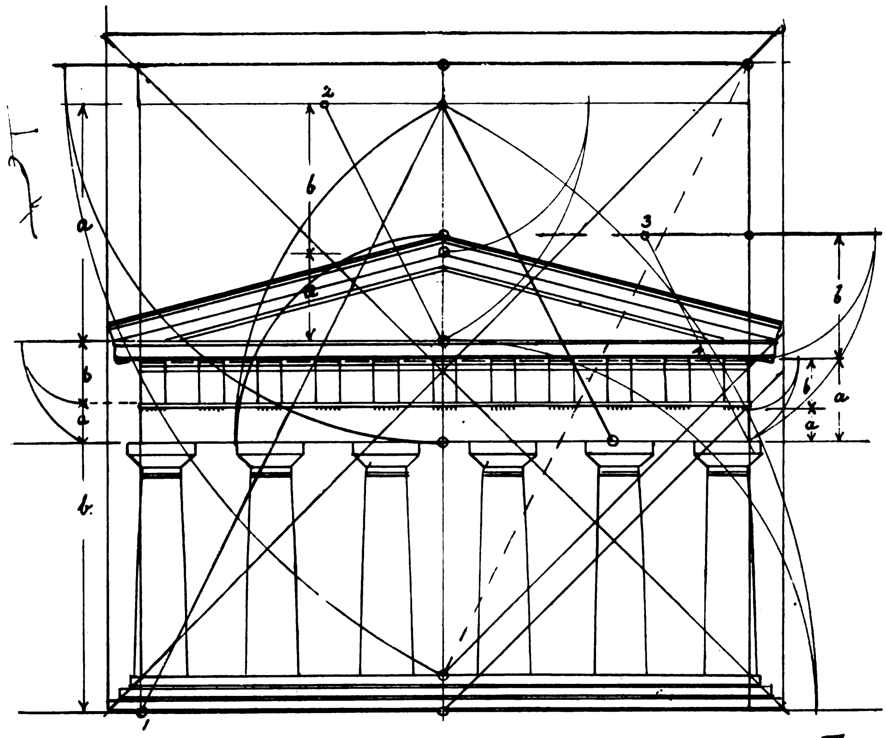


Fig. 182.—Temple of Poseidon, Pæstum. Analysis of the front.

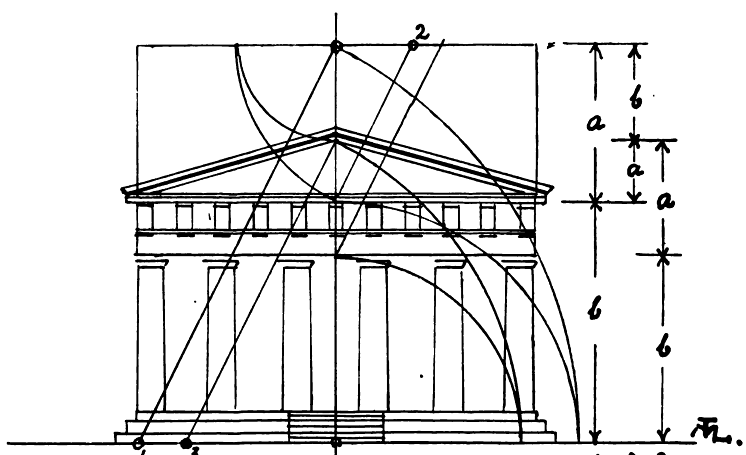


Fig. 183.—Temple of Athene, Ægina.

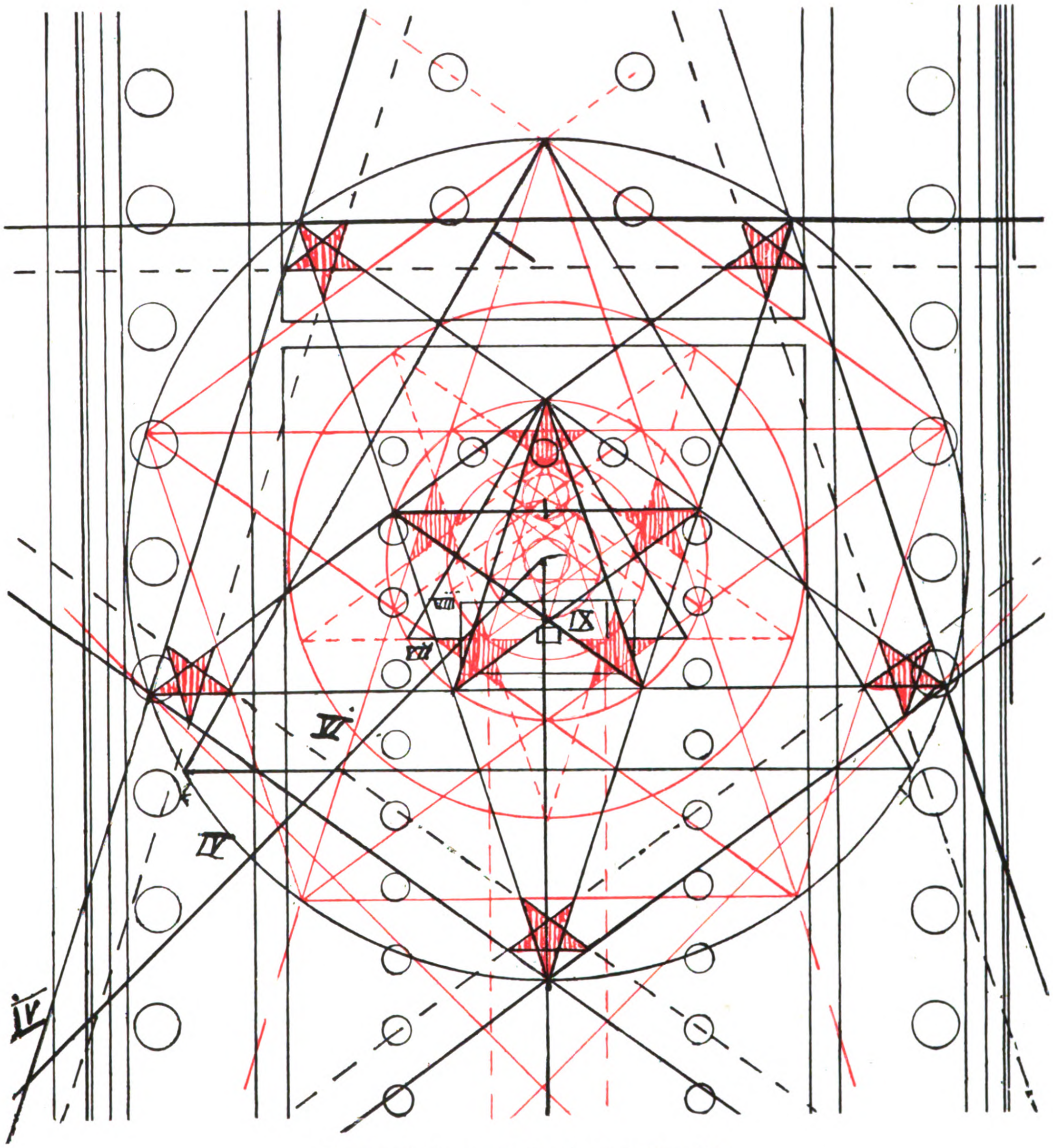


Fig. 181.—Parthenon. Section of plan on Plate XVI.

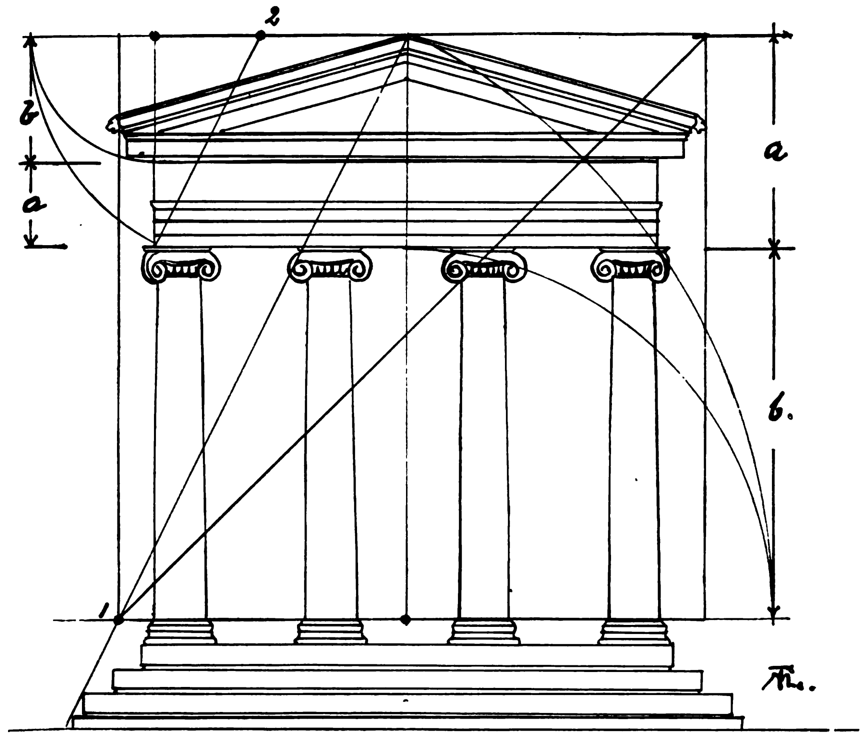


Fig. 184.—Temple of Nike Apteros in Athens. Analysis of the front.

These two temples are worth, therefore, more than ten evidences. It might be interesting, however, to see other examples. We give them according to age:

Fig. 182 shows the method used for the *Temple of Poseidon of Pæstum.*

Fig. 183 is the method used for the *Temple of Athene in Ægina.*

Fig. 184 is the method used for the *Temple of Nike Apteros in Athens.* In this Ionic temple we see finally the front rise *ad quadratum* as in a medieval church.

At Vitnum ok Gognum skal Mál Manna döma; any explanation is superfluous, the drawings speak for themselves.

The Modulus

Now that we have produced an explanation of the system which creates harmony in antique temple architecture, it remains to consider the *unity* which runs throughout the proportioning of this architecture.

We have already proved unity in the Temple of Concordia, and the same has been done with the Parthenon, from the fact that a dimension was found which is = the radius of the circumference of the columns.

This discovery leads us back involuntarily to the well-known *modulus* of Vitruvius.

Vitruvius was a contemporary of Julius Cæsar, and consequently wrote his work on architecture from 400–500 years after the time when the geometrical system first ruled the architecture of the Greeks. If we are to judge from his book, this theory once so living, must have become, already before the days of Vitruvius, a mere rule of thumb, practised mechanically. The book demands a great deal of the architect's knowledge, education, and intelligence; whether Vitruvius himself fulfilled these requirements seems doubtful, nor does it interest us, but he quotes some older Greek philosophers and architects to support his building theory, and his quotations have a great value because the quoted writings do not exist any more. Unfortunately, he never does this directly; he relates the working method of the Greek builders, but his account of their theory is unclear, perhaps because he did not understand it fully himself or more probably because he, or rather his Greek authorities, were unwilling or did not dare to unveil the secret of the principle hidden in the theory of sacred architecture.

This applies undoubtedly to the teaching of unity, which we find propounded in dry rows of figures in Vitruvius. But behind its schematic form, which he gives as a guidance without any explanation, we can detect a scientific theory, and his account possesses on this very point a great historical value which is enforced by the evidence of the buildings themselves.

On the building of a Doric temple, Vitruvius writes in his fourth book, third chapter, that "a front with four columns must be divided in 27 * parts, one with six columns in 42 parts. Of these parts one becomes the *modulus*, or the unit of measure which in Greek is called *embates* (kothurne), and by making this modulus the fundamental measure, unity in the whole building is produced by calculation. The thickness of the columns is composed of two moduli, their height including capital of fourteen moduli, etc., a purely schematic formula.

In the second chapter of his first book on the fundamental principles of architecture he writes: "Architecture rests upon the following principles: (1) arrangement or planning, which in Greek is called *taxis*; (2) designing, called in Greek *diathesis*; (3) *eurythmy* (rhythm and aspect); (4) *symmetry* (interplay of proportions); (5) expediency; and (6) economy." Every one of these principles is then more fully developed.

On *taxis*, or the "instrumentalising" fundamental drawing, he says that it consists "in determining the various parts, and in proportioning the totality of a building according to dimension and purpose; this planning is determined by a given proportion of quantity which the Greeks called *posotes*, and which is a scale taken from the division of the building itself, a uniting result produced by the various single parts of the whole."

He says further, on *symmetry*, that "it is produced by the divisions of the building itself, that it is a result which harmonises with these divisions, and consists, therefore, of a corresponding proportion between one (main) part calculated according to the lesser parts, and the totality. Just as, in the human body, the symmetrical and eurythmical proportions are determined by the forearm, the foot, the palm of the hand and other parts, the case is the same with a building. In a *sacred* building the symmetry and the eurythmy are determined either by the *thickness of the columns*, or by the *triglyphe*, or the *embates*."

In another place again he says, first book, first chapter, fourth part: "The cost of a building, and the division of measurements, are calculated by means of arithmetic, while the difficult question of the eurythmic (harmonious) proportioning is solved by the laws and rules of geometry."

Furthermore, in the third book, first chapter, first part, he says: "The planning of the temple depends upon conditions of symmetry, the laws of which the architect must be thor-

* 27 must be a copying error instead of 26 (XXVII instead of XXVI), as we shall prove below.

oughly acquainted with. The conditions of symmetry are produced by the proportionality which the Greeks called *analogia*. Proportion means the accord of all the single parts with the whole, from which the law of symmetry has sprung. Without symmetry and proportion in the plan, a temple cannot be made perfect; like a perfectly shaped human body, it must possess in itself, a fully developed law of proportion."

It must be admitted that these short extracts taken from Vitruvius on the Greek science of building temples agree with one another, and they get additional clearness from the interpretation of the Greek words which fortunately Vitruvius has quoted: *taxis*, *diathesis*, *embates*.

Taxis means to place the pawns in good order on the board, soldiers in rows, individuals according to their rank in the kingdom, ideas according to their value.

Diathesis signifies to separate from each other goods which are for sale, the clauses in a contract or in a will, a part in a lecture; it corresponds, therefore, to the Latin word *disposition*.

Embates signifies a cothurnum, the high shoe, in which the tragic actor stepped forward in solemn, measured steps; by *embaterion* is meant a marching song with firmly marked rhythm. When Vitruvius translated the Greek *embates* by the Latin word *modulus*, measure, he had surely in his mind the verb *modulari*, which is derived from it and signifies to sing or play in time, and the adjective *modulatus*, which corresponds to what we understand like the Greeks by the word "rhythmic." By using the right embates, or unit of measure, the architect obtains *modulation*—that is to say, cadence and rhythm in his work.

This very instrument of modulation, modulus, or embates, is what we have rediscovered in the two analyses given above of the Temple of Concordia and of the Parthenon, and which reveals the fundamental "instrumentalisation" of Doric temple architecture.

From the main proportion, and the length and width of the Temple of Concordia, we showed the laws for the division and for the raising of the temple. In figs. 169, 175, 176*a*, and 176*b* we found in various ways what had been the instrument of architectural modulation, this very modulus. In the drawings of the Parthenon, the measure used in the geometrical analysis allowed us to draw the radius of the circumference of the column, as well as the width of the triglyph (see in the central axis of the front the triglyph drawn with the help of the geometrical auxiliary construction, fig. 180). The analysis of the plan of this temple (fig. 181), like the one of the Temple of Concordia, offers a complete harmonic progression of figures, according to the proportion of the sectio aurea, growing from the smallest red circle round the centre, signifying the thickness of the column, right on to circle I, which circumscribes the whole plan of the temple.

Already the "instrumentalising" construction of the Parthenon, which in elegance does not come up to the Temple of Concordia, shows that the method was becoming a routine; but precisely for that reason our analysis of the clear and simple composition of the Parthenon elucidates the statement of Vitruvius, that the builders of Doric temples used as unit the thickness of the column or the width of the triglyph (modulus, or embates) for fixing the proportions of the parts of the building and determining the whole according to these parts tuned to one another; the drawings of our auxiliary constructions certify that the Latin author *quotes* his Greek authorities correctly. On the other hand, the written evidence of Vitruvius gives an additional confirmation to our argument that it is this very method of the Greek temple-builders which we have succeeded in discovering, through our geometrical analysis of their buildings, as they stand to-day.

This analysis has revealed how its smallest element used as unit of measure (modulus) fixes all the parts of the Doric temple in proportion to one another according to law and harmony, from the diameter of the columns to the dimensions of the total width, length, and height of the building. It is precisely what the Greek references of Vitruvius seem to demand

according to his quotations. But Vitruvius himself makes wrong use of this knowledge, because he uses it in a quite schematic way.

His Latin book became, as we know, the catechism of architecture used by the architects of the Renaissance, at a period when Gothic art had long ago degenerated into a stereotyped handicraft; a mechanical obedience to the commandments of this catechism caused the misunderstanding of classic art which many of the architectural enormities of this "new birth" reveal—for instance, the Church of St. Peter in Rome. These pompous efforts were made to awaken the soulless, imitative building art of the political, military, and practically mercantile Roman times; that restless age, over-productive, but really poor of ideas. These efforts leave us cold, in spite of all their dazzling show of wealth, whereas, face to face with Gothic cathedrals and the classic masterpieces of Hellas and of Greater Greece, surrounded by their great silence, we feel the life-giving radiance of the sacred mystic spirit manifesting itself through the eurythmic constructive evolutions of the endlessly creative and eternal laws of geometry.

The architect of the Parthenon had the choice of two ways for the planning (taxis) of his temple: he may have begun either from the inside or the outside. If we suppose the first, it must have been with the diameter of the innermost and smallest circle as chosen modulus. In this he has drawn a pentagon, the sides of which he lengthened outwards so that they intersected one another, and formed a pentagram. By inscribing this in a new circle and by continuing to draw pentagons and to develop them into pentagrams, he reached finally circle V, whereby he found the length of the stylobate.

The taxis of the Temple of Concordia shows that the method gives scope for variations when it is employed by a geometrician schooled in the Greek way of thinking.

Vitruvius gives also the Greek word *teleion* for the division of the unit of measure.

In third book, first chapter, fifth part, he says: "They [the Greeks] had also fundamental measures which seem necessary in all buildings. These are taken from parts of the human body, such as inch (finger), palm, foot, ell (length of the arm from the elbow); and they divided them, taking as basis a perfect number which they called *teleion*. The Greeks fixed ten as the perfect number, because the hands have ten fingers." Vitruvius claims for this the support of Plato; but he adds that "mathematicians do not agree, as they consider six to be the perfect number."

This quotation is correct. For the Pythagoreans, six was the perfect number. We have not had time to find out how old the Greek system of temple architecture can be, but we are inclined to attribute to the Pythagoreans the honour of its invention. We think the Temple of Concordia of Akragas to be the oldest existing Doric temple; it was built in the sixth century B.C., at the time when the famous Pythagorean philosopher and artist, Empedocles, was an honoured citizen of that town. Is it not just possible that this man of genius could have been the master to whom we owe the taxis of this building? The Temple of Poseidon in Pæstum dates from about the middle of the fifth century. The Parthenon is a little later, built between the years 447 and 434 by Callicrates and Iktinos. The Ionic temple of Nike Apteros is the latest, erected 440–410. Plato was born in 429, therefore much later than the time when the system of the Doric temple originated.

The idea of 6 as the perfect number is purely Pythagorean, and is closely connected with that very geometry, shown to be the foundation of the taxis used in sacred architecture. The reason why 6 should have been considered perfect was because it is the first in the natural progression of numbers which equals the sum of its factors—that is, $1 + 2 + 3 = 6$; $6 : 3 = 2$; $6 : 1 = 6$.

We have remarked above that the Greeks considered the irrational as sacred and its geometrical image ἡ τομή, "the section," as a mysterious symbol; it is natural, consequently, that they should give the name "perfect" to the first arithmetical expression of the mysterious

sectio aurea, 6 being at the same time the sum and the product of 1, 2 and 3, the lowest introductory terms in the progressional proportions of the sectio aurea 1 : 2 : 3 : 5 : 8 : 13 : 21, etc.

In connection with this it is also worth considering that the Greek word for the "perfect" number which gives the division of the unity of measure, τέλειον (teleion), is derived from the verb τέλειν (telein), to finish, to fulfil, to fill the measure, which moreover was the expression for consecrating and for initiating into the mysteries. Another derivation of the same root, ἐντελεκεια (entelecheia), signifies the will power existing in the universe, continually creating, continually shaping, therefore an abstract expression of what is mysterious and divine, the symbol of which was seen in the irrational τομή (tomé) or what we call the sectio aurea, having its first and lowest three terms summed and multiplied in the number 6.

This choice of 6 as the perfect number agrees fully with the Pythagorean tendency to symbolise, because 6 possesses in itself, so to speak, the keynote of the endlessly form creating geometrical progression according to the proportion of the sectio aurea, as teleion or dividing number for the unit of measure (modulus) which should serve as cothurne (embates) to the temple builder for the correct (eurythmic) marching cadence, during his planning (taxis) of the fundamental proportions of the future shrine—a problem of which he was granted the key on being initiated into the mystery.

Vitruvius relates, however, that in his time the number 10 was used as teleion. It is true that it was held sacred in another connection, although it cannot be regarded as "perfect" in the same sense as number 6. His statement proves that inside the building guilds a practical teleion was used, which supplanted the original and real one. We understand that the Greek authority of Vitruvius, for the use of this later teleion, belonged to the post-Doric period.

Historians of art have found a contradiction in the fact that Vitruvius gives in one place the radius and in another the diameter of the circumference of the columns as modulus. This contradiction in reality is only apparent, but it is nevertheless of great historical interest, because it shows how the untheoretical author is led to alter his expression for the unit of measure on account of the schematic arithmetical method which he uses constantly; for instance, when he states that a Doric front with four columns must contain twenty-seven, and one with six columns, forty-two units of measure.

Let us give in the descending progression of this number, 42, the arithmetical expression of the proportion of the sectio aurea: 42 : 26 * : 16 : 10 : 6 : 4 : 2; we find in the smallest term of the progression precisely the diameter of the column. The progression is produced by doubling the numbers of the original progression, starting with the unit 1 : 2 : 3 : 5 : 8 : 13 : 21, the radius of the column being unit of measure. In practice it has been found more convenient to operate with the thickness (diameter) of the column, instead of half the thickness (radius), and it was forgotten at last that the unit is number one, and rightly so. The apparent contradiction of these different quotations is only the result of Vitruvius' lack of penetration when it concerns theory, and his preference for schematic practical rules.

The geometrical and philosophical theory of sacred architecture which shines clearly through the unscientific treatise of Vitruvius agrees, however, with the result of our geometrical analysis of Doric temple architecture given above. The quotations of the Latin book taken from ancient Greek architectural science confirm that it is this very science which we have rediscovered in showing the proportioning of the Temple of Concordia and of the Parthenon, according to the law of the sectio aurea, in an interplay logically carried out, of tetragons and heteromekeias, through all the parts of the buildings.

* This shows that 27 (xxvii) is a copying error for 26 (xxvi), as we supposed above.

Our account proves that the art of building a Doric temple was from the beginning a mystery into which only the scientific and fully educated master was solemnly initiated. The temple was not to be produced as the "personal," "original," or "inspired" work of an artist, but it was to be a creation of the unreckonable and always harmonious play of law-bound nature with the contrasts of rational and irrational, in endlessly changing conformity, in its totality and in all its parts, as a symbol of the universe, a Cosmos kept in equilibrium by the proportion of numbers. It was only allowed to begin the task of such a creation after having fully acquired the elements of the universal philosophy of the time concerning the eternal laws of geometry; and if the initiated had proved himself quite worthy to be admitted into the higher gnosis which gave the key to the secret of all life, he was permitted to produce the picture of the irrational, the unreckonable—the divine section, ἡ τομή, the magic formula of the eternally varying but self-contained harmony between tetragons and heteromekeias.

He only who had been entrusted with this key was the master of taxis, by means of which the temple was combined within hexagons and pentagons, proportioned in its various parts by all the regular polygons which compose the picture of the world, the self-creating Cosmos.

It does not seem to have been an accident, but a consciously intended symbol when we see that the circle inscribed in the square (fig. 173, Pl. XII, see fig. 175, Pl. XIII, and fig. 179, Pl. XIV), fixing the length of the cella, surrounds the sanctuary of the temple by a ring of small pentagrams. To emphasise them we have drawn a small circle round each of the five pentagrams which are formed in the corners of the pentagon inscribed in circle IV; the diagonals of this pentagon play with the diagonals of the pentagon in circle II. If we imagine these two pentagons turning upon each other, a constant kaleidoscopic play of small stars being lit and extinguished would be produced (see fig. 185).

We find again the same phenomena in the Parthenon (fig. 181). Here are two rings of stars, an outer one which is formed by the pentagrams in circles I, III, and V, and an inner one formed by the pentagrams in circles V, VI, and VII. The outer one, with a diameter equal to the width of the temple, encircles the sacred room of the cella, the inner one encircles the most sacred of all, passing by the pedestal of the statue of Athene—that is to say, also just over the head of the goddess.

Does not this bring to our memory the teaching of the Pythagoreans on the belt of stars of the universe with its ten circles, in the middle of which dwells the unit, *one*, and with musical intervals of chords between the circles round the centre of the unit? It will be noticed that the construction holds just ten circles. It is undoubtedly a reflection of these thoughts which we found in the analysis of Cologne cathedral, where we saw the star pentagons light up over the altar over the crossing and between the spires of the west towers (Plates IX and XI).

Another peculiarity about the Parthenon may arrest the attention in connection with this: a small rectangular open cavity exists in the fundament of the statue; the measures taken of the sides of this cavity stand in proportion to each other exactly according to the sectio aurea. Can it be possible that this heteromekeia cut into the stone corresponds to the reliquary of the Christian altars? possibly containing a pentagram in gold on the scale of the unit of the temple?

It was intended that the temple should stimulate the imagination and cause that astonishment which creates humble reverence in the intelligent man and fear in the unintelligent. It was not possible for whomsoever to start measuring the temple with a foot-rule, as if it were a house intended for personal use. Art archæologists with callipers and diopters have noted down seconds of angles and rows of decimals painfully tabulated, in order to find out the secret of the noble, but unexplainable harmony of the Doric proportions, until now quite in

vain. The road to the mystery would have stood wide open if, instead of going to work with modern methods, they had trusted to the guidance of the spirit which ruled the architecture of those times; it would have shown the initiated how to bring the star riddles of the universe down from the inaccessible sphere of the unreckonable, and to transform them into harmony, in evolutions of rational and irrational, in a logical interplay within the laws of geometry.

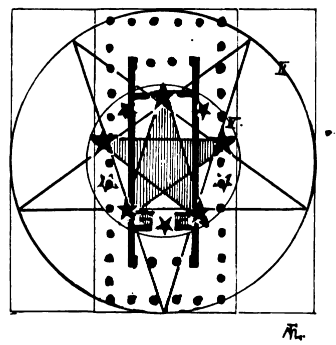


Fig. 185.—Temple of Concordia, Akragas. Plan and wreath of stars.

CHAPTER XIII

THE SECTIO AUREA IN MEDIEVAL ARCHITECTURE

WE have now proved convincingly that the Greeks used the square as a medium for proportioning both the plan and the elevation. In our investigation of the system, as used in Greek temple architecture, we started from the supposition that the opposites in the Pythagorean table of categories, tetragonon and heteromekeia, used as conditions of harmony, had to be something more than a mere symbolic play of words. We found that these two figures solved the great riddle which the Greek temple has been to all who have tried to study it, this means that it proves indisputably the correctness of our conclusion.

These two words signify, therefore, a reality, and they contain not only the spirit of the Greek cult of beauty, but we venture to say, without exaggeration, the essential in the conception of the mystery of the world as taught in Greek philosophy and for which the temple itself was to be the symbol.

We can understand now why Hippocrates of Chios committed an outrage when he earned his living by teaching geometry. He gave an embates, an enteleion, to the uninitiated, which introduced him into the mystery of the temple and of life, while not yet sufficiently qualified. By being thus laid open to everyone, the temple was robbed of its secret, and this was synonymous with desecration, it was *Asebema*.

It is also probable that a book was never written in Greek on sacred architecture, because it would have revealed its secret principle. The superficial manner in which this is treated by Vitruvius points to it. We do not know either the Greek expression used for the science of harmony in architecture, in the table of categories, as we mentioned above; if somebody has said " kata tretagonon kai kata heteromekeian," or " kata pentagonon," it is of no interest to us.

But what is of importance is that this use of the square as means of harmony—besides being used by the Greeks—dates back to a primitive way of building in the East, as we find it in the Biblical description of Solomon's temple, where the square has been clearly used in plan and in elevation. The plan had two squares in length, therefore a proportion of 1 : 2 (figs. 9*a* and 9*b*). We find also the same proportion given by Vitruvius in Book IV, chapter 4, when he writes that the length of the *temple* is to be divided in such a way that the width is half of the length.*

We saw that this statement, supported by the sanction of the Bible, had determined the church architecture of the Benedictines.

* "Distribuitur autem longitudo ædis uti latitudo sit longitudinis dimidiæ partis" Vitruvii, *De Architectura*, ed. Valentin Rose et Herman Müller-Strübing, p. 94 (Teubner, Leipzig, 1867).

Moreover, we can again trace the Greek use of the square of the principle *ad quadratum* in elevation, just as it has been preserved in writing in the Milanese document from 1392. We are convinced that this principle is everywhere the foundation for cathedral building in the Middle Ages, several centuries before that document, and we are compelled to acknowledge that there is a close connection between the Greek method and the medieval one. This connection was kept up partly or perhaps mostly by tradition and routine inside the guilds, where the method had been gradually adapted and developed, so that it could be employed in the high three-storied churches, but partly also by theoretical development, without which

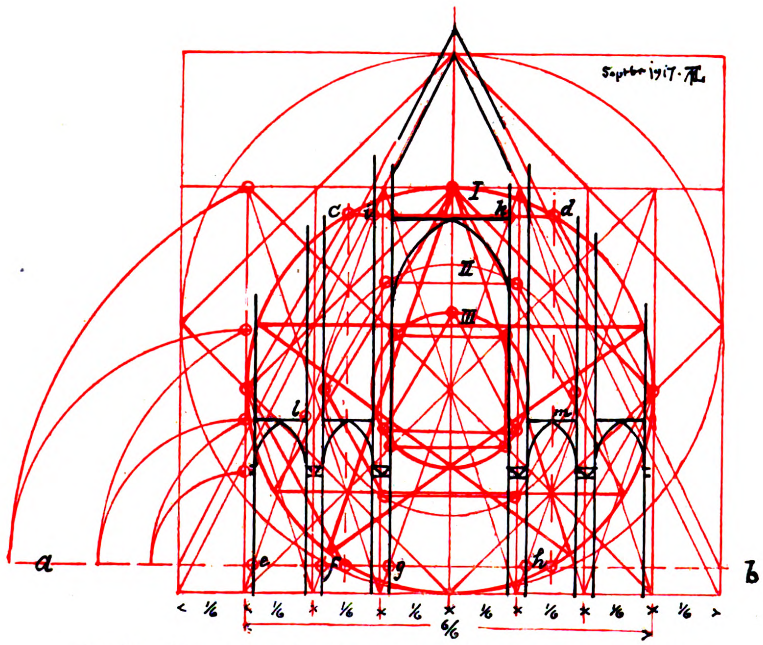


Fig. 186*a*.—Theory of the five-aisled cathedral, *ad quadratum* and *ad pentagonum*.

it could hardly have been used with such subtility as we see it, for instance in its perfect form, in Cologne cathedral.

We consider the knowledge of the sectio aurea, or, if preferred, of the pentagram, as the criterion of this truth. This knowledge was certainly employed in the guilds; but, although we do not in any way underestimate them, we must ascribe to Boëtius the honour of making the Middle Ages acquainted with Greek science—long before Arab influence.

After having analysed the Temple of Concordia and the Parthenon, and having learned to know the constructions which are at the base of the proportioning of these temples, it will be admitted that these constructions throw a light on the words of Boëtius, when he says that geometricians draw an equilateral and equiangular pentagon around and inside a given circle, and when he says, further, that everything which is expressed by numbers exists by the intrinsic proportion of numbers and that they are produced proportionately by one another.

The Latin translations of Boëtius were the main source from which the Middle Ages obtained its knowledge of Greek science, especially of Aristotle (Stahr, *Aristoteles bei den Römern*, p. 196, etc.).

We reminded above that his mathematical writings were used as text-books in the medieval grammar schools. Old Norse literature from the twelfth century shows that his works were also found in the library of the archbishops of Nidaros.*

Viollet-le-Duc says that the people of the Middle Ages had seen more of classic architecture than those of the nineteenth century.

When we consider that hundreds of buildings have disappeared since the Middle Ages, we are entitled to think that ancient MSS. of which we are ignorant have also existed and disappeared.

Our investigations will show that the Middle Ages had, in any case, a greater knowledge of classic science and its spirit than the modern history of science has been willing to acknowledge or clever enough to admit. This knowledge does not only limit itself to the principle of building *ad quadratum*, but it contains the whole geometrical science; this is manifested by the temples of Akragas and of the Acropolis in Athens. It means, then, that the Middle Ages understood the secret of the sectio aurea and its power to create unity and modulation. The medieval architects had their embates, just like the Greeks, and understood, as Boëtius says, that values are derived from one another.

Before demonstrating this, we must first see how the mere use of the square and its division into six or four rectangles produces already proportioning after the sectio aurea, as regards the main divisions.

In fig. 186*a* we see the five-aisled cathedral according to the division of the square into six parts. As Cologne cathedral represents the logical rendering of the theory, this figure gives so far the theoretical rendering of this cathedral.

We have explained above that the dimensions of the walls are produced by the intersecting points of the diagonals of the rectangles, the line above the plinth, *a b*, with the corresponding line, *c d*.

As regards this demonstration we have only emphasised three diagonals marked at their intersecting points with the plinth-line by the letters, *e*, *g*, and *h*. By vertical lines through these points and the corresponding one, not marked, we find the constructive outer lines of the walls. Where the diagonals again intersect these outer lines, the height of the stories is determined, *l* and *m* for the aisles, *g* and *k* for the nave.

In the analysis of the diagram of the three-aisled cathedral (fig. 186*b*) the corresponding diagonals are marked *e g* for the aisle and *f d* for the nave. This theoretical transverse section corresponds with the transverse section of the nave in the cathedral of Nidaros. Consequently we have included it in the theoretical diagram of the *front* of this cathedral—as the theory of that front is dictated by the plan of the church, as will be seen later.

Constructions of the sectio aurea are carried out in both drawings (figs. 186*a* and 186*b*). It will be noticed that the division of the sectio aurea corresponds with the division of the stories, as pointed out above, as it also corresponds with the division produced by the pentagrams introduced in the circles inscribed in the square of the church in both figures 186. Both drawings being made at the same scale, circle I in fig. 186*a* is identical with circle I in fig. 186*b*; it follows that their geometrical functions give identical results. Consequently we can treat the two transverse sections under one head. The principle for the introduction

* This is proved, among other places, in *Historia de Antiquitate Regum Norvagiensium*, by Theodorici Monachi, dating about 1170. Thjodrekr quotes, besides Boëtius, several Greek authors, and shows also that he knows Greek. From the same century we have the approximately correct degree of latitude of Nidaros and Bjorgvin.

of the circles is the same as in the Parthenon. The diameter of circle II = the major of the diameter of circle I, and the diameter of circle III = the major of the diameter of circle II. In this way the diameter of circle III = the minor of the diameter of circle I, while the diameter of circle IV = the minor of the diameter of circle III.

It is evident that there exists the same connection between the polygons inscribed in the circles as in the Greek temples, as we shall see later.

An equilateral triangle is introduced in circle I (fig. 186*a*). Its side = the height under the vault, in both figures. The diagonal of the pentagon in circle I is equal to the interior width of the church. The side of the decagon in the same circle is obviously = to the side of the hexagon in circle II. The side of this hexagon gives the interior width of the nave measured from wall to wall (fig. 186*a*).

The diameter of circle II (fig. 186*a*) = the distance between the pillars across the aisles. Finally it is clear that the side of the triangle in circle III = the distance between the axes of the pillars across the nave. The side of the square in circle III = the distance between the outer surface of the columns under the capitals of the vault (fig. 186*a*).

In fig. 186*b* a similar phenomenon occurs, as the square in circle II marks the inner width of the nave similar to what occurred with the square in circle III (fig. 186*a*). This distance = also the side of a heptagon in circle I.

The interior width of the aisles in fig. 186*b* = the side of the hexagon in circle III, while the side of a decagon in the same circle = the width of the aisles in fig. 186*a*.

The height of the aisles in both figures = the diagonal of the pentagon in circle III. The interior of the church lies between the parallel sides of a hexagon and a decagon.

The thickness of the walls, such as it is produced according to the explanation given above, is in fig. 186*a* = the side of the pentagon of second order in circle IV. In fig. 186*b* this is = the diameter of the circle which circumscribes the pentagon of second order in circle IV.

According to this it will be understood that, similar to the Parthenon and the Temple of Concordia, we find here also in this cathedral the same inner harmonic connection between the various divisions of the building and its dimensions.

It could be objected, with a certain show of reason, that this had not been done on purpose and that it is only the result of the principle *ad quadratum*, which leads always, when used judiciously, to the proportions of the sectio aurea and its power to create an inner harmonious accordance.

Such an argument would be quite correct if this manner of proportioning did not go further than the main parts, most of which have been indicated here. But in the more perfect cathedrals it goes systematically throughout the whole fabric and determines, not only the subordinate architectonic and ornamental divisions, but it gives the very diameter of the columns, that is to say, the modulus of the ancients, such as we found in the Temple of Concordia and in the Parthenon. But this modulus cannot be produced by the division into six and into four of the square and the diagonals of the rectangles resulting from these divisions; it can only be found by means of the sectio aurea in the manner of Euclid, by sectioning the square, or by the pentagram as in the temples of Akragas and of Athens. It is this last method of the pentagram which was used in the Middle Ages, and of which we shall give below a striking proof. When pentagrams are found in old churches they are certainly no stone-cutters' marks, but marks of guild, or, if preferred, the marks of a system, a key in view of future restorations for the Magistri Lapidum initiated in the secret of the temple, the Grjötmeistarar, in old Norse.

It will be interesting at present to compare the results of the theoretical transverse section of some churches with the results of their real section.

As Cologne cathedral is built according to the theory to full perfection, we will analyse its transverse section (fig. 187) in relation to the theory in fig. 186*a*.

In accordance with our analysis in fig. 71 (Pl. VI), we now lift the theoretical auxiliary construction above the plinth-line. We observe at once that the line *a*—*b*, which in fig. 186*a* is the starting-point of the construction, and which in the older cathedrals also represents the

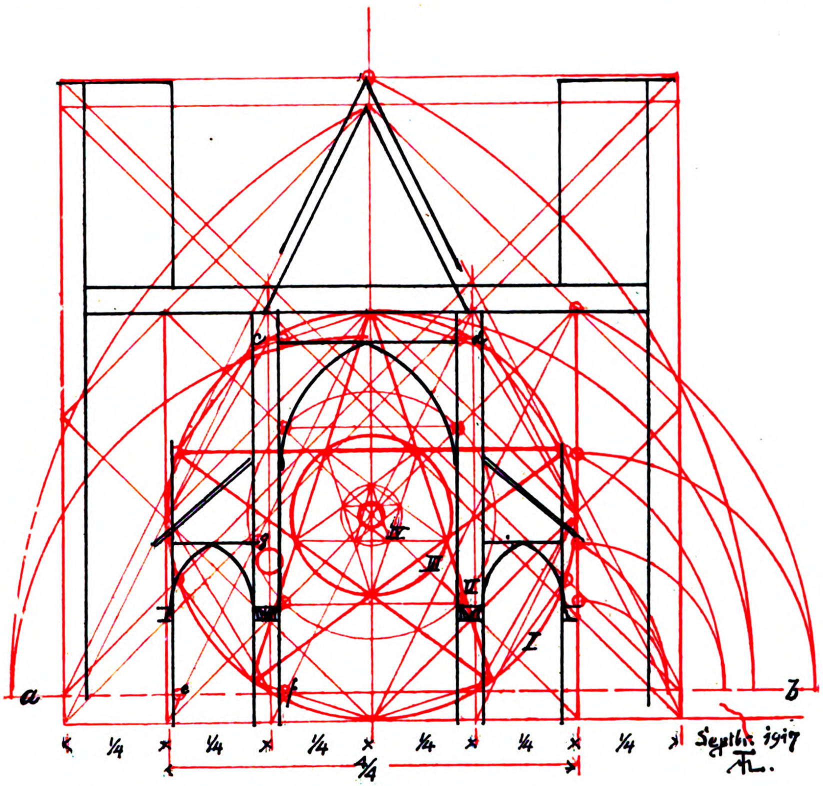


Fig. 186*b*.—Theory of the three-aisled cathedral, *ad quadratum* and *ad pentagonum*.

line above the plinth, is here in fig. 187 (*a*) (*b*) lifted so as to be on a level with the first division of the buttress. This extremely interesting phenomenon shows that, as regards the exterior, the whole lower division under the line (*a*) (*b*) has been considered as a high pedestal for the high elevation of the building.

In consequence, we have made our analysis of the proportioning of the exterior above this line. To the left the constructions of the sectio aurea are drawn in red. They need no explanation. It will at once be evident that not only the main proportion of the building—the division of the stories—is according to the proportion of the sectio aurea, such as shown in the theoretical drawing, fig. 186*a*, but that the purely decorative architectural divisions also fall within the same proportioning.

This appears to be the case equally with the proportioning of the interior of the church. The measurements regarding this are put in red on the right, and marked from 8 to 11. Of the four measurements shown, the proportioning appears in two cases (8 and 9) to have started from the floor, while in 10 and 11 it has started from the line above the plinth of the columns. As it will be seen from the construction of the sectio aurea No. 9, the arcades of the nave are the minor of the high columns under the transverse arch of the vault. The statue on the column in the nave is also placed according to the sectio aurea, therefore this proportioning is not a free result of the geometrical function produced by the co-operation of the rectangular division of the square with the diagonals, but it is the result of a *conscious* use of the sectio aurea as a harmonising principle. Our analysis of the interior proves this.

For that purpose we have here also inscribed the circles in the square (fig. 187), such as it was done in fig. 186*a*, and in the classic temples. The circles are numbered according to their order.

We have inscribed in the circles some of the regular polygons.

As regards the connection between these polygons and the whole building we will limit ourselves to pointing out a few characteristics.

The diagonal of the pentagon in circle I = the height from the line over the plinth to the keystone of the groins the vault. This is demonstrated by the broken red line marked R^1. The arc of a circle is drawn with this as radius. By following the course of this arc it will be seen that, after passing through the corner marked *x* in the decagon inscribed in circle I, it passes the point of butment of the webs of the vault in order to go after this through the keystone, where the lines of the webs of the vault intersect one another. From this it follows that the intrados of the vault is determined by the diagonal of the pentagon as radius. We have marked with small red circles on the diagonals of the pentagon coming from zenith of circle I the points where these diagonals intersect circle II. The arc of a circle having as radius the distance between these points and the lower edge of the webs (each radius marked *r*) corresponds exactly with the curves of these webs. A little below, between circle II and circle III two new points are marked on the same lines. A line marked by a small *r* from one of these points to the lower edge of the transverse arch is the radius of the curve of this arch. The precision with which the curves of the construction coincide with the curves of the vault excludes any idea of accidental coincidence. The diagonals of the pentagon are thus used as means of determining the vault construction.

There is another characteristic: the radius of the webs (marked *r*) = the side of a dodecagon in circle II, while the radius of the transverse arch (the cursive *r*) = the minor of the diagonal of the pentagon in circle II or the side of the pentagon in circle III. If we measure now the height, from the line above the plinth to the keystone of the transverse arches, we get the interesting result that this height = 3 sides

of a decagon in circle I = 5 sides
of a decagon in circle II = 8 sides
of a decagon in circle III = 13 sides
of a decagon in circle IV = 21 sides
of a decagon in circle V, therefore according to the scale of the sectio aurea, 3 : 5 : 8 : 13 : 21.

As regards the exterior it can be mentioned that the height from the line above the plinth to the ridge = 7 sides of a decagon in circle II = 9 sides of an octagon in circle III.

The height of the pinnacle over the walls of the nave from the line (*a*) (*b*) over the pedestal = 6 sides of a decagon in circle II, and from the plinth = 16 sides of a decagon in circle IV.

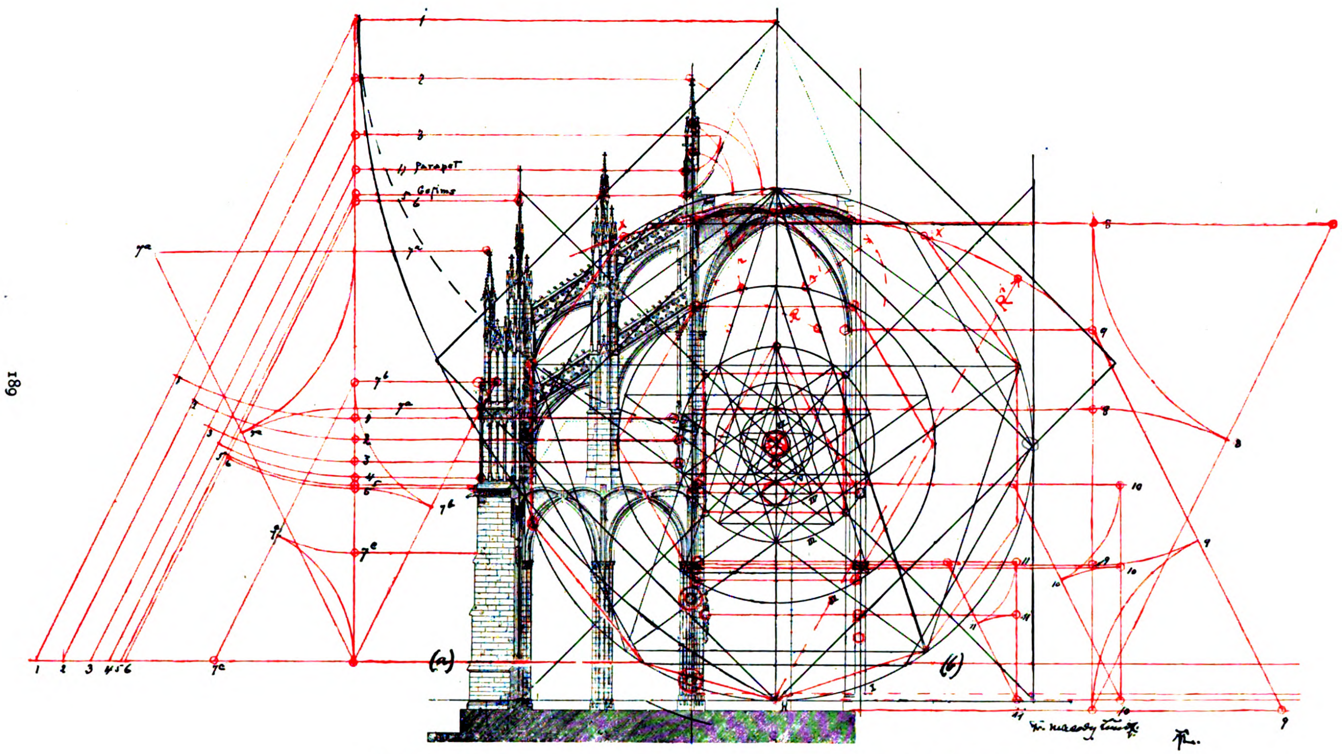


Fig. 187.—Cathedral of Cologne. Transverse section *ad quadratum* and *ad pentagonum*

We consider it superfluous, after these demonstrations, to undertake any more comparisons of a similar nature. Those we have chosen here at random are sufficient to prove that the various divisions of Cologne cathedral stand in intimate harmonious relation to one another and to the whole, as well as to all the regular polygons which can be inscribed in the four circles introduced in the square.

We see from this that there exists certainly a unity going through the whole scale of this symphony, an *embates*, or modulus modulationis.

This embates is the side of the decagon in circle V or the diameter of the circle drawn in red between circles IV and V, and also the circle drawn in red which is inscribed in the small pentagon in circle VI.

This diameter is the total thickness of the columns on the lower floor,—the arcade of the nave. Circles with this diameter are drawn in two places on the columns of the nave. The diameter of this circle includes the whole group of thinner columns of which the entire column is composed, therefore also the three slender columns which carry the groins of the aisle and the three columns which carry the groins, the transverse arch and the groins of the vault of the nave.

Inside the larger circle of the pillar there exists a smaller circle, the diameter of which is the diameter of the core of the bunch of columns which supports the intrados of the arcade. This diameter = the side of the decagon in circle VI. As a geometrical outcome of the auxiliary constructions, it is similar to the greater diameter of the column drawn in red in the construction just under circle VI to which it belongs. As the diameter of circle VI = the minor of the diameter of circle V, it follows that the side of the decagon in circle VI is the minor of the side of the decagon in circle V. The dimensions of the group of columns of the arcade and its core, are therefore determined by the proportion of the sectio aurea.

The square within which the church is planned contains 22 diameters, while the square of the interior, in height and in width, contains 21 diameters of the entire thickness of the group of columns, or 42 radii of the columns.

The height of the column, from the floor to the lower edge of the capital = 6 diameters, and from the top of the plinth to the top of the capital, 6 diameters. From the top of the upper division of the plinth to the floor of the triforium the height represents 9 diameters; from the floor of the triforium to the floor of the clerestory 3 diameters are contained. The column of the window of the clerestory from the upper edge of the plinth to upper edge of the capital contains 5 diameters, and the rose tracery at the top of the window measures 5 diameters.

One could continue measuring and introducing other units of measure; for instance, the diagonal of the pentagon in circle VI, or its side or other values in the geometrical system of harmony, and, as in all the other measurements previously made, it will be found that all the divisions in the building are in a perfect harmonic and unreckonable accord, which never can be created without the help of the sectio aurea or without the Greek "instrumentalising" auxiliary method, which we have called *ad pentagonum*.

Just as in the theoretical five-aisled cathedral in fig. 186*a*, it will be seen that in the real cathedral of Cologne the lines of the interior of the nave as well as the longitudinal walls, the vault and the plinth-line, lie within a decagon, while the walls of the nave correspond with the square in circle III.

The *side elevation*, as it can be measured with dividers, or by comparing fig. 72*b* (Pl. VII) with the measurements introduced in fig. 73 (Pl. VIII), is divided also according to the proportion of the sectio aurea.

As regards the west front, fig. 188 shows that the architectonic division of the parts is

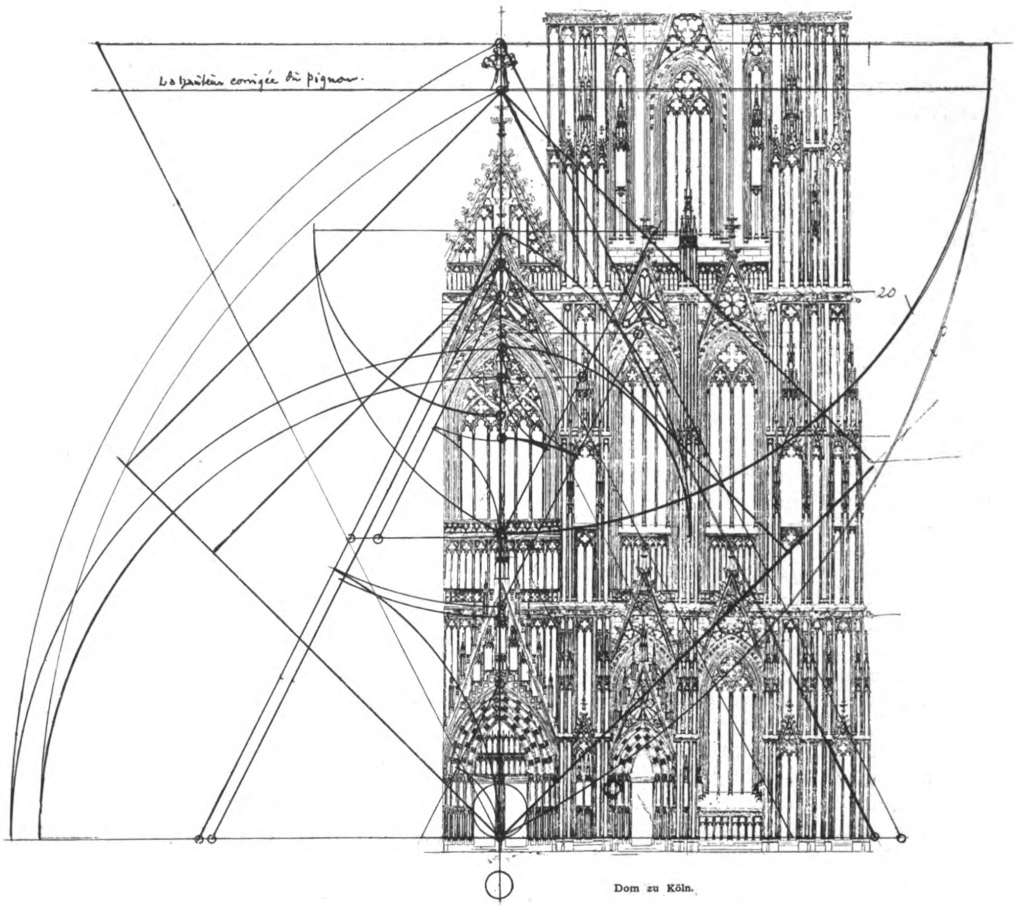


Fig. 188.—Cathedral of Cologne. West front with demonstrations of the sectio aurea.

a logical continuation of the proportioning of the main divisions, which we saw in fig. 76 (Pl. IX) marked by the line numbered in Roman figures I—V, and the unmarked line to the right of that figure. The analysis of fig. 188 speaks for itself without further explanation. Underneath this drawing we have introduced the diameter of the columns. The opening of the centre porch is 2 diameters wide and 2 diameters high over the plinth-line.

As it has been shown, therefore, Cologne cathedral is in every respect proportioned according to the sectio aurea. The truth of what we saw above must be acknowledged: that this cathedral, more than any other of the Middle Ages, is related to the Parthenon.

The Cathedral of Nidaros

Fig. 190 gives the transverse section of the nave towards east. The walls of the aisles are medieval. The columns of the arcade of the nave have been re-erected according to the existing half columns west and east, that is, towards the gable of the nave in the west, and towards the central tower in the east, wherefore the whole lower story can be considered as medieval. The church is rebuilt to the floor of the clerestory by the late architect Christie with great insight and unfailing respect for the medieval forms. The clerestory and buttresses with their flying buttresses are according to the reconstruction proposed by us based upon the remains and upon the geometrical system according to which the cathedral appears to have been built. It must be added that the part of the line of the roof shown by NB on the western wall of the tower is a groove, or trace in the wall, left by the medieval roof, therefore a clear guide. Later on in a special analysis we will make clear the importance of this fact.

We have included this cathedral here in our documentation to show the use of the pentagram and of the sectio aurea in medieval cathedrals. It may be of interest, in connection with this, to say that the cathedral of Nidaros belongs to the very oldest Gothic cathedrals, as its octagon and chancel are previous to 1179, the walls of the aisles previous to 1188, its nave from about 1200, and the west towers as well as the west front from 1248.

Its chronology will be given later. The time of its construction coincides with the older French Gothic cathedrals. It offers, therefore, an interesting example of the use of the principle *ad quadratum* and *ad pentagonum* considerably older than Cologne. The auxiliary square in this case is not, as in Cologne, lifted to the line over the plinth.

It will be seen at once that the cathedral of Nidaros agrees with the theoretical demonstration of a three-aisled church in fig. 186*b*, its proportioning being also carried out by means of the sectio aurea.

We notice thus the square in circle II with the measurement of the width of the nave. We notice, furthermore, that the side of an equilateral triangle in circle III gives the opening between the pillars of the central tower.

Finally, in the side of the decagon in circle V, and in the diameter of the circle inscribed in the innermost pentagon, we find the diameter of the thickness of the columns.

The square of the church contains then, here also 22 diameters of columns, of which 11 fall on the distance between the axes of the nave, and $5\frac{1}{2}$ on the distance between the axes of each of the two aisles.

The height from the plinth to the top of the wall is 21 diameters.

The interior width is also 21 diameters, as in Cologne.

In the nave, the height up to the vault is 20 diameters—that is to say = 20 sides of a decagon in circle V = 12 sides of a decagon in circle IV.

The height of the lower story = the side of a pentagon in circle II, and the height from here to the summit of the canopies above the clerestory windows corresponds in measurement with the side of the pentagon in circle I; it is, therefore, according to the proportion of the sectio aurea. It must be noticed that this part, as well as the whole clerestory with the system of abutment, is according to our reconstruction, and based upon the fragments found. It may be explained, further, that the greatest external width from the outside surface of the buttresses = 2 sides of the pentagon in circle I.

Finally, it is also interesting to state that the centres of the transverse arches of the vault of the nave, such as this has been planned here according to the geometrical system of the church, fall exactly at the very summits of the star pentagon in circle III (radius marked R).

The height up to the vault of the aisles = 8 diameters of columns or sides of the decagon in circle V. Moreover, the same height = the diagonal of a pentagon in circle III = the side of a pentagon in circle II just as in the theoretical church in fig. 186*b*.

The arch under the tower = 2 diagonals of a pentagon in circle III = also the side of a square inscribed in circle I, reckoned from the line over the plinth.

Finally, similar to the theoretical drawing, fig. 186*b*, the interior opening of the nave marked by two small rings on the line of the plinth = the side of a heptagon in circle I.

The height to the ridge, as it is given by the medieval trace of roof = 2 sides of a square inscribed in circle I. The case is the same with the theoretical church in fig. 186*b*, the height up to the vault in this and in the cathedral of Nidaros = the side of a triangle in circle I.

We have ascertained, by this data, the conformity between the theory of the three-aisled church and the cathedral of Nidaros, which we shall treat more exhaustively later on.

After these comparatively extensive demonstrations of the use of the sectio aurea in the

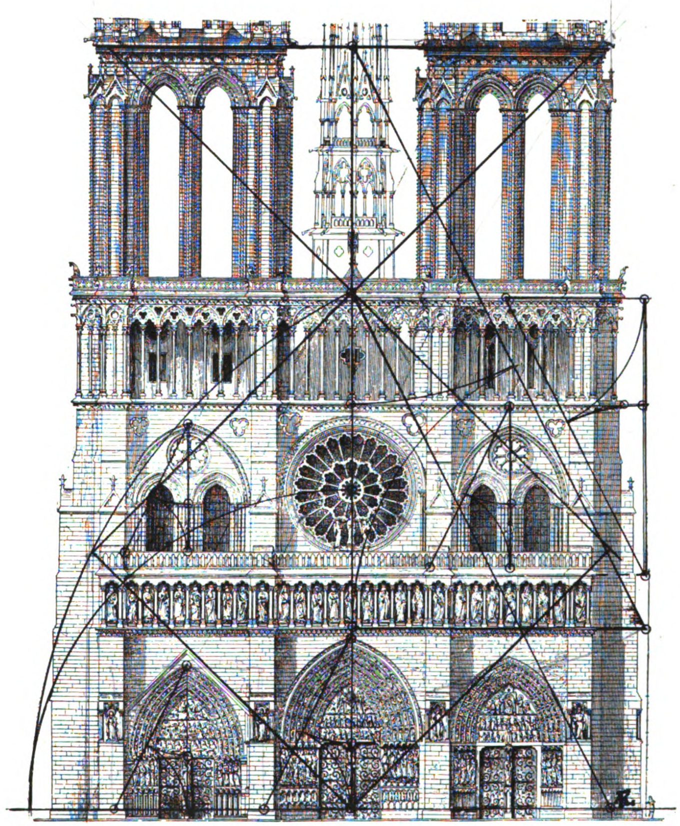


Fig. 189.—Cathedral of Paris. West front with demonstrations of the sectio aurea.

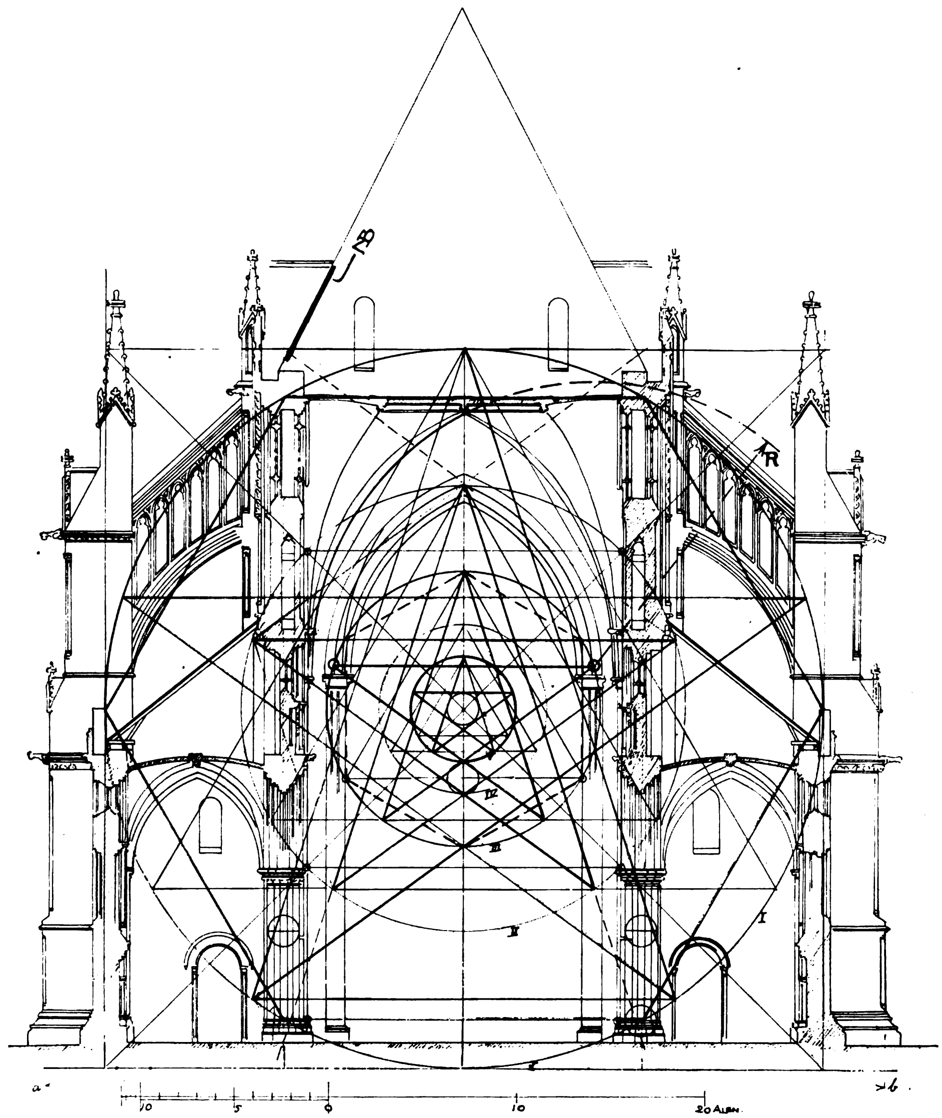


Fig. 190.—Cathedral of Nidaros. Transverse section of Macody Lund's design, proportioned according to the sectio aurea.

construction of the cathedrals of Cologne and of Nidaros, we consider it superfluous to explain the following examples, as the drawings speak fully for themselves.

In fig. 189 we give the front elevation of *Notre-Dame of Paris*. This was begun, as mentioned, in 1163, and finished in 1220; it is planned, therefore, about three generations before Cologne. Both the main and the secondary divisions of the front, as well as the placing of the rose window, are determined by proportions of the sectio aurea. In fig. 59 (Pl. IV), it can be seen, by the construction of the sectio aurea put in red, that the level of the centre of the rose in the large window of the gable in the northern transept is determined in the same manner because the radius of the rose is the minor of the height of the window. This cathedral also has its modulus as shown in fig. 52.

Fig. 191 gives the front elevation of the *cathedral of Laon*. This is about contemporary with Notre-Dame. Similar to this, it is one of the most beautiful examples of the use of the principle *ad quadratum* and of the sectio aurea. The proportioning is demonstrated by the punctuated lines on both sides of the drawing of the front. The modulus is also employed in Laon, but we cannot demonstrate it in the present work.

In the examples from England, we have shown in figs. 92 and 93, and fig. 94 that the sectio aurea has been used to proportion the *cathedrals of Wells and of York*.

As we could continue indefinitely, we must do as for the classic, and limit ourselves to the typical examples here given.

In the planning, the pentagram appears also to have been employed. The previous analysis of the plans of the *cathedrals of Amiens, Beauvais, and Cologne* is a fully convincing proof of it: Figs. 66, 67, 68*a*, 75, and 79 (Pl. XI).

* * *

We saw, in the previous explanation, how the discovery of the geometrical system of the cathedral of Nidaros has given us the key to a number of problems hitherto unsolved in sacred architecture. Its various epochs, the classic and the Christian medieval, have been considered, until now, as quite independent of each other, the opinion being that there was no direct connection between them.

It was held, furthermore, that the mathematical science of the classics was unknown in the earlier part of the Middle Ages, until the Arabs made Western Europe acquainted with it via Spain. Our investigation has shown that this is a myth. We have succeeded in proving its uninterrupted connection and its continuation from the classic to the Christian medieval time.

We have seen, moreover, that sacred architecture is not, as our time chooses to see it, a "free" art, developed from "feelings" and "sentiment," but that it is an art strictly tied by and developed from the laws of geometry.

A number of attempts have been made in our days to solve the obscure riddles which are contained in the ancient temples and in the heaven-reaching medieval cathedrals. The temples have been, through generations, the object of intense studies; special expeditions have been sent fitted out with the finest instruments, they took measurements under certain atmospheric conditions, made long mathematical calculations, through which they hoped to find the principle according to which these sacred buildings of antiquity had been planned, and in what manner they had been carried out. In a similar way, medieval cathedrals have been the object of an exhaustive study; but in this case, as in the other, quite in vain. Every new variation has created a new theory, having no connection with the previous one.

This discovery of the geometrical system of the cathedral of Nidaros, as we have shown previously, has given us the key to the great simplicity, the depth and the marvellous elasticity of the method according to which such different buildings as the Parthenon on the

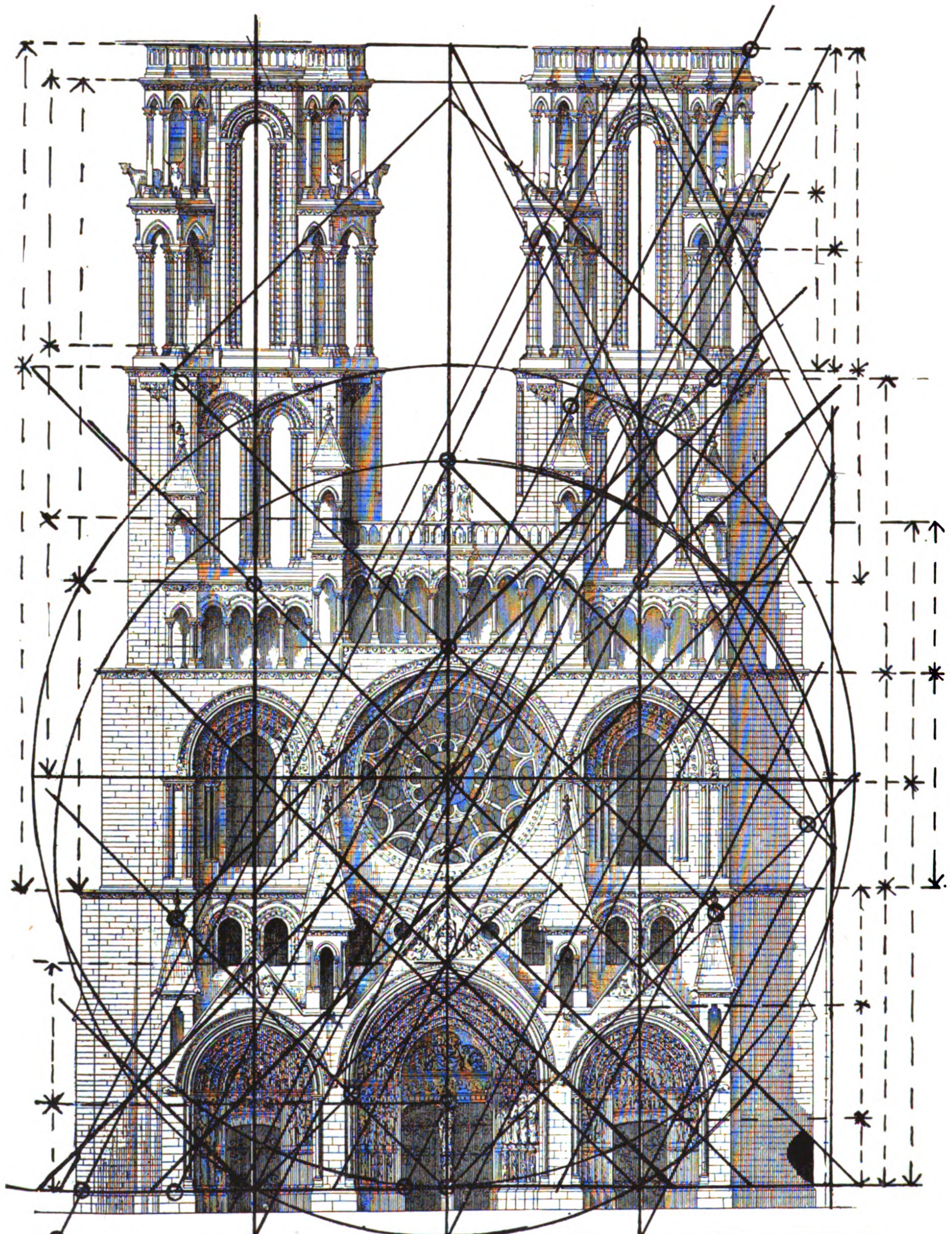


Fig. 191.—Cathedral of Laon. West front with demonstrations of the sectio aurea.

Acropolis and the cathedral of Nidaros, indeed even the Norwegian "stav" churches, have been built. We have obtained full confirmation of the truth of the words of the French architect during the discussions which took place in Milan in 1398: "Ars sine scientia nihil est." We have come to understand how it was a crime in ancient Hellas to initiate the layman into the secrets of geometry, also why sacred architecture had to be a secret art, cultivated only by the few initiated, and how the very essence of this art, its creative principle was accessible only to the *magistri lapidum*. After the Reformation these initiates were no more required. The dark hatred towards "Popery" and the interest of German politics to swell the importance of the Reformation as a "Lossagung von Rom" (a deliverance from Rome), to use a modern expression, that is to say, a national liberation, and on the other hand, the contempt of the rationalistic eighteenth century for the "age of superstition," and the consequent criticless admiration of the Renaissance have faithfully worked together to throw the Middle Ages in darkness. "Je grösser in der Neuzeit die Unkenntniss, desto finsterer ist das Mittelalter" * ("The greater the ignorance of modern time, the deeper becomes the darkness of the Middle Ages.")

Under this "liberation" and "enlightenment," the medieval science of temple-building was bound to disappear, while later times got only the empty shell, the artisan's external routine without theory.

We return now to the building which was the starting-point of our investigation, and which, by its harmonious composition with fixed principles, has given us the key to the greatest problem of architecture, and the most difficult of access—this is the cathedral of Nidaros.

* Hazak, *anf St.*, p. 268.

CHAPTER XIV

HISTORICAL PRELIMINARIES OF THE FOUNDATION OF THE CATHEDRAL OF KING OLAF KYRRE IN NIDAROS

IN the previous chapters we gave a theoretical analysis of the geometrical system of the cathedral of Nidaros, supported by historical research and evidence. We have thus brought back to light this very ancient principle of sacred architecture, strictly put in practice already in the time of the Pythagoreans and severely continued to its perfection in the cathedral architecture of Christian medieval times. Its precise and scientific employment culminates in the planning of the cathedrals of Rheims and of Amiens, and, above all, of the cathedral of Cologne.

We return now to our starting-point, the examination of the cathedral of Nidaros, in detail; it can be done in a clear methodical manner, supported as it is by the genetic and theoretic analysis given above.

As already suggested, and as it will be proved further by notes, this metropolitan church of Norway is one of the oldest Gothic buildings in Europe. In its original pre-Gothic state it coincides also with the oldest types, called rightly, Benedictine pilgrim cathedrals, such as we found them already at an early stage in Chartres, later in Cluny, and, later still, in the rebuilding of the cathedral of Canterbury after 1096, with a central tower as the centre of a plan composed of two large squares.

When mentioning Durham cathedral (begun in 1093) we said that this building was undoubtedly influenced by the cathedral of Nidaros.

The following general and historical account will justify the argument, apparently so contrary to the usual view taken by historians of art and culture, regarding the position of Norwegian national architecture in the development which took place in the Middle Ages.

The importance of the Northmen (Norwegians) as state-forming and constructive power, their organising talents, their aptitude for renewing the spiritual and material development of the place where they settled, and how they introduced new principles of construction and a new style in the art of building wherever they went, this is already recognised so generally and so long ago in the historical science of Europe that it does not need any further demonstration on our part. In connection with this we can give such names as Henri Martin, Freeman, Viollet-le-Duc, Ruprich Robert, and many others, besides the Danes, Worsaae and Sophus Müller, and the Norwegians P. A. Munch and J. E. Sars.

In spite of this acknowledgment we see that opinion and appreciation have practically been the opposite on the part of Northmen in their own country, especially as regards Norwegian *national* architecture. In this self-contradiction we cannot discover anything else

than a reflection of the accounts written by the old historians on the depredations of the Northmen. Coming as conquerors, it is understandable that, from a foe's point of view, they were looked upon as barbarians or savages, who threw children on spearpoints, burnt churches and cloisters, without respecting, or rather without understanding, the civilisation of the conquered countries. Therefore, whatever there exists of valuable remains of earlier architecture in Norway—remains witnessing to high culture and highly developed art—cannot be reconciled with the inherited opinion concerning Norwegians, except through the misunderstanding, that whatever they have achieved was in imitation of what had been produced already in Europe. For instance, the rich wood carvings of pre-Christian time in Norway were supposed to come from Ireland, the religious ornamental designs, to be borrowed from England and France; in other words, Norwegians were always supposed, in their national architecture, to follow, to borrow and to imitate, while the same people abroad, whether noticed or not, were foremost in creating and remodelling.

In Norway these questions have not been subjected to any exhaustive scientific investigation. If one excepts the archæologist and architect, H. M. Schirmer, and the Professor of History, J. E. Sars, all the writers who have partly touched our building chronology, have taken for granted the general opinion. They seem to have lacked certain scientific qualifications necessary to come to an independent opinion on this subject, which, besides imagination and talent, requires deep and wide knowledge of history and of the history of art, as well as developed insight into technical architecture. Therefore they have taken the already mentioned opinion as an axiom, and it is on this that the authorised chronology of Norwegian architecture is based. It will be argued, against our previous statement, the improbability in a country lying so far north as Norway, where Christianity was introduced only towards the end of the tenth century, that already in the eleventh, cathedrals like the one in Nidaros should have been built, belonging to such a type of construction and with such a developed Norman ornamentation that it should have been taken as model for the cathedral of Durham already planned in 1093. There is nothing unreasonable in this, however. It is true that, before the introduction of Christianity, architecture in northern lands must be considered as being limited to wood only, and that it was under the influence of the Roman Church and through its liturgic demands regarding the main forms of the temple that stone architecture was introduced in Norway; but, on the other hand, the acknowledged aptitude of Norwegians for creating and renovating must presume an old civilisation at home, an aptitude which needed only an impulse to find new ways and create new forms.

In northern countries Christianity met an old, high civilisation handed down from the common Germanic tribes before the great migration, and further developed independently as it came in contact with Frankish, Anglo-Saxon, Irish, and Byzantine development of classic and Christian inheritance from the Romans and the Greeks. While the "dark centuries" of the Carlovingians and the Merovingians threw their shadow over a chaotic Europe, where agriculture and handicraft were the usual pursuits of life, there existed in northern countries the same secure, well-balanced conditions which prevailed on the south side of the Baltic before the upheaval of the great migration. The original tribal civilisation was developed constantly and broadly under the fresh and fertilising influence of the Viking times, due to their travels in far-off lands, between the Bosphorus and Icolmcille, and, so far as concerns the Norwegians themselves, beyond any known land, across the ocean, to Iceland, Greenland, and America. A strict social order and a quiet civilisation at home, exploits and adventures abroad, the imagination stirred by the loneliness of the sea—these were elements enough for the development of an original culture in a race possessed of natural gifts equal to their neighbours both south and west. At that time Norwegians had settled so much all round Europe that they felt at home wherever they travelled. In the ninth and tenth centuries the north of France

and part of the west, large parts of England, Scotland, and Ireland, as well as Gardarike (Russia),* besides many other places, were so populated with Norwegians that when a northerner goes now through lists of names in documents from those countries dating from these times he feels that he is in homely surroundings. From their farms at home in the Orkneys, or in Iceland, the families of Norwegian chiefs could count friends and relations in Northumberland, Scotland, Ireland, Normandy, Novgorod, and Constantinople. The three first Christian Kings of Norway had been baptized in London, in Winchester, and in Rouen. It is characteristic of that time, that a man like the famous Norwegian king *Harald Haardraade* went in his youth with a great suite of compatriots through Gardarike to Greece, where he remained for fifteen years, from 1030 to 1045, of which twelve were spent as army chief in the service of three Greek Emperors. During that time he went to Sicily, to Asia Minor, Babylon, Palestine, and Egypt. In Apulia and Sicily he came across the same French Northmen (Normans) who, a few years before, had begun to visit South Italy, as they also began, in 1016, to go on pilgrimage to Jerusalem. This French-Norman and King Harald's Northmen's journey to Jerusalem took place more than two generations before the French began their first crusade. The fruitful results of those crusades is well known. All round the Mediterranean Harald Haardraade had seen with his own eyes the masterpieces of classic architecture. For instance, he was in Athens in 1040, after having conquered the Piræus. The Runic writing on the marble lion from that place, now in the Arsenal of Venice, is undoubtedly, in spite of Oscar Montalius, an inscription in memory of this event. Harald Haardraade was indisputably the most travelled monarch of his time, just as his compatriots were the first " tourists " then, as Englishmen are now.

Harald brought back with him a well-earned reward of Byzantine treasures, and he had Greek and Armenian ecclesiastics among his followers. Later, as King of Norway, from 1047–1066, he displayed a great building activity. Moreover, it must be considered significant of the relation of the Norwegian chiefs towards the mother country, when the Norman Duke *Roger* (*Rodgeir*) *Bursa of Apulia* considers the Norwegian King *Sigurd Jorsalafare* (" Jerusalem Traveller ") as his lord, from the fact that he is proclaimed King by Sigurd on January 1, 1110, on the way during his crusade.

As it is not without interest for our demonstration, we shall remind that Sigurd sailed from Bjorgvin to this crusade in 1107 with 60 ships and 10,000 men. He visited first the Norman King Henry I and his magnificent court in England, where he made rich gifts to the Church; on his further journey he beat the Moors near Lisbon, in Njorvasund (Straits of Gibraltar), and at the Baleares. Finally, after sailing for three years—the longest sea-journey since the days of *Pytheas*—he reached the Norman King *Baldwin* of Jerusalem, whom he helped to take Sidon. From there he sailed to Miklagard (Constantinople), where he left his ships and a good number of his men with Emperor *Alexios*, in order to travel home on horseback across the Balkans and through the valley of the Danube, through Hungary to Bavaria and Swabia, where he visited the Emperor *Henry V*; he continued north through Saxony, Sleswig, and Denmark, and finally reached Norway. He brought back, among many other things, a present from King Baldwin of Jerusalem to the church built on the tomb of King *Olaf* the Saint, a piece of the cross, and from Constantinople an altarpiece made of enamels, and a MS. of the Bible on purple parchment with gold lettering. Both these travelling kings, Harald and Sigurd, found their consorts at the Court of the Grand-Duke of the Varjags, in Novgorod and in Kiev.

But, apart from these remarkable royal travels, which are naturally recorded in the Sagas, we have evidence in our ancient law-books that these were not unusual for private Nor-

* In Gardarike the Swedes were more numerous.

wegians of that time. The older *Gulathingsret*,* which contains laws and regulations right from the tenth century, has, for instance, in chapter xlvii, a regulation of how a man's legal rights are to be looked after when he is in Greece. Furthermore, we have in chapter vi of *Farmannalogen*,† evidence of the general long-reaching travels of Norwegians in the statement of punishment concerning the desertion of sailors in English, Irish, and Greenland harbours, and also "austr i Gordum," that is, east in Russia. The Mediterranean was to them a well-known sea during the whole eleventh and twelfth centuries, just as the European coasts of the Atlantic, even from the ninth century, were like their second home.

Such writers as Ruprich Robert (*L'Architecture normande*, vol. i, p. 239), Viollet-le-Duc (*Dictionnaire*, vol. iv, p. 125), and Henri Martin (*Histoire de France*, vol. ii, p. 502) have already put stress on the aptitude of the Normans to assimilate the deep and lasting impressions which they had received in these continual and usual travels, into an individual and richly developed art. The last-mentioned writer says: "The Gospel conquered the Scandinavians, and hardly were they Christians than they rushed with all their energy at the head of Christendom, of young France, and of the civilisation born anew. They took the initiative everywhere; they gave up their language, ‡ as they gave up their gods, in order to take up the Romanesque tongue and make it the instrument of a new poetry. Art, literature, architecture, they had destroyed all, but they contributed powerfully to rebuild everything."

Without bringing with them certain qualifications it is difficult to imagine how these colonising "pirates" would have been able to play such a decisive part in bringing about the new development in the Empires of Charlemagne and of Alfred the Great. We grant that, when masters of Dublin and other Irish harbours for 300 years, Norwegians received lasting impulses from the people with whom they lived—from their art of illuminating, and of their stone and metal ornamentation, which was the pride of Ireland, and possibly they were also influenced by the highly developed art of the bards of the Gaelic *Filed* and *Allamh*. But the Irish had no monumental art. Their boats, made of skin stretched on wicker, could only raise pity in the Vikings, who came sailing on their long, sea-worthy ships. In 999 already, King *Olaf Trygvason* built the celebrated ship *Ormen lange*, for a crew of 600 men. Nor could the humble houses of worship of the Irish, built like bee-hives, of rough slabs, nor their low-roofed royal halls, with their walls made of wicker filled with clay, although of large proportions, have impressed the Norse warrior-kings, who at home possessed large, well-timbered halls decorated with carvings and towering gables.

A decisive proof of the technical perfection which Norwegians attained already in Christian times in wooden architecture is found in the two burial-mounds of Vestfold (near Oslofjord), from which have been lately excavated the hulls of large ships from the oldest Viking time, and also in the door-posts and frames from heathen temples incorporated into Christian churches. The origin of the ornamentation which predominates in the carvings of these pre-Christian antiquities agrees entirely with the common Germanic style of the iron age, which we find fully developed on metal ornaments, and on Runic stones from the Viking period; we mean the dragon pattern, which in Norwegian carving reached its highest and most original development, and which reminds us, with the exception of some particularities, of contemporary Celtic, Romanesque, Moorish, and Byzantine ornamentation.

This artistic originality, however, is only a testimony of secondary importance, if it is compared with the constructive excellence of the builders of the Viking ships who found its equal only in the Norse Vikings' unsurpassed science of navigation and in their love of bold

* *The Laws of Gulathing* (ancient collection of Norwegian laws).

† Farmannalogen: laws concerning trade.

‡ This remark of Henri Martin is not exact; it is proved that Norwegians kept their language for several generations, as the Dane, Worsaae, has already pointed out.

Fig. 192.—The ship of Oseberg.

adventures. An English expert on the subject has said of the *Gogstadship*, which, after a thousand years' rest in its mound, was brought to light in 1880: "It is quite evident, from her construction, that her builder possessed the greatest experience, and that her designer, whoever he may have been, thoroughly understood the art, which was subsequently lost, to be revived in modern times, of shaping the underwater portion of the hull so as to reduce the resistance of the passage of the vessel through the water." *

Another equally enthusiastic English expert exclaims in the following terms of admiration: "It is the opinion of experts in naval architecture that, for model and workmanship, this vessel is a masterpiece, nor for beauty of lines and symmetrical proportions could she be surpassed to-day by any man connected with the art of designing and building ships." †

In order to prove conclusively their constructive superiority and the well-known artistic originality of the carvings, we mention another Viking ship (fig. 192) excavated in 1903 in Oseberg (also in the neighbourhood of Oslofjord), in size and construction a sister ship of the previous one, but much richer in carvings and actually loaded with carriages and sledges having a corresponding princely outfit, with fantastic, beautiful ornaments.

A work of art never appears spontaneously. It is always the product of a long previous development. The sumptuous treasure of art represented by the ship of Oseberg is the result of a strong old national art, conscious and restrained in its perfect technique, fertile to the uttermost in its bold fancy; in its massive, free rhythm it surpasses completely the mild and schematic, symmetrical, and trifling productions of the Celts.

A remarkable written evidence of how the Norwegians of the Middle Ages considered their independent position in the history of architecture during the "Norman" period is preserved in a small Saga, "Hákonartháttr Hárekssonar," valuable also for the study of legends (published in the series of *Fornmannasógur*, vol. xi, p. 422, etc. (Copenhagen, 1823.) Having been looked upon until now as a legend, this account contains, when compared with reliable dates of the time it deals with, information having a completely reliable historical stamp. We are told here of the son of a wealthy Norwegian Hákon Háreksson, who, having squandered his fortune, is received by the Danish King Svein Ulfsson (Estridsson) and apprenticed to a smith. His ability gets him promoted as silversmith, goldsmith, and "enamelsmith"; finally he is educated as "grjötsmeld"—"stonesmith"—that is to say, architect; in each of these trades he is declared superior to the Danish masters because of his artistic originality. King Svein advises him now to go to England, to try his fortune at the Court of Edward the Confessor, and to build him a new hall, saying that "they still build halls over there in the old-fashioned manner." ‡ The Norwegian architect followed the advice, and was ordered by the English King to pull down the old hall and build a new one in the new style. As a test of the truth of the story, it must be remembered that King Svein was Edward the Confessor's cousin on the mother's side; her second husband was Robert le Diable. Svein was educated at the English Court, and in consequence he knew well the conditions in England. It must be also remembered that Edward the Confessor built Westminster Hall in the new Norman style. It is well known, besides, that King Svein had several notable Norwegians at his Court, some of whom remained probably from the time of his predecessor, the Norwegian King of Denmark, *Magnus the Good*, son of St. Olaf.

Among these Norwegians it might be of interest to remember Svein Norbagge, who, after extensive studies in foreign seats of learning, became Bishop of Roskilde. He showed great activity in his bishopric. Besides the cathedral of Roskilde, he built also the church of *Our*

* Sir George C. V. Holmes, "Wooden Sailing Ships," p. 60 (*Ancient and Modern Ships*, Part I, London, 1908).
† E. Keble Chasterton, *Sailing-ships and their Story*, p. 118 (London, 1909).
‡ *Dvi at Hallir i Englandi eru á fyrnsku Hátt*, p. 430.

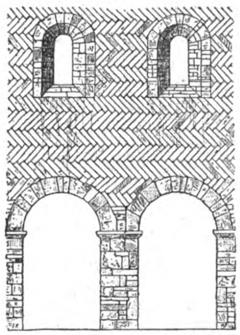


Fig. 193.—Church of Our Lady in Roskilde. Part of interior wall.

Lady. Some of its Norman parts are still left; it was built, as shown in fig. 193, with the ancient herring-bone wall. Bishop Svein died in 1088, that is, at the time when King *Olaf Kyrre* was building the cathedral of Nidaros.

The first church of St. Olaf, in Nidaros, was a generation older than the Danish church just mentioned, built by a Norwegian; it was founded by Magnus the Good, son of St. Olaf, who died in 1047; it was completed before 1050 by his successor, the above-named brother of St. Olaf, King Harald Haardraade. Of this first church the plinth only remains, and is consequently the portion of a building dating previous to 1047. As shown in fig. 194, there has been grafted on the old national dragon pattern a Christian ornament, the Romanesque vine motive. The excellently executed engraved star ornament, as well as the profile of the plinth of the nave in imitation of the antique, shows the same kind of grafting.

We have given earlier, with the ship of Oseberg, an example of Norwegian ornamental sculpture of the eighth century, and we shall supplement it further from the objects found in the ship; we give in fig. 195*a* the shaft of one of the sumptuously ornate double harness sledges. Besides the sure and beautiful ornamentation, we must point out that the shape of the shaft, its curve, the placing and the development of the ornamentation, express obviously and underline the mechanical function of the shaft, viz. pulling and directing; a shape which could be obtained only by a workman having complete mastery over the material, and under-

Fig. 194.—Church of St. Olaf in Nidaros. Part of plinth.

standing the artistic requirements of its functions. Fig. 195*a* is a perfectly accurate copy of the whole shaft, while 195*b* gives only a piece of the original. It is decorated with silver nails and bands, underlining the decorations, which, no doubt, were beautifully painted. In fig. 196 we give the decorative extremity of one of the four corner posts of the sledge; this is also an accurate copy of the original. The objects shown here are only a choice made haphazard among the numerous and various contents of the ship of Oseberg. Each object, in its unparalleled beauty, its firm lines, its shape, its delicate execution, is an evidence of a highly developed and refined art, such as an old civilisation alone can produce it. We do not find here a barbaric love of ornamentation, or an ungoverned fertility without meaning, but a critical grouping; and in the distribution of the principal and secondary parts of the decoration there is the same fastidiousness, the same critical demand of balance and harmony, therefore the same emotional power and solemn greatness which characterise the older Norwegian poetical work, the *Eddakvadenes* (the songs of the Eddas), and the rhythmic form of the legendary and heroic poems. The ornamentation of the objects repro-

195a.—The ship of Oseberg. The pole of a sleigh.

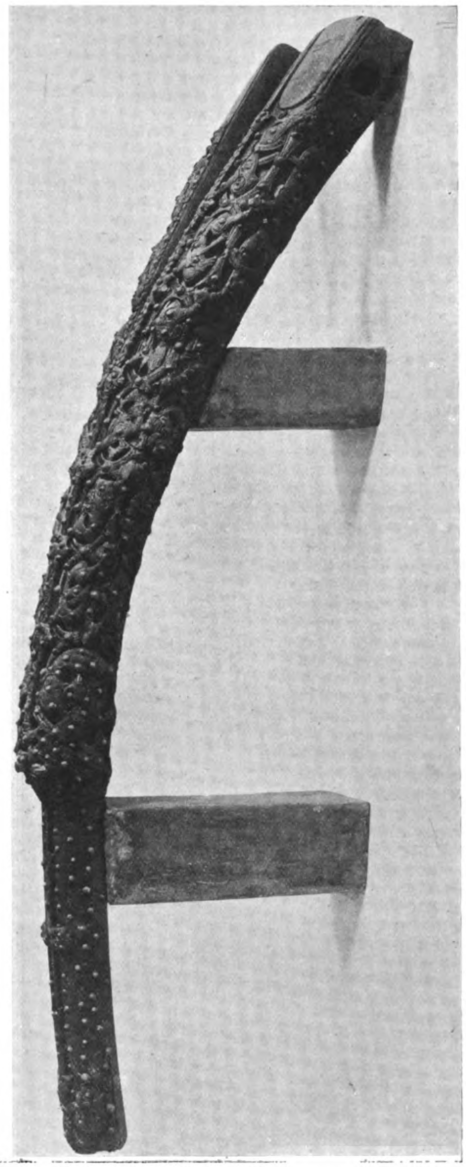


Fig. 195b.—The ship of Oseberg. Part of the pole.

Fig. 196.—The ship of Oseberg.—Ornamented corner post of the sleigh.

duced here, as of most of those in the ship of Oseberg, confirms the statement of Viollet-le-Duc, when, referring to the design on the door-post of the monastic church of Souillac in the south-west of France, he finds there an influence of "monuments scandinaves, nord-européens, islandais, ou à ces manuscrits dits saxons" (*Dictionnaire*, art. Sculpture, vol. viii, p. 198).

We have also an admirable example of sacred art in Norway from heathen times in the remains of ornaments from a heathen temple incorporated in the mentioned "stav" church of Ornæs (fig. 197*a*, 197*b*, and 197*c*). We must equally notice the porch of the later

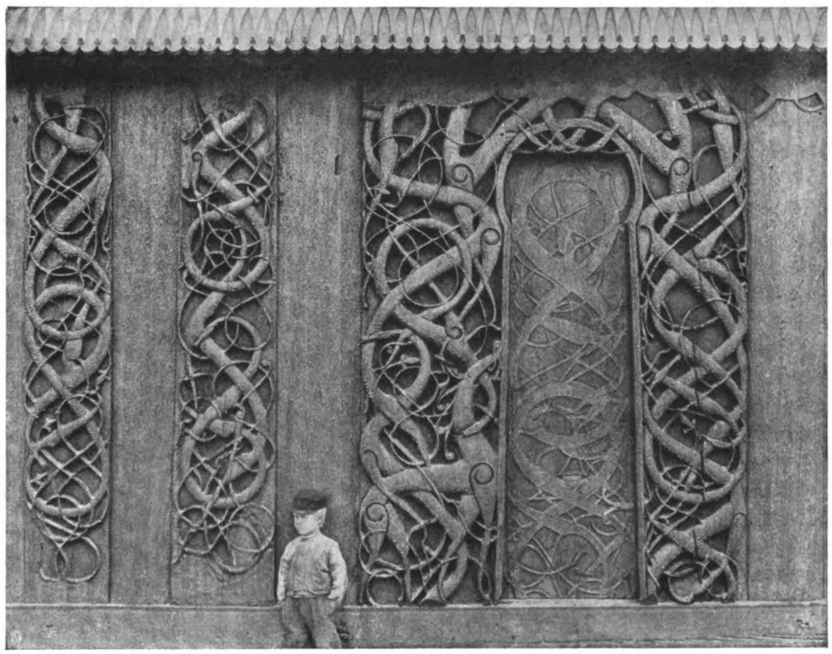


Fig. 197*a*.—Church of Ornæs. Part of north wall.

church of Borgund in Lærdal from about 1100, where the heathen and Christian national and foreign motives are worked out together in the most dexterous manner in an admirable carving (fig. 198).

We have from Denmark an example of wood carving, related to the ornament design of Ornæs—on a piece of old board—taken from a church (fig. 199). We see here how the pagan ornament design is carved on one side of the board, while the Christian (of the eleventh century) is painted on the other.

It is understandable that when stone architecture was introduced in Norway with Christianity, the high artistic ability trained already through many centuries should find a new field of development. Such a peculiarly suitable material as the soap-stone (steatite), found on the spot, must have prompted still more to it. With its plasticity, its toughness, and its

Fig. 197*b*.—Church of Ornæs. Corner post of the north wall.

Fig. 197*c*.—Church of Ornæs. Lower part of a door-frame, north wall.

resistance to weather, this stone must have tempted the old inherited talent to transfer itself from heathen times art and splendour into new development. For this reason we find at an early date in Norwegian Christian ornamentation a rich, firm and living design, which can be compared with later productions of the same kind in Europe, where this plastic stone was not found.

Figs. 200, 201, and 202 give illustrations of fragments from destroyed churches in Nidaros dating from the first part of the eleventh century. These fragments have been used as material for post Reformation repairs of the cathedral.

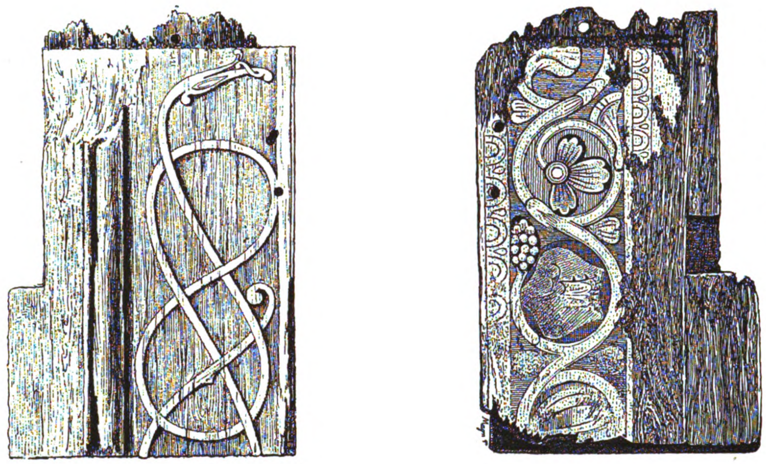


Fig. 199.—Church of Horning, near Randers. Pieces of ornamented board.

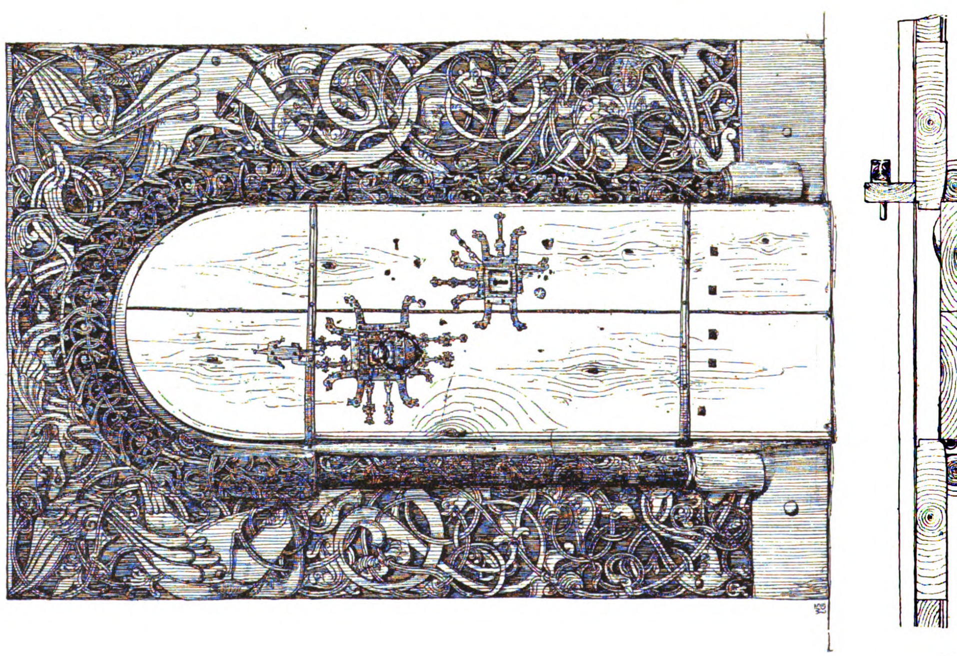


Fig. 198.—Church of Borgund, the porch.

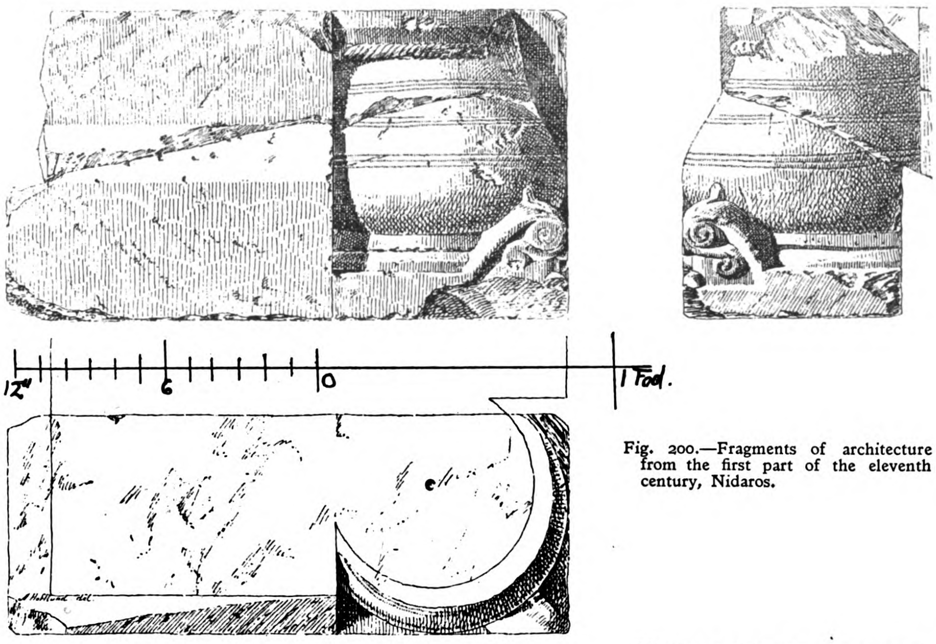


Fig. 200.—Fragments of architecture from the first part of the eleventh century, Nidaros.

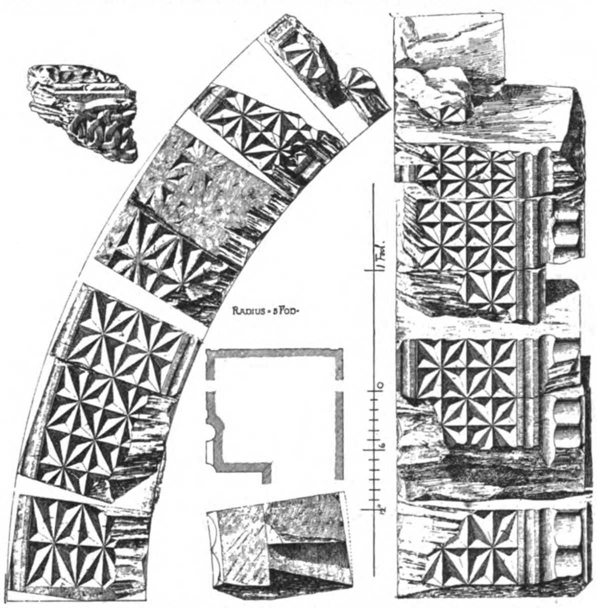


Fig. 201.—Part of ornamented archivault from the first part of the eleventh century, Nidaros.

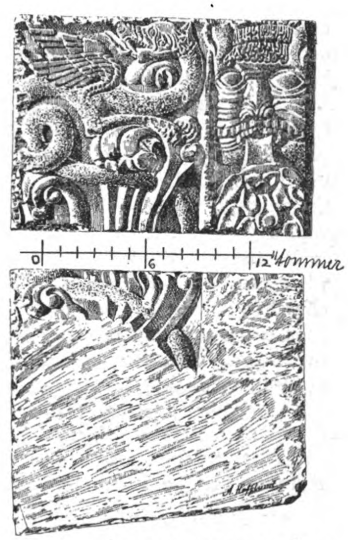


Fig. 202.—Ornamented corner cap, from the first part of the eleventh century, Nidaros.

Fig. 200 shows in the form of the plinth a clear transition from wood to stone architecture. It is also particularly remarkable that the transition between the plinth and the base of the column is occupied by the corner ornament, which had been generally considered as a mark of the twelfth-century style, except by Viollet-le-Duc (*Dictionnaire*, under word "Griffe," vol. vi, p. 48).

We see in fig. 201 the richly developed star ornament, and finally, in fig. 202, a corner capital of a developed Romanesque form, of excellent execution. The examples given here are taken from the rich collection of fragments from the oldest church of Nidaros, and now preserved in the well-stocked museum of the stones of the cathedral. They all show a completely developed technique of ornamental design and a perfect mastery of the new material.

We find just the same remarks made as to Norman sculpture by the French savant, *André Michel*, who was well acquainted with Norwegian medieval art (*Histoire de l'art*, Paris 1905, Part 2., vol. i, p. 657). "Scandinavian influences upon sculpture are more precise; here we find again in Norman sculpture the process of wood technique of Scandinavian importation; the result can be seen in some rather coarse wooden capitals in our abbeys, in some types of corner capitals in Bayeux, on the flat-headed corbel stones which decorate all the Romanesque churches, perhaps also in some bizarre subjects, some strange figures which might possibly be remembrances of Scandinavian legends, as also in the interlaced decoration." *

"One has tried to recognise recently on the capitals of our castles, and of our churches, in Falaise and elsewhere, the representation of some myths of Doric Greece, the goddess with the lions, who is supposed to have come to us from Greece through Scandinavian paganism or Odinism, where it is to be found again on pieces of furniture. The dragons of the corner pilasters of Bayeux could just as well be of Scandinavian as of oriental origin." †

What has been said here concisely is enough to prove that, at the time when the first bishoprics were created, Norwegians had set themselves the task of building a monumental cathedral on the tomb of their national saint, and in his honour they prepared to raise an edifice which could compare in holiness with those they had admired in their relations' new home in the north of France.

Stone churches had been erected in Norway already in the time of Olaf Trygvason (997–1000). One or two have been preserved entirely, and a few are in ruins (fig. 130 and possibly fig. 133). In the town of Nidaros, where the national saint was buried, and where there existed after his death, in 1030, besides the above-mentioned church dedicated to him, the plinth of which we saw in the illustration (fig. 194), there were already before the cathedral was begun, at least five churches, of which two owed their existence to the brother of the saint, the already mentioned traveller to Constantinople, Harald Haardraade, whose poems, which are preserved, give sufficient evidence of artistic talent. It is obvious that all the churches in Nidaros which were raised by this highly talented and powerful monarch have been excellent constructions. The learned *Adam of Bremen* mentions Nidaros in about 1070 as the capital of Norway, possessing several churches.‡ There were also several royal residences built in stone.

* * *

After having demonstrated the *artistic* antecedents which Norwegians possessed, enabling them to build a large and monumental temple, we shall briefly indicate the *social and inner*

* Henri Prentout, *Essai sur les Origines et la Fondation du Duché de Normandie*, p. 269 (Paris, 1911).

† Lecture given at Caen, by M. Lefèvre-Pontalis on "Henri Prentout, *Essai*," etc.

‡ "Metropolis civitas Nordmannorum est Trondemnis, quæ nunc decorata Ecclesiis, magna populorum frequentatione. In qua jacet corpus beatissimi Olaphi Regis et Martyris. Ad cujus tumbam usque in hodiernum diem Dominus operatur sanitatum miracula, ita ut a longinquis illuc confluant regionibus, qui se sancti Martyris meritis sperant posse juvari."

historical preliminaries leading to the martyrdom and canonisation of Olaf Haraldsson, as well as the reason which made him the acknowledged symbol of the country's freedom and independence, which symbol had necessarily to find its material expression in a cathedral, representing the people's feeling towards their saint.

We must bring back to mind the fact that, in the twelfth century, the French Kings were the first to build large cathedrals, because they saw their power threatened by powerful fendatories and no less powerful abbots who also possessed fiefs. To counter-balance them, the Kings tried to tie the Bishops and the towns to the Crown, principally through raising great cathedrals. The same dynastic object, which made itself felt to the French Kings, had also been determining in Norway for Olaf Kyrre (1066–1093). Towards 1070 he created bishoprics and laid the foundations of the cathedral of Nidaros. Like his contemporary, *William the Conqueror*, Olaf Kyrre invited the Bishops to reside in the new towns, whom he took under his protection for political reasons. We find him exchanging friendly letters with the great reformer in the papal chair, *Gregory VII*, with whom he discusses sending young Norwegian noblemen to Rome to be taught the new rules of the Church.

We find an authentic evidence of the political importance of raising a cathedral in honour of the holy member of the dynasty, in an account of the *Orkneyingasaga*, where precisely the same inner political occurrences had been the cause of building large cathedrals as political means of power.

After Jarl Paal (Earl Paul) of the Orkneys had killed Jarl Magnus, he kept from the young Jarl Ragnvald, his nephew, his rightful part of inheritance to the earldom in the Islands. Jarl Magnus was recognised as saint and entombed on December 13, 1135. In 1136 Ragnvald went to his father, the powerful and astute "Lendermand" (baron, vassal of the king) Koll Kalason of Agder, in the south of Norway. After having his rights to the earldom recognised by the Norwegian King and the lord of the Orkneys, Harald Gille, Ragnvald held a council with his men, as he was lying at Herö on the Norwegian coast with his small fighting fleet on the way home. There he explained how badly Jarl Paal had treated him in keeping his inheritance, which the King of Norway had recognised legally and he declared that he would sail back and win his rights or die. His men thought what he said was right, and promised their faithful help. Then his father stood up and spoke: "We have heard from the Orkneys that every man will rise against thee, and with Jarl Paal they will try to prevent thee getting thy possessions. It will be a long enmity, my son; but now this is my advice: to seek help where it is found in plenty, and let him help thee to the kingdom who possesses it justly. I mean *Jarl Magnus the Saint*, thy mother's brother. My wish is that thou shouldst turn thy prayers to him, that he should grant thee the inheritance of thy ancestors, the same that he left; but thou must promise that when thou hast won the land, thou wilt erect a stone minster in Kirkejuvaag (Kirkwall) in the Orkney Islands, more splendid than any church in that country, and dedicate it to thy kinsman, Jarl Magnus the Saint, at the same time giving so much land to it, that the Bishop's see gains in importance when it is removed thither with the relics of the saint." This was considered a good advice, and they all agreed to it.*

The saga relates further how the advice was followed, as we shall see later.

This wise counsel of chief Koll of Agder, who was a frequent and honoured guest of the Norwegian King, had been borrowed, so to speak, from the politics of the Kings of the dynasty ever since the death of the saint at Stiklestad.

In order to understand better the reasons which led to the raising of a cathedral such as

* See *Orkneyingasaga* ed. Gudbrand Vigfusson, p. 119 (London, 1887); and *Flateyjarbók*, vol. ii, p. 455 (Christiania, 1862).

that of Olaf Kyrre in Nidaros, it is necessary to give a brief reminder of the early constitutional state of Norway.

Before the introduction of Christianity, Norway was a kingdom, it is true, but it was not a society. The old Norwegian paganism had not, like in Greece and in Rome, raised itself into the society's religion round a common religious idea, a holy fatherland. It consisted of a social agglomeration of families, or rather of clans, the extent of whose possessions was determined by geographical conditions and which alone the thought of defence assembled in the fylke (province). One independent clan stood against another independent clan, each for its own rights. The individual was not valued according to his worth, but according to birth, wealth, and his achievements as warrior. An offence against one member of the clan was not only an offence against a person, but against the whole clan. Therefore it was a society steeped in family revenge, in vendetta and continual strife, asking life for life, father for father, brother for brother, cousin for cousin, until the seventh generation.

The chief foundation of this independence was unalienable possession, that is to say, the family's indisputable claim to its members and its property. Family without property and property without family was unthinkable. The founder of the dynasty, Harald Haarfagre, had already attacked this independence when he opposed the clans and exacted, beside taxes, Thegngilde also—that is to say, compensation to the Crown for the loss of a subject, or, more exactly, he demanded for the Crown a part of the compensation which before was the sole right of the clan. This King's encroachments and infringement of the privileges of the clans caused the emigration of chiefs from Norway, and this was the reason that Iceland, Faroe Islands, Shetland, the Orkneys, the north of Scotland, the Hebrides, the Isle of Man, Northumberland, Cumberland, Ireland, and Normandy were colonised by the best Norwegian families and their Varnadarmenn, or subjects. It has been counted that 40,000 people emigrated to Iceland alone. Religion in that society was also a religion of clan. There was no heaven, but a Valhalla—that is, a heaven for the strong, the brave on the field of battle; there was nothing for the weak, and absolutely nothing for the thrall. Thus it was a heaven not according to a man's worth, but according to his value in society.

St. Olaf came with a heaven for all, a heavenly Father for all, for the small as for the great, for the poor as for the rich, for the bondman without lineage as for the noble and free warrior, *patrem omnipotentem, creotorem coeli et terræ*. His introduction of Christianity was necessarily to be regarded as a more revolutionary and disastrous encroachment on the power of the clans than the infringement of their privileges by Harald Haarfagre. The teaching of Christianity was like a wedge driven into the heart of the old society.

It has been said, quite rightly, that Norway, more than any other kingdom, owes its political existence to Christianity.

The work of bringing about national unity and independence for the Norwegian people had been, until then, a question of external politics only. Right from the time of Harald Haarfagre until Magnus the son of St. Olaf, there had been fighting for supremacy in northern countries. The Kings of the Lodbrok clans in Sweden and in Denmark did not recognise Norway as a kingdom, but as a gathering of small kingdoms under the rule of the sons of the Lodbrok clans. The descendants of Harald Haarfagre were looked upon as rebels, and it was easy for the King of the Lodbrok to make an open or secret alliance with the dissatisfied Norwegian chiefs. It is characteristic of the position of these clans towards the power of the Crown, that, when the small King of Oplandene, Rörek, answered St. Olaf's appeal for help through the small King Sigurd Syr, he said that they would prefer foreign Kings, who would be satisfied with a tribute, but who, after all, lived so far away that they left people alone.

The independence they had enjoyed as regards foreign interference, and which was the

work of Harald Haarfagre, who united the different parts of the kingdom, was a moral and practical good, the value of which they were unable to understand. But when the clans who fought for their independence, which was then identical with their religion, slew the Christian King Olaf Haraldsson at the battle of Stiklestad, in 1030, and when the victorious nobles had surrendered to the foreign domination of the Dano-English King Canute, with whom they had secretly made an alliance, the foreign rule brought a number of laws of conquest and a great increase of tribute, which was a much greater encroachment on their independence than the Thegngilde of Harald Haarfagre had ever been.

Dissatisfaction grew with a bad conscience, legends formed, and they gave birth to the picture of a king wrongly slain, whom the people, in its short-sightedness, had deceived and betrayed. The defeat of Olaf "Digre"* was the victory of King Olaf—wherefrom St. Olaf rose.

The power of the clans was not destroyed on that account. This was the opportunity for the son of the king, Magnus, to do as the pretender of the Orkneys did later, to found his dynasty and sit it securely on the throne by binding the Church to the Crown and turning the people's repentance into attachment to its founder. We see, therefore, that one of the first things Magnus did when he came to the throne was to build a dignified church on the tomb of his father, the Church of St. Olaf, the beautiful plinth of which was shown in a previous illustration (fig. 194).

It is clear that this had a great importance for the dynasty, when we see that each successive King who fought against the clans, whose politics obviously did not alter with the changes on the throne, always found the previous church on the grave of the Martyr-King too small. The successor of Magnus Olafsson, Harald Haardraade (1047–1066), St. Olaf's step-brother, who had himself been in the battle of Stiklestad, raised his Church of St. Mary close to the royal palace and removed the relics to its altar, at the foot of which he was himself to be buried; he also built a church, consecrated to Mary, in the town of Oslo founded by himself; this was done from national motives against the intrigues of Sweden and Denmark. Here, for dynastic reasons also, he allowed his nephew, Halvard, to be chosen as local saint for Viken—the south-east part of the country.

After the death of Harald Haardraade, however, new and very serious difficulties cropped up again for the dynasty.

The great plan of the conquest of England had to be given up after the fall of that King at Stamford Bridge in 1066, where at least half his large army, consisting of the flower of the Norwegian youth, was cut down, owing to the daring of the King, used to be always victorious, and unused to the stratagems of the English. The army alone, when it started from Norway, consisted of 240 ships, besides transport and small vessels, with about 25,000 picked men, not counting the troops he got from the Orkneys and the Norwegian possessions on the west coasts of Scotland and England. The saga tells that a better army never left Norway. At least half of it perished with the King. This loss was so great that, in proportion to the present population of the country, it corresponds to 50,000 men. The memory of Norwegian bravery in that battle, as it was remembered in the sagas, relating Eystein Orres and his followers' heroic fight at Stamford Bridge after the King's fall, could hardly cover the disappointment, the bitterness, and mourning at the terrible loss which the King's too great courage had caused the country. Under these circumstances the accession of his son, Olaf Kyrre, to the throne was far from easy. The conditions were all the more difficult for him that he had to share the crown with his step-brother, Magnus. P. A. Munch sees rightly in this a step back in the principle of monarchic unity, and he thinks it

* "Digre," the thick, was the epithet used with insulting meaning, by the adversaries, for King Olaf.

likely that the clans had been for something in this arrangement.* Moreover his father, Harald, had not been popular previously, mostly because of his severity, as shown by his name,† and we have a glimpse of the depressed feeling which the expedition to England had created in the answer which King Olaf gave to King Canute Sveinsson of Denmark, accompanying his refusal to help him in the English war against William the Conqueror; he gives as reason that the country was still suffering (1084) under the loss of its best youth fallen in the battle of Stamford Bridge.

These circumstances explain why it must have been of great importance for Olaf Kyrre to ameliorate the threatened position of the dynasty by strengthening still more the power of the Church, which he did towards 1070 by creating permanent bishoprics. In this he had an example near by, in Denmark, where the King, Svein Ulfsson, had done the same under similar circumstances. Later, in 1075, William the Conqueror used the same means in England under like circumstances. Olaf Kyrre built in Oslo a cathedral (fig. 140) consecrated to the local saint, St. Halvard. Furthermore, he laid out the foundation of a large cathedral in Bjorgvin, to which he had himself given the right of township after having started a magnificent cathedral in Nidaros, which was to be a worthy resting-place for the Martyr-King. He had all the more reasons for founding a new cathedral, that both the Church of St. Olaf, of Magnus Olafsson, and the Church of St. Mary, of Harald Haardraade—which we shall mention later—had become too small for the ever-increasing stream of pilgrims.

Fig. 203.—Cathedral of Nidaros. Statue of St. John from the west front.

The resting-place of the Martyr-King had become gradually a miracle-working place of European fame, second only to the Church of St. Peter in Rome, to St. James of Compostella in the north of Spain, to the cathedral of Chartres, or to St. Denis, near Paris; Canterbury had not yet its Thomas à Becket. The fame of St. Olaf had spread in the Christian world far outside the wide limits in which Norse language was spoken — Norway, Iceland, Greenland, Faroe, Shetland, Orkneys, the Hebrides, the Isle of Man, and Ireland—farther on to Scotland, England and Normandy, to Denmark and Sweden with Finland, to the north of Germany, to Hungary, to Russia, and even to Constantinople; everywhere in those countries new churches were erected and consecrated to St. Olaf.

Already a few years after his death in Stiklestad, 1030, *Sigurd Jarl of Northumberland* (died 1055), built to the memory of his deceased friend a church near York, later "St. Mary Abbey."

* P. A. Munch: *Det norske Folks Historie*, vol. ii, p. 377.

† "Haardraade" means "hard judgement."

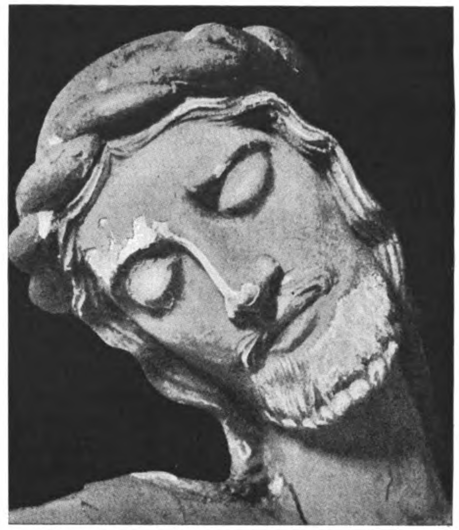


Fig. 204.—Church of Mosvik. Head of a crucifix.

Hardly twenty years after the death of St. Olaf, the contemporary Bishop of Exeter, *Leofric*, entered in his missal, now in the British Museum, the Office of St. Olaf, which is found in Scandinavian missals; and a shorter Mass for the Day of St. Olaf (July 29) is found to have been introduced in what is called the "red book of Derby." This Mass was founded in Winchester in 1061. Pieces of the cloak of St. Olaf were among the gifts which Bishop *Duncan* gave to the cathedral of Dublin, then recently founded by "Austmændene" (Norwegians in Ireland) already in 1038. Besides York and Exeter, churches of St. Olaf are found in Chichester, in Chester, and in London; this town has no less than five. From the mud of the Thames, a small hatchet of lead was brought up, and is now preserved in the British Museum; it is the usual memento of pilgrimage which people brought home from a pilgrimage to Nidaros. Adam of Bremen relates towards 1070 "that the Lord works in these days wonderful cures at the grave of St. Olaf wherefore pilgrims stream from far-off lands in the hope of help for the sake of his services." *

The five previous churches of Nidaros were comparatively small, except the church of St. Mary of Harald Haardraade, and were unsuited to receive the constantly increasing crowd of pilgrims who came to worship before the shrine of St. Olaf. This reason only, apart from the dynastic one, was enough to render necessary the erection of a large and beautiful church of a shape suitable to the actual requirements; this was found in our previously demonstrated cruciform cathedral. This form of church, with its procession-path, was suitable to the service, and to the great ceremonies inseparable from a pilgrim church of this rank, the biggest in North Europe. It is evident, then, that it had to be bigger and more stately than the cathedral which Olaf Kyrre was building at the same time in Oslo for a merely local and even princely saint, who did not attract pilgrims in such crowds, not even from the diocese of his own country, and who, although he was of some importance for the dynasty, was not nearly of such tremendous value as the Martyr-King, who after death was victorious over his opponents at home, and drove the foreign usurper from the kingdom.

The ideal purpose of the new edifice demanded a building of the highest order inspired by the new spirit of the times, which distinguished men from Norway, Iceland, and Orkney Islands had learned to know during their frequent journeys to Rome, receiving at the same time impressions from the great architecture which was a development of this spirit. We have already mentioned the relations existing between Olaf Kyrre and Pope Gregory VII.

The cathedral which was erected in honour of St. Olaf became a symbol of the final inclusion of Norway into the common civilisation of the time, and a uniting centre for the future ideal and moral development of the nation: away from clans towards national consciousness, from the restlessness of the Viking times and the ancient cult of the "blot,"

* Quoted in the note ‡, page 212.

upwards towards clear thinking, expressed in a later generation, witness of the crowning of the work, in the *Song of Lilja,* the profound wisdom of *Kongespeilet,* in the emotion of *Draumkvædet,* in the gracefulness of the statue of St. John (fig. 203), and in the deep and intense feeling of the crucifix of Mosvik (fig. 204).

* * *

This cursory examination of the circumstances which led to the foundation of the cathedral of Nidaros must be sufficient to create a perfect understanding of the reason why, just there in Nidaros rather than in another place in northern countries, and at an earlier date, it was necessary, out of political and religious reasons, to raise a majestic temple, larger and more splendid than any other church in those parts, and on a level quite as high as the best contemporary churches in Europe.

Such a church alone could give the dynasty the necessary united power and—or rather if possible—surpass the expectations with which the crowd of pilgrims from distant lands looked forward to, and longed for, the holy place.

But, for the Norse people, the cathedral was to be the keystone of the nation's union, for the love of the great martyr's memory: he who had destroyed paganism and introduced the principles of human right of the new age, and who was, through his church, the uniting symbol of the kingdom's freedom and independence, its law and right, so long as Norsemen lived under the rule of Norway from the frontier of Sweden in the east, to Iceland and Greenland in the west.

CHAPTER XV

THE RITUALISTIC REQUIREMENTS OF A CATHEDRAL AND OF A CHURCH OF PILGRIMAGE

AFTER having given an account of the general historical and social conditions, as well as the artistic development which led to the foundation of the cathedral of Olaf Kyrre, we shall now indicate the particular requirements which were part of a cathedral and a church of pilgrimage in the eleventh century, and which determined the dimension and character of the plan.

In a previous account of what we called "fylke" churches we have already brought to notice that, before the permanent establishment of bishoprics, they represented an intermediate stage between a cathedral and an ordinary parish church, and that, consequently, they were designed for an important ritual and to accommodate a large ecclesiastical assembly. This assembly did not consist only of the Bishop and his suite, but of priests from all the parish churches of the province, who came with the communicants. The old Norwegian law does not mention how large this suite had to be; but it could not have been small, judging from the regulations for the suite of the Bishop when he consecrated a fylke church. In summer it had to consist of thirty men and thirty horses; in winter, of fifteen men and fifteen horses. From the literal interpretation of this rule, found in the *Saga of Sverre*, chap. cix,* this was intended for all the official travels of the Bishop.

The churches of Akr, near Oslo, and of Gran are both of the eleventh century, the first is mentioned in the church regulations of Olaf Kyrre, dating from 1070–1080. Both these churches (figs. 133 and 134) are quite good examples of the accommodation of a fylke church in the south-east of Norway. They were large enough for the ordinary divine service, and were probably used as a meeting-place for priests and for ecclesiastical gatherings, and perhaps also as hall for the fylke council assembly; their central position making them useful for that purpose. The fylke churches were raised on the places where the old fylke councils were held and where the old pagan fylke temples stood.

It is evident that, for a cathedral, many more considerations must have entered into the choice of its size and character, than for a fylke church. The Bishop was the minister of his own spiritual domain; his church was its centre and came first. This is why cathedrals were often called head churches—just as the King, in his temporal kingdom, had his Court and his vassals, serving in turn, so the Bishop had his Court consisting of his dignitaries and

* Ed. C. R. Ungers, p. 121 (Christiania, 1873).

of his canons; the priests of the parish church were his vassals, who had to come and serve their lord in turn.

In the discussions concerning the first cathedral of Nidaros, our archæologists, with a few exceptions, have taken an absolutely Lutheran point of view, as if discussing a meeting-hall or a Protestant assembly church. They forgot that a Catholic church is, above everything, a hall for permanent celebrations, the house of God, built in His honour, and in festive memory of the holy men of the Church.

It was not only the regular Drottinsdage, Day of the Lord (= *dimanche* = *dies dominicus*), as the heathen Sunday was named by the Church, which was in festive commemoration of the Saviour's life and sorrows on earth; but, so to speak, each day had a ceremony to commemorate one or another holy man who had contributed to spread God's Kingdom, or, in other words, who had contributed to Christian civilisation. Therefore, the Church was the Pantheon of civilisation, where all who had laboured and suffered for its sake had their commemorative place. Instead of building a theatre to Shakespeare, as in our days, or a memorial to Goethe or Ibsen, in the Middle Ages they erected a church or an altar in memory of superior individuals, an Augustine, a Gregory, a Benedict. These altars of the saints, with their caskets and reliquaries, stood round the church and its chapels, as holy evidences that men possessed an unshakable conviction, a belief in life; the indifferent "liberal" talk of our day on the freedom of faith was unknown. Truth was only to be found on one side. People could not be duped by clever paradoxes, like the one which they swallowed from George Brandes, in the eighties, representing as the foundation of a free, untrammelled thought the idea that, for example, two and two could very well make five in another solar system! In the Middle Ages there was but one truth and one holy religion, and its disseminators were God's holy men—but they were not gods, worshipped on altars raised in their honour. On these altars stood the Host; it was only on special days that processions went to the saint's altar, such as on the anniversary of his death or of his martyrdom, if he were a martyr who had sacrificed his life for his faith or for the salvation of the faithful. The liturgy was great or small according to the saint's importance regarding that particular church.

Researches have shown that it is quite an exception when the traces of at least three altars are not found in a Norwegian parish church. These churches had at least three priests. The high altar, the altar of Christ, or of the Cross, as it was called, was the altar of the parish priest. He was the official of the church, and remunerated from its landed property and from the legal contributions of the people, while the others, such as the "Högendis" priests, in Norway, were remunerated from funds attached to altars which pious donators had endowed.

It is quite easy to understand that already, from the very beginning, a cathedral had many altars, towards the raising of which wealthy men of the diocese and ambitious well-to-do parents and relatives of novices had given the necessary means. Consequently, several priests were always attached to each cathedral, even when it was not a collegiate church, such as was the case with all the Norwegian cathedrals outside Norway proper, as on Iceland or on the Orkneys—that is to say, those cathedrals which were not authorised to choose their own bishops. The plan of the cathedral of Kirkjuvaag (fig. 147, p. 120), which, besides the Bishop, had only an Archdeacon, shows that nevertheless room had been required for a numerous ecclesiastical assembly. The plan of the cathedral of Oslo points to the same (fig. 140). The older part of this was founded from sixty to seventy years before the Norwegian cathedral had proper elective assemblies, if it had not already been built under Harald Haardraade; but of this we shall say more hereafter.

Councils not possessing the right of choosing their Bishops were attached to all the

cathedrals of Europe before the establishment of chapters. This council consisted of what was called "Elders"; they were leading priests of the diocese, that is to say, priests from the various parish churches who served in turn at the church. As regards Norway, our historical sources do not give any evidence of this. But from certain passages in the Icelandic sagas of Bishops, as well as from the plan of the cathedrals of Kirkjuvaag and of Oslo, and from the parish churches, it would appear that there had been a similar arrangement in Norway before the establishment of chapters.

The Bishop's Court, apart from the higher ecclesiastics, consisted of a whole group of officials of the most varied kind, of lower clergy, to which were added singers, choir-boys, and church beadles. There were also the *magistri lapidum* of the church—that is to say, the architect, with his masons, besides menials and others. All these had to have a place in the chancel according to their rank and position. There was the King and his numerous Court as well as visitors of high degree.

Moreover, this fact must be borne in mind, that the Catholic Church does not "hold" divine service, with a somnolent "sermon"; but it *celebrates* Mass in a solemn ceremonial, where each part is a symbol. Great and pompous processions accompanied this celebration, at the time of great church solemnities, especially during Easter week, and what was called "Gang-dager" (procession days), *dies rogationum*, and more important saints' days, which were held from "Nönstimen," (three o'clock p.m.) the day before. These processions were all of different character, and had a different road, according to the ceremony and its meaning. In the planning of the cathedral these had to be given convenient room.

We have already mentioned that the Danish King, Svein Ulfsson, who died in 1076, established permanent bishoprics in Denmark, as stated by Adam of Bremen. The cathedral of Lund is the oldest existing Danish cathedral, founded in the last decade of the eleventh century; but as early as 1085 a chapter was established. Fortunately there exist remains of an older cathedral previous to the time of the establishment of chapters in Denmark; it is the cathedral of Dalby, in Skaane. This was founded by Svein Ulfsson, and, after his death, the building was continued by his successor, King Harald Hein, who was buried in the chancel in 1080.

After an exhaustive study of the remains obtained from sure sources, the Swedish professor, C. G. Brunius, from Lund, has given an account of it.* Owing to this we have been able to give its plan in fig. 205. As shown by lines of the system drawn in, it has been a perfectly regular church according to its type. The hatchings showing the square in front of the west gable represent a crypt lying half above the ground, which seems to be the trace of a tower, no doubt added when the cathedral was superseded towards the end of the eleventh century already, and turned into a church for the monastery of Dalby.

In the plan of the latter we have the evidence that a northern cathedral, in the eleventh century already, was entirely planned according to the requirements of the ritual which the Catholic Church demands necessarily everywhere of an episcopal church.

The centre points of this ritual were the altar and the Bishop's throne. The chancel was designed and arranged according to this. Right up to the end of the twelfth century the high altar stood in the centre of the semicircle of the choir, and the high throne of the Bishop, to which led several steps, stood at the back of it on the longitudinal axis of the church (see in fig. 205 an arrow marked N B). Round the walls of the apse were seats for the older ecclesiastics and other priests summoned, as suggested in the plan. This place—"Sanctuarium," "Sancta-Sanctorum," "Retro-choir"– was only used by the higher clergy and, later, after the establishment of the chapter, by its members only; therefore it was also called presbytery. The

* *Skaanes Konsthistoria för Medeltiden*, p. 56, etc. (Lund, 1850).

choir was contiguous to it—bas-choeur, chorus psallentium, where the lower clergy with the singers, the psalmists, the readers, the taper-bearers, the choir-boys, etc., had their place, as well as the high laity and the various lay officials of the Bishop. In the choir, the King and his suite had the first and best places. This is shown by a crown on the drawing.

But as the number of altars, and consequently the number of priests, canons, brothers of the Cross (as the members of the chapter were called in Norway) increased, the place became too small; consequently, already at the end of the twelfth century, in Nidaros, and towards 1200 in France, the presbytery had to stretch lower in the choir; accordingly it was pushed towards west in the church. If the arm of the choir or of the chancel was not long enough the transept and one or two bays of the nave were used as choir.

Thus it came to consist of three parts: (1) the retro-choir, with the high altar and places for the high clergy; (2) the presbytery; (3) the choir. If the remains of a great saint were kept on the high altar, this became an independent part, and with it the retro-choir itself, so that the choir and its three parts were pushed still more towards west. The retro-choir was al-

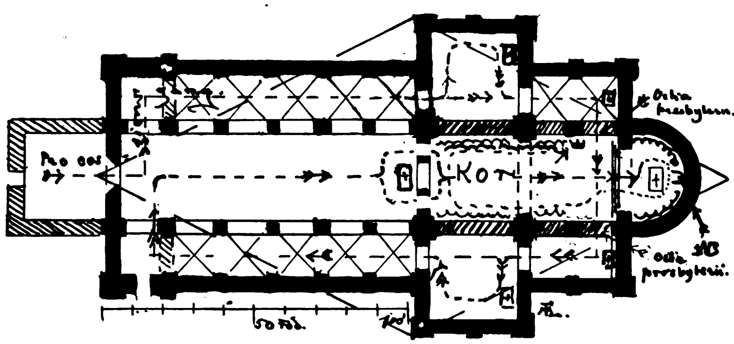


Fig. 205.—Cathedral of Dalby. Plan.

ways strictly divided from the choir by regularly three or five steps—Gradus Presbyterii. The steps were always odd in number, because, when going up, the priest had to step with his right foot on the first step and on the floor of the retro-choir.

The whole of the place allotted to the clergy, with its primarily two and then three parts, was first separated from the church by a low barrier. In the twelfth century it became a screen having a height of six to eight ells.*

But whether or not the presbytery coincided with the retro-choir, or, as was the case towards the end of the twelfth century, when it was put west of Gradus Presbyterii, account had to be taken that church processions were able to reach, or at any rate to pass in front of the high altar.

In fig. 205 we have explained the old manner of the eleventh century. The partition walls round the choir are in hatchings. The doors for the priests, on the south and north sides nearest the presbytery, are marked by an arrow and by their medieval name, Ostium

* All this divided place, with its parts, is always called in Norwegian by the popular name of "koret," in the existing old Norse MSS and documents. Sometimes the screen which limits the choir towards west, and on which finally the organ and the singers were placed, was named "koren." This has created the most ridiculous misunderstandings among Norwegian historians, when, in their Protestant ignorance of the Catholic Church, they have discussed the cathedral of Nidaros.

Presbyterii. Processions had to pass through these doors; these processions were, as mentioned already, various in character and solemnity, according to the occasion, as fixed by the rules. To demonstrate it, we shall give a description of the procession on the occasion of the reception of the King.

The Middle Ages looked upon everything in a religious and therefore poetical way. Just as the Bishop, by the grace of God, had responsibility through being His representative in the spiritual world, so was the King, by the same grace, given the responsibility of governing the temporal world. In looking thus upon the two dignitaries as vassals of God's grace, Christianity created a moral responsibility and a spiritual dependence of the Church—for the Bishop as for the King—to the great advantage of a humanising legislation.

A dependent condition was always present in the mind as a result of the religiousness of the time, even when the Kings, as in Norway and England in the eleventh and twelfth centuries, renounced, or at least tried to renounce, the spiritual hegemony of the Church, which at last happened unfortunately.

In the ceremonial of the Church, the King was always treated as representing the opposite power, but by the grace of God. His visit to the church had to be announced in advance, and he was received with strict ceremonial. As example, we shall recall a reception on an especially solemn occasion, when the King "bar Korunu," as it was expressed in the Middle Ages. The Bishop and his priestly court sat together in the choir, where they placed themselves two and two according to rank and went with the cross, standard-bearers and choir-boys at the head of the procession, out of Ostium Presbyterii in the choir's south screen and down the south aisle to the centre porch, which, as a rule, only opened for important processions and on great celebrations. After the Marshal had knocked on the door and the formalities were over, just as if he were entering a town, the door was opened to the King and his suite, they were received by the Bishop, who escorted him under a dais, and they formed the centre of the procession. The cross and standard-bearers led the way. They were followed by the singers, who sang a hymn suited to the occasion, and by the choir-boys, swinging the incense-burners. There were as many priests before as behind the dais; the highest dignitaries were nearest to it. Then came the Court, each man escorted by an ecclesiastic of correspondingly high standing. The way is shown in fig. 205 by a broken line. The procession went solemnly up the north aisle, bowing at the various altars, until it reached Ostium Presbyterii in the north screen of the choir. The procession then entered the choir, passed the high altar with a genuflexion, and went out again through the priests' door in the south screen. It came down the south aisle, turned in the nave, and walked through the respectfully retreating crowd, up towards the altar of the cross, where the procession divided and met again in the choir so as to presently reach the presbytery, where it went, after a genuflexion, round the high altar; from here again down the choir, where the King was escorted by the Bishop to his seat, standing highest on the northern side (shown on the drawing by a crown) * while the courtiers were shown to their places by the Bishop's courtiers, as the guests of God. Then the Bishop, with his higher ecclesiastics, walked up the presbytery.

The whole took place while hymns were sung and incense-burners swung and smoked, and Mass began according to the great ritual. The psychological power of this thrilling, solemn ceremony was enough to restrain a restless and fighting monarch. The Norwegian Hirdskraa, or ancient regulations of the Norwegian Kings on Court and ranks, describes similar pompous ceremonies which show a distinguished and poetical mind.

* When the presbytery was moved lower down in the chancel, this became the Bishop's place, because this side is the Gospel side, or the right side; this rule, however, is not fixed—sometimes the Bishop's seat is on the south side, so that he can sit with the altar on his right and be on the left of God.

The secular followed the procession on ordinary fête-days, but, after having bowed before the high altar, at the bottom of Gradus Presbyterii, they had to go out through the priests' door in the south screen and hear Mass from the aisle of the chancel; if it were a particular saint's day, the whole procession went to the altar of the saint with the priests at the head.

It can be seen, from the plan of the cathedral of Dalby (fig. 205) that this was designed in view of the ritualistic requirements. The case is the same with the cathedral of Oslo (fig. 140). But the cathedral of Dalby was no church of pilgrimage, or at any rate, as such, it would have been imperfect. It seems that the cathedral of Oslo had first, in the twelfth century, the fully developed type of a church of pilgrimage, as shown in fig. 140.

In all churches possessing relics of a great and famous saint, the conditions made necessary certain arrangements for the large affluence of pilgrims. They showed their rank as cathe-

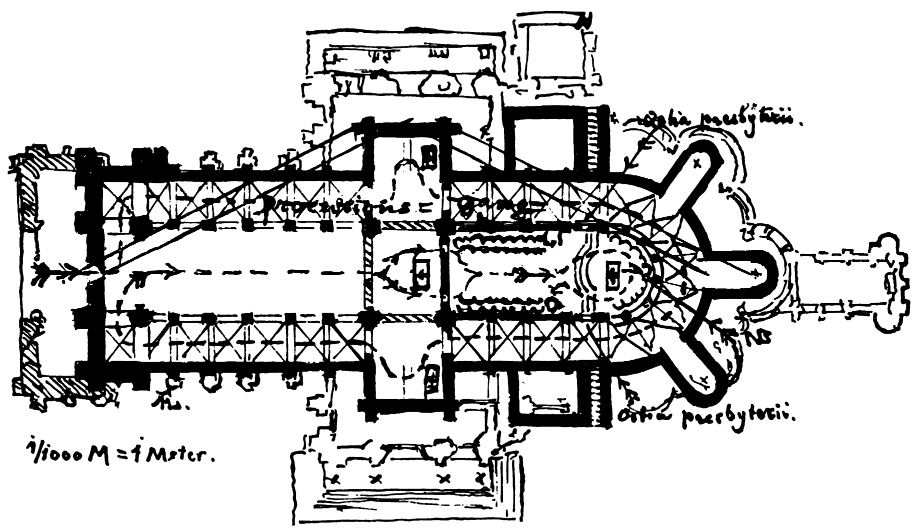


Fig. 206.—Cathedral of Chartres. Plan.

drals and pilgrim churches. The day and night before the saint's day the pilgrim church was full of far-travelling groups, who during Vigilia (in Norwegian Vaka, for example: Olafsvaka, Jönsvaka, commonly made to Olsok, Jönsok) remained in the church. The healthy and the sick, the lame and the blind, the poor and the rich, crowded in prayer on the grave of the saint. Therefore, it had to be of easy access; but, as a pilgrim may also be a weak man, tempted by the holy relics or by their value, they had to be guarded, not only by secretly posted watchmen, but by screens surrounding the holy place. Through the open work of the screen the pilgrims saw the reliquary and relics. Sleeping or praying, they were either lying round the tomb or at the various altars, but in the church of the saint, under his roof.

In the plan of the abbey church of Cluny (fig. 17), and in the cathedral of Toulouse (fig. 21), we see two typical pilgrim churches from the end of the eleventh century. As a still older and more famous pilgrim church, we give in fig. 206 the plan

of Chartres Cathedral, as the remaining foundations show it to have been with certainty. It was France's greatest international pilgrim church, early in the eleventh century. Consequently, after the fire of 1020, Archbishop Fulbert could ask for help towards its reconstruction not only from the King of his country, but from all the other Kings of Europe. It can be interesting for Scandinavians to know that there still exists the Archbishop's letter of thanks to King Canute the Great of Denmark and England for his important gift. The thick black line shows the church of Fulbert reconstructed in 1020 to 1037; the thin outer lines indicate the church after its extension in the twelfth century.

The procession path is marked by a broken line. As this shows, large processions could go round the altar without it necessitating the removal of a single chair in the chancel, just as pilgrims, night or day, could visit the shrine, unhindered, without disturbing the worship of the priestly college or their conferences in the choir. The apse of the presbytery at Chartres was about 23 m. 5 long, in the eleventh century, and it must then have had room, besides the large episcopal throne, for twenty-two to thirty seats for the members of the chapter; twelve to fifteen on the right and left of the Bishop, in a semicircle. The choir for the lower ecclesiastics, the singers and the guests, had room for eighty fixed stalls, which stood in two rows on both sides of the choir-walls as well as transversely along the choir-screen facing west. For great ceremonies some 200 ecclesiastics could be accommodated in the choir, besides the distinguished guests.

When the Bishop with his canons moved down in the choir in the thirteenth century, he had his place farthest up on the southern side; this seemed to be the rule in France, while the seat of the King—according to Viollet-le-Duc—was over on the northern side—that is to say, where it had been in the eleventh century. Our opinion is that it had been farthest up in the lower choir—that is to say, not in the presbytery. Through this removal the square under the central tower had to be included in the chancel, the new screen of which is in hatchings on the drawing. Now the screen stands again in its original place, between the pillars of the east tower; but in the Middle Ages it must have stood necessarily between the two western pillars. There must have been room, thereby, for 120 fixed choir-stalls besides the Archbishop's and the King's raised thrones. As a result of this extension of the choir towards west, the altar of the cross was removed to the first bay of the nave, west of the central tower.

Through the removal of the presbytery, the high altar, as already noticed before, happened to stand far off from the daily service. This took place eight times in twenty-four hours; they were called Horæ Canonicæ (in old Norse, Tidir), when the members of the chapter had to attend. These "tidir" were: (1) Ottusöngr hinn fyrri (12 a.m.); (2) eptri Ottusöngr (3 a.m.); (3) Midsmorguns Tid (6 a.m.); (4) Dagmálatid (9 a.m.); (5) Mids Dags Tid (12 p.m.); (6) Eykartid, or Nön (3 p.m.); (7) Aptansongs Tid (6 p.m.); and (8) Náttsöngr (9 p.m.). It was not only in monastic churches, but in cathedrals also, where particular architectural conditions made the distance to the altar inconveniently long; this fact, and the insufficient light during evening service, had the result of bringing a new altar into the choir, the matutinal, or "ottesang" altar. Owing to this a new sanctuary or presbytery was made, one lower besides the high one before the sanctuary proper.

The high altar and the place of the high choir happened in this manner to stand outside the daily presbytery. This is what the English have called retro-choir, or feretorium; it became a separate chapel, where the reliquary of the saint (feretrum) was placed at the back of the original and real high altar. We shall see a typical example of this in the cathedral of Nidaros. In this way the pilgrims could go round the whole of the sanctuarium. But in churches where an extension towards east, or some other architectural or economical conditions, made such an arrangement difficult, the high altar was put, for the

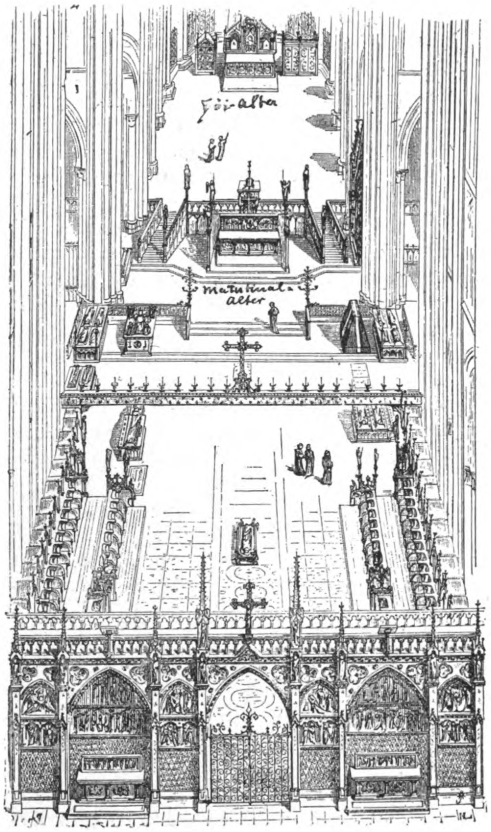


Fig. 207.—Pilgrim church of St. Denis. Interior of chancel, after Viollet-le-Duc.

sake of the pilgrims, on the place of the altar of the cross, west of the choir—that is to say, in the nave, such as, for example, in Winchester. In fig. 207 we reproduce the drawing in perspective, taken from Viollet-le-Duc, of the choir of the famous pilgrim church of St. Denis, near Paris, as it was in the Middle Ages. We see first the matutinal altar, with reliquary at the back, and, farther on, the high altar proper with the chief saint's reliquary placed at the back of it. The burial-place of the French Kings was in the choir. Over the screen (so called from the barrier round the relics) of the choir (in old Norse "Koren," in French "jubé"), there rose the large cross. In German it is called "Lettner," because the Gospel was read on the north (right) side of it and the Epistle on the south (left) side, during the service at the altar of the cross, west of the screen. Sermons and lectures were also delivered from here to the members of the chapter. Everywhere in Europe the same ritualistic requirements determined the design of the plan of the cathedral, just as we have seen that everywhere the regulations for religious architecture had symbolically determined the proportioning of churches, even of "stav" churches.*

Finally it may be useful to indicate the size of the cathedrals shown here, as examples from the eleventh century. The cathedral of Dalby was 196 feet long, and the cathedral of St. Halvard in Oslo 182 feet, after its extension it was 220 feet; while Chartres was a little more than 390 feet long.

As explained, this cathedral was, in the eleventh century, the most famous pilgrim church of Central Europe. As the introduced lines of the system show, the central tower stood, as it still does, in the middle of the church, and the length of the nave and chancel is the same as the length of the transepts. This cathedral of pilgrimage has undoubtedly served as example for the plan of other newer cathedrals and churches of pilgrimage, such as for the cathedral of Nidaros, and for the enlargement of Canterbury twenty years later.

* Translated literally, "stav" means a log placed vertically, or a post.

CHAPTER XVI

THE CATHEDRAL OF OLAF KYRRE

IN order to explain the conditions of sacred architecture in Norway at the time of Olaf Kyrre, 1067–1093, we give in Plate XVII the plans of some older, some contemporary, and some later churches, national and foreign, at the same scale.

The following five churches were, as mentioned, erected in Nidaros before the cathedral:

(1) The church of St. Clements, built by King Olaf Trygvason, dating from the years preceding 1000 (later rebuilt by St. Olaf); it has disappeared since the Reformation, no information existing as to its size.

(2) The chapel which was erected soon after 1031 on the tomb of St. Olaf. Its size is not known either. The archæologist and architect, H. M. Schirmer, is of opinion that the so-called "chapter house," situated on the north side of the chancel in the cathedral, is this very chapel.

(3) The church of St. Olaf, started by King Magnus Olafson, before 1047, and completed by his uncle and successor Harald Haardraade previous to 1050, and of which richly carved plinth we gave a reproduction before. This church was not large, measuring only about 110 feet long (fig. 228, Pl. XVII).

(4) The stonehall of King Magnus Olafson, which was converted by his successors into the church of St. Gregorius. Size unknown.

(5) The church of St. Mary, erected by Harald Haardraade before 1066, in honour of his step-brother the Martyr-King. According to Snorre Sturlason it was a large church ("mikit Musteri"); it was removed between 1176 and 1182 from the neighbourhood of the royal palace, and of the cathedral, across Nidelven to the Augustine monastery of Helgesætr, and they both disappeared subsequently. Some of its masonry was used in the sixteenth and seventeenth centuries as building material, partly for the extension of the late church of St. Mary, dating from the end of the twelfth century, and partly for repairing the Archbishop's residence. According to the foundations which he measured and sketched on the site in 1773, the historian Gerhard Schöning describes it as "an important building with a nave 48 ells long and 33 broad, having an aisle on each side and a chancel 26 ells long and 21 ells wide." These figures give the interior measurements, as shown by his drawing.* We reproduce on a smaller scale, in fig. 208, the finished plan (No. 1) of Schöning and the preliminary sketches of it (2 and 3). No. 2 is undoubtedly the sketch he used while measuring the place, No. 3 is done afterwards, and No. 1 is the finished drawing reproduced in his book. The pillars are placed irregularly in all three drawings; it is the same with the south porch. They contribute, however, all together to help form a correct idea of the plan. In Nos. 2 and 3 he has placed columns in the chancel. It is evident that

* Gerhard Schöning, *Reise gjennem en Del av Norge*, 1773–5, vol. i, p. 8, etc., published by the Royal Norwegian Society of Science, Trondhjem, 1910.

on the walls he has found bases or trunks of pilasters, as indicated in No. 3. As he was ignorant of the ways of building, he believed from this that the chancel had been three-aisled and with pillars. It is impossible that this chancel could be the one of Harald Haardraade, but it must have been rebuilt when the Birkebeiner burnt the church in their pursuit of Duke Skule in 1239.*

The chancel must then have been altered and extended, and given a Gothic vault, although in a different manner than as indicated on sketch No. 3. The saga tells us, in the same place, that the church had a tower, when it says that the Duke's men shot from the tower on the Birkebeiners (" Thegar sem Birkibeinar kámu at Klaustrinu, *skütu Hertuga Menn at theim ör Stoplinum* "). In the finished drawing we have indicated the original chancel, such as it must have been, as well as the tower, and from sketch No. 2 we have removed the pillars into the nave, whereby the normal proportions between nave and aisles are established.

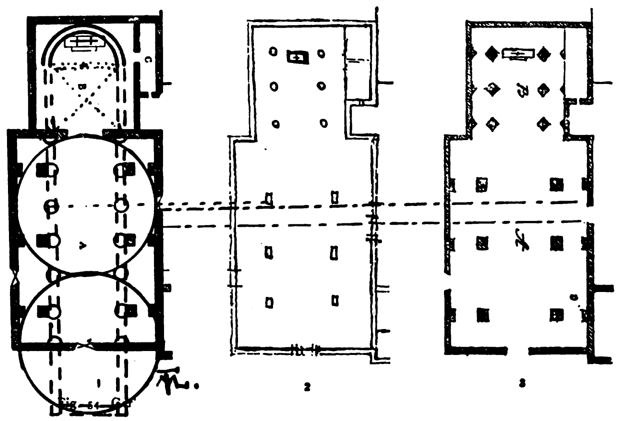


Fig. 208.—Church of St. Mary of Harald Haardraade in Nidaros, after Schöning.

Between the three pillars of Schöning the two missing pillars are introduced, making five in all. In order to recognise them, ours are drawn circular.

Through this rectification according to the rules of religious architecture and the additions based on authentic remarks in the saga of the Kings, the careful measurings of Schöning, as he says himself, produce the completely regular plan of a large parochial church. This has absolutely the same proportion and the same distribution as the plan of the somewhat earlier church of Stavanger, made later into a cathedral (fig. 141). In the church of St. Mary of Harald Haardraade there were six bays in the nave with a longitudinal intercolumniation of 8 ells, in Stavanger there were the same number of bays with an intercolumniation of not quite 8 ells. The exterior length has been 180 feet, the width fully 66 feet, against 175 and 64 respectively in Stavanger. Compare figs. 228 and 233. This interesting, but not surprising result, which justifies the claim of Gerhard Schöning to accuracy, shows also that Snorre could with reason call the church of St. Mary of Harald Haardraade in Nidaros " mikit Musteri." Not only did he see it himself, about forty years after its removal to the Augustine monastery of Helgesætr, but, in his scientific, critical, and methodical way of working

* "Saga Hákonar Hákonarsonar" in *Konungasögur*, p. 395, published by C. D. Unger (Christiania, 1873).

—which the present time appreciates *—he had heard it described by men who had seen it on its original site in Nidaros, and he was able to tell, therefore, how well it was built. Snorre says: "Magnus Konungr Olafson lét gera Olafskirkju i Kaupangi; i theim Stad hafdi náttsett verit Lik Olafs Konungs, that var thá fyrir ofan Böinn. Hann lét thar ok reisa Konungsgardinn. Kirkian vard eigi alger ádr Konungr (vitz. Magnus) andadist (1047). Lét Haraldr Konungr fylla that er á skorti. Han lét reisa af Grundvelli Mariukirkju uppi á Melnum, nær thvi er Heilagrdomr Konungsins lá i jördu hinn fyrsta Vetr eptir Fall hans; that var mikit Musteri, ok gert sterkliga at Liminu, svá at varla fekk brotit, thá er Eysteinn Erkibiskup lét ofan taka. Heilagrdomr Olafs Konungs var vardveitr i Olafskirkju, medan Mariukirkja var i Gerd. Haraldr Konungr lét hüsa Konungsgard ofan ifra Mariukirkju vid Ana thar sem nü er; en that sem han hafdi Höllina látit gera, lét hann vigja Hüs til Gregoriuskirkju." † ("The King, Magnus Olafson, caused the church of St. Olaf to be built in the town; on this spot the body of King Olaf had been laid during the night; it was then above the town. He caused also the King's palace to be built there. The church was not completed before the death of the King (1047). King Harald completed what was not finished. He raised the church of St. Mary from the foundation, up on Mælen, near the place, where the King's holy body was laid in the ground the first winter after his death; it was a great minster and masoned so strongly of good lime that it could not easily be broken, when Archbishop Eystein caused it to be pulled down. The holy body of King Olaf was kept in the church of St. Olaf, while the church of St. Mary was being built. King Harald caused a house to be built in the royal residence above the church of St. Mary near Aaen [the river] where it stands now; but there, where he had caused the hall to be built, he consecrated the house to the church of St. Gregory.") Moreover when, in chapter xviii of *Magnus Barfots Saga*, Snorre tells of the engraved marks on the northern porch, which showed the heights of Harald Haardraade, of Olaf Kyrre, and of Magnus Barfot,‡ he must have seen personally what he had read earlier in *Morkinskinna*.

We see that under Harald Haardraade many large buildings of stone were raised, religious and civil, both in Nidaros and in Oslo. This far-travelling, highly gifted king was a fastidious poet, as we know, and a builder as fastidious and as exacting. The stone hall of King Magnus the Good Olafsson, the son of his brother, was not found large enough for a king by the Constantinople chieftain, and he turned it into a church, and, although he finished his church of St. Olaf, he did not find it large enough and worthy to be the resting-place of the Saint-King, as it was, with only one nave, 47 feet wide and 125 to 130 feet long, including an unimportant little tower; a basilica of important appearance ("eit mikit Musteri") such as the drawings of Schöning show us, could alone fulfil the King's great aspirations.

As the sixth church of Nidaros, we can include the church of St. Margaret, of the brothers of the Guild, built under Olaf Kyrre. It has disappeared without leaving any trace.

We have mentioned that there exist in Nidaros some richly ornamented architectural fragments from 1030–1066, evincing a *highly* developed art of building and of ornamentation.

In Plate XVII, figs. 225 and 226, we have given, for comparison, the plan of the fylke churches of Akr and of Gran, both dating from the eleventh century; they are three-aisled, with a central tower and transept indicated. The first is 125 feet long. As shown by the elevations given previously in pp. 112 and 113, these churches, like all those belonging to that part of the country, are built in an especially plain early Norman style with a barrel vault,

* Among others the Professor Gustav Storm, Snorre Sturlason's *Historieskrivning* (Copenhagen, 1873), as guide to English readers, reviewed by E. W. Gosse in *The Academy*, June 6th, 1873, and by Ph. Schweitzer, *Gesch. der Altskandinavischen Litteratur*, vol. i, p. 206, etc. (Leipzig, 1885).

† "Saga Haralds Hardráda," *Heimskringla*, chap. xxxix, p. 576, published by C. R. Unger (Christiania, 1868).

‡ Ibid., p. 654.

on account of the material, which is a kind of hard limestone. In spite of this, there can be found here and there in the stone walls of the fortress and palace of Akrshus, some fragments from older churches of Oslo, showing that in that part of the country equally, the stonemasons were able to develop a fine, shapely ornamentation, even in this hard and difficult stone. Some of these date as far back as the oldest churches in that town, of which there remain only the foundations, apart from the detached pieces mentioned.

Fig. 227 reproduces the church of St. Mary, built by Harald Haardraade near the royal palace of Oslo, dating from 1050. It was 145 feet long and 78 feet broad.

We have already examined the cathedral of St. Halvard, begun by Olaf Kyrre, 1067–1093 (fig. 233). It shows a developed cathedral plan—three-aisled, with central tower and transept 93 feet long and chapels opening from it. The total length of the church was 185 feet, and the nave was 74 feet broad.

The cathedral of Bjorgvin, begun by Olaf Kyrre, was put aside to undertake the cathedral of Nidaros, and therefore it was not completed until the twelfth century. It has entirely disappeared, but, as we said, it was cruciform in shape and three-aisled, both as regards the nave and the chancel.

The later cathedrals of Kirkjuvåg, of 1137 (fig. 235), and of Hamar, 1152 (fig. 236), were, like the cathedral of Dalby, previous to 1076 (fig. 237) and of Lund, about 1087 (fig. 238), also three-aisled and cruciform; even the small single-aisled cathedral of Greenland from about 1120 (fig. 151), had a transept as a sign of its dignity.

The plan of churches inherited the transept from the Roman profane building called basilica, which formed a cross like a T, with an apse continuing the nave. The first church of St. Peter in Rome had this shape, as the drawing shows (fig. 209), and it served undoubtedly as model to the first episcopal churches. We said that the Middle Ages saw a symbol in everything. "With this sign thou shalt be victorious" were the words of the Scripture. The cross was the sign of victory of the Church and the symbol of its teaching. This is why churches in the East, in Armenia and in the Byzantine Empire, had at an early date the shape of the Latin cross; they were designed in one large square, with a dome built over the central square out the five small ones forming the cross. This was adopted early in the south of France, where it was afterwards incorporated in the long churches with the two squares system, and became the general rule for the churches of Western Europe. The cruciform shape is, therefore, the shape of all episcopal churches.

As already mentioned, the chancel became longer and longer, as the number of priests increased, and as other reasons made it necessary. Through this the transversal beam of the cross was gradually pushed towards west, until it coincided just with the middle of the church. This need of a long chancel was felt early in churches of pilgrimage; it is not easy to know when exactly; but already in the tenth century the cathedral of Tours is supposed to have had this cruciform shape with ambulatory for pilgrims. The international church of pilgrimage of Chartres (about 1024) seems to have served as example to St. Denis, Cluny (1089), and, let it be said once for all, to the later French cathedrals. This plan came to England with the famous Norman philosopher—born in Lombardy—Bishop Anselm and with Prior Ernulf, who enlarged Canterbury cathedral in 1096.* This must also have served as example for the long cathedral of Olaf Kyrre.

As shown previously King Olaf Kyrre (1067–1093) found it important, for psychological and political reasons, to strengthen the dynasty by strengthening the Church. Under his peaceful reign the country prospered in its economic life, while he encouraged and protected urban

* In the general examination of this cathedral, p. 17 etc. (also in the description of fig. 27), through carelessness, Lanfranc has been given as its builder, as also the date of its building 1070–77.

and social development. All towns increased, new towns were founded. All the sagas without exception give prominence to a trait of his character: he loved splendour and ceremonial pomp. He provided the rooms of the royal mansions with fireplaces in the wall instead of the ancient open ones in the middle of the floor; he increased and reorganised the Court, according to a pompous regulation. When a prince with such tastes decided to build a cathedral in the old capital of his kingdom, which should serve as the lasting resting-place of the founder of his dynasty, the Martyr-King, suited to receive and attract in constantly increasing number a stream of pilgrims from near and far, it is obvious that he must plan a building which can put the two former memorial churches in the shade and surpass also the two newly founded cathedrals of Oslo and Bjorgvin. This construction was intended to be a pilgrim church of the highest rank.

We shall give convincing proofs that the foundations of this cathedral still exist in the main, also as far as concerns the pulled down nave and the chancel, while the two lower stories of the transept with few changes, remain untouched, and the east wall of the chancel is partly existing.

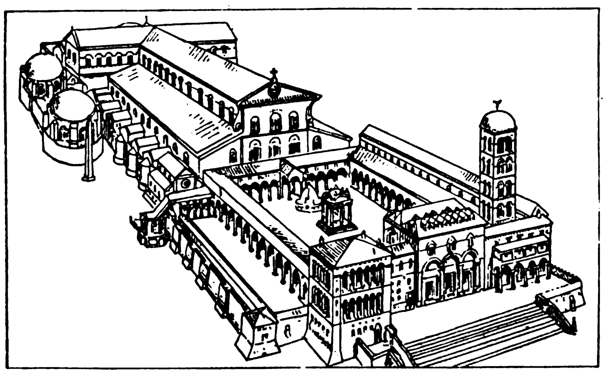


Fig. 209.—The old church of St. Peter in Rome. Bird's-eye view.

According to the available remains, this must have had, like Canterbury cathedral, enlarged later, a plan worked out after the model of the pilgrim church of Chartres, with a central tower in the middle of the church, and a single-aisled transept—but with a three-aisled chancel and nave and west towers projecting from the sides just as it also happens to be the case with the cathedral rebuilt by Archbishop Eystein, as it can still be seen. The end of the chancel towards east is difficult to ascertain because of later modifications; the probability points to an apsidal termination, having an architectural communication with the well of St. Olaf lying south of the apse, to which the pilgrims had controlled access from the aisles, as we have indicated in fig. 231 of the plate. This plan of Olaf Kyrre had a length of about 290 feet.

This opinion is shared in its main points by the professor, P. A. Munch, already towards 1840; it was founded on the interpretation of certain historical documents and on the church itself; it was held with greater conviction after 1855, when he worked in collaboration with the learned architect, H. E. Schirmer, senior, who had himself studied the plan of the cathedral as much as it was possible at the time; he came to the conclusion, however, that the chancel of Olaf Kyrre could only have been one-aisled, an opinion which he emitted as liable either to be confirmed or corrected when the pulling down necessitated by the work of restoration, bringing all the material to light, would decide the matter.

In fig. 211 we find the general drawing of the plan of Munch. He bases his opinion as to the length of the chancel on the saga's certainly correct statement that the altar in the cathedral rebuilt by Archbishop Eystein in the twelfth century, stood on the same spot as in the cathedral of Olaf Kyrre. He found an evidence of the missing aisles of the chancel in the *Saga of Sverre*, where it says, relating the battle of Kalvskinnet in 1179, that "Lik Erlings

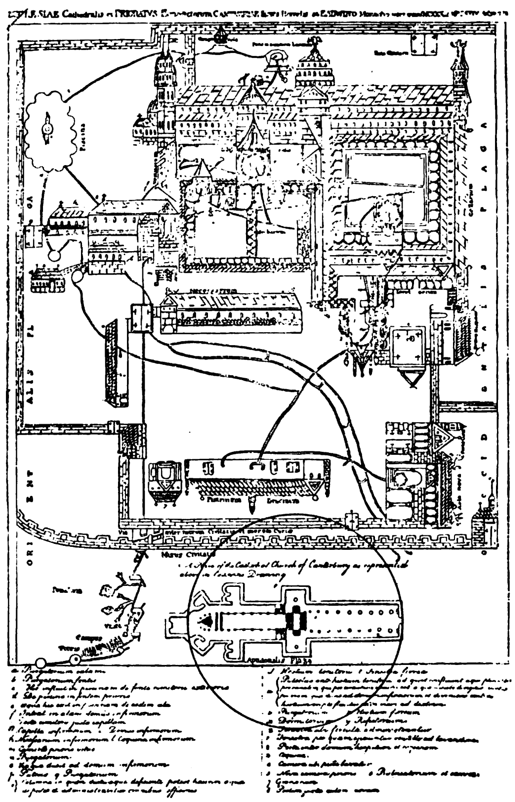


Fig. 210.—Cathedral of Canterbury. Plan from the drawing, dating about 1165.

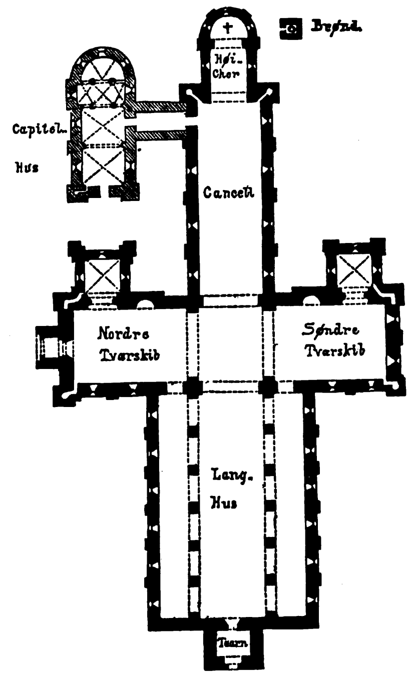


Fig. 211.—Cathedral of Nidaros. General plan of P.A. Munch of the cathedral of Olaf Kyrre.

Jarls var borit til Graptar fyrir sunnan Kirkju, enn nü er Gröptr hans i Kirkju" * ("The body of Jarl Erling was carried to his grave before the church, but now his body is in the church"), and finally a proof of the tower outside the west gable is found in the account of the battle in the same saga. † As this spot is of importance for the chronology of the cathedral, it requires a closer examination.

Jarl Erling had gone with his men, from the harbour near the mouth of the river situated in the lower part of the town, *up* to the church, where they met King Magnus and his suite coming out of the royal palace, east-south-east of the cathedral. After the Jarl and his son Magnus, with his little troop of 600 men, had gone towards west, on the south side of the cathedral—(the description of the place points to the latter, as between the cathedral and the church of Harald Haardraade there was just a small passage only a few ells wide)—and

* "Sverris Saga," in *Konungasögur*, chaps. xxxiii and xxxiv, p. 41, etc.; chap. xxxv, p. 43; ed. C. R. Unger (Christiania, 1873).

† Herman M Schirmer, *Kristkirken i Nidaros*, p. 165 (Christiania, 1885), p. 165 in the original text of the book of plates *Throndhjems Domkirke*, by Munch, written 1855–57, published as an addition to the book of Schirmer.

come "ut um Stöpul," that is to say, past the tower, the Jarl looked on both sides, that is, up and down (toward south and towards north), to see if his troop kept together: "hvar erum vir nu allér?" "Nu foru Birkibeinar *utan* at Bönum ok fundust á Akrinum *fyrir ofan Stöpul*," viz. King Sverre and his group came from without against the town and they clashed together (with the enemy) *above* the tower.

In the fact that there is only *the* tower (in the singular), "ut um Stöpul," and "fyrir ofan Stöpul," Munch finds it a plausible evidence of there having been but one tower in front of the gable, as his drawing indicates, and that consequently the battle went on west of the tower, as he makes "fyrir ofan" to mean, in the direction towards west. This conclusion must be considered altogether as rash, arbitrary, and incorrect; because, although one never says, to go round the *corners*, but the *corner* of a house, no one would think of denying that a house has more than one corner, so Jarl Erling did not go "ut um Stopla," but "um Stöpul," viz. one of the two flanking towers, proved by the existing foundations.

Now, as in both instances one tower only is mentioned, the next expression, "fyrir ofan Stöpul," proves doubly to be, when read with attention and with due regard to the topography of the place—that this tower has been the south-west tower of the cathedral.

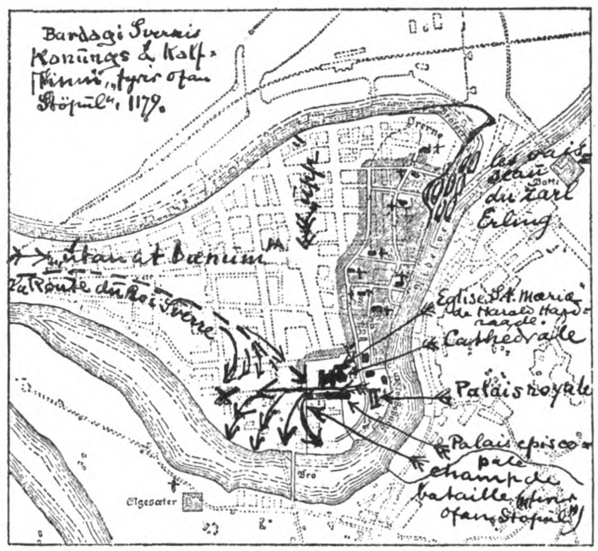


Fig. 212.—Sketch-map of the battle of Kalvskinnet.

King Sverre would not have been the clever strategist acknowledged by all war-historians without exception if he had not given its direction to the battle, such as the saga relates it precisely and briefly. He came ("utan") from Steinbjörg, across the narrow Nidareid with a force which was as three to five against the Jarl. The latter and his son, King Magnus, came from the royal palace and the tower, where, sheltered by the cathedral, the palace, and the town, they could reach their ships. The task of Sverre was then clear: to try to approach and surround them—viz. east of the Jarl—to cut off their retreat and drive them into the river. Under his encircling pressure from west and north he came in contact with the Jarl "fyrir ofan Stöpul," towards where the battle had gradually moved from the beginning, that is, south-east, and then taken place and obtained its conclusion south of the cathedral, between this and the river, where the western extension of the Archbishop's palace was built later. Through this decisive movement the Jarl was obliged to use his next and last alternative, to reach the bridge over the river, lying south-west of the west front of the church, which he did not succeed in doing, because with his right flank Sverre surrounded him. When the Jarl had fallen, most of his men, instead of reaching the bridge, ran into the river and were drowned, while Magnus succeeded in escaping through the town.*

Upon the added sketch-map (fig. 212) the stroke lines show the movements of Sverre

* *Sverre's Saga*, chap. xxxiv, p. 43, 1 c.

and the continuous lines those of the Jarl. Down on the north of the river the position of the ships is marked, whence the Jarl came and where Magnus escaped. The saga was dictated by King Sverre himself, as all know. The adverbs used to give the directions are the same as those still used at present; they give concisely and clearly, in Sverre's manner, the whole action of the battle. Contemporary readers could not have the slightest doubt as to the meaning: "fyrir ofan" meant south of the Cathedral, and "ut" the movement towards west. Compare with the quotation, p. 228, on the church of St. Olaf, which was originally lying above the tower, viz. south of the tower.

But it is no less rash to take the following statement of the saga: that Jarl Erling was carried to his grave "before the church, but now his body is in the church," as a proof that Olaf Kyrre's chancel had no aisles; it can very well be, concerning this grave, as it was for the grave of King Magnus the Good, of which *Morkinskinna* and *Fagrskinna* say, that it was laid outside the chancel, while *Flateyjarbök* adds that now it is lying inside the chancel in front of the episcopal throne—it is possible that it *had been moved*, as Munch says himself in the book of plates before mentioned.* Therefore it is uncertain, not to say inconsequent, to base the chronology of a great edifice on a single grave. In the south aisle, near the extreme eastern bay, a medieval grave was found five feet under the ground.† It was thought that this was the grave of Jarl Erling which became included in the church without being moved, through Archbishop Eystein having added the aisle. In the Roman Church it was everywhere considered as the most distinguished place to be buried under the dripping eaves, the "Upsardropa," as it is called in old Norwegian church laws. But Jarl Erling was not the only distinguished man in Norway; one ought to have found several medieval graves of famous men at the same depth, if the existing aisle had been "undir Upsardropa," that is, outside the original chancel, but not a single one has been found during the work of restoration. Jarl Erling was more than usually high-born; he was related to the legitimate royal house through his mother—his son was crowned King by Archbishop Eystein, who was himself of royal descent, and who protected him. It is quite reasonable to think that Archbishop Eystein, or his successor Eirik, who certainly in their hearts entertained no doubt as to King Sverre's or his family's illegitimacy, may have removed the body of Jarl Erling inside the church,

It can hardly be doubted, that if Munch had known the history of the development of the cathedral types to which precisely the cathedral of Nidaros belongs, not only would he have found the length of the chancel in no way remarkable, but without aisles he would have found it unsuitable for the ritual of a pilgrim cathedral. He would also have found it remarkable, and even unthinkable, if he had known or made himself thoroughly acquainted with the description of the cathedral of Dalby, by Professor Brunius, that this ordinary church should have been designed according to the requirements of a cathedral, while, for the first church of pilgrimage of Northern Europe, Olaf Kyrre was satisfied with a chancel not having the necessary aisles for the convenience of the pilgrims. But, however one-sided this opinion of Munch was, it gave at least the impression of a cathedral as regards size, which coincides somewhat with the historical events leading to its foundation and the indisputable evidence of the edifice itself, so cleverly interpreted in his collaboration with the great architect, Herm. E. Schirmer, in his account of the church.‡

But during a visit to Copenhagen in 1858 his opinion underwent a change, owing to the persuasion of Höyen, professor at the Academy, a quite ordinary exponent of popular art

* P. A. Munch, H. E. Schirmer, *Throndhjems Domkirke*, p. 20 (Christiania, 1859).

† *Annuary of the Society of Antiquaries*, 1866.

‡ See the original text in the above-mentioned book of plates, as well as the account in Lange's *Norsk Tidsskrift* 1848, p. 52, included in P. A. Munch's collected accounts, published by Gustav Storm, vol. i, p. 333, etc. (Christiania, 1873).

history, without any special knowledge of the subject, and who, having never visited France nor England, had never seen a West European cathedral with his own eyes, nor had he ever visited Norway, still less seen the cathedral of Nidaros. He was deficient, moreover, in all the qualifications needed to understand Norwegian medieval art. It is quite beyond comprehension how Munch, who was a critical scholar, thoroughly acquainted with Norway's historical past, acquired through a profound insight into history and a solid knowledge, could allow himself to be influenced by this incapable foreigner and proclaim on his own authority what had been infused into him. According to them, the cathedral of Olaf Kyrre would only have been a small parochial church, without aisles, reduced to the size only of the central aisle of the later chancel, having a small square chancel and a semicircular apse, and probably with a simple small tower in front of the western gable—the same " Stöpul " from *Sverre's Saga*, appearing again—a tower of which, in spite of the most exhausting search on the site, not the slightest trace has been found.

Fig. 230 shows the plan of the cathedral according to the theory of Höyen and Munch. Its total length—if one reckons also the imaginary tower before the west gable—would have been 145 feet, but without tower 123 feet, and its width on the east 47, and on the west 42 feet.

It can be seen, from the plate, how much inferior is Höyen's plan of the royal cathedral raised in memory of the founder of the dynasty and Martyr-King, the widely venerated national saint, compared to the cathedral of Dalby, 198 feet long and 88 feet wide (fig. 237), and to the two cathedrals already planned, that of Oslo, 185 × 74 feet (fig. 233), and of Bjorgvin, as well as to the later cathedral of Stavanger, 180 × 65 feet (fig. 234), but it is still far below the then existing church of St. Mary, 180 × 67 feet (fig. 229), situated only about 15 to 20 feet distant, north of " the cathedral," which it was to supersede as the resting-place of the saint. The " cathedral " of Höyen is again put in the shade by the fylke church of Akr, without tower 125 × 58 feet (fig. 225), and it can just compete with the small fylke church of Stjördalen, in Værnæs, 130 × 40 feet (fig. 232) and the fylke church of Mæren, 142 × 40 feet!

The antiquary Nicolaysen gave up his former opinion, on the authority of Munch, and later the Professor L. Dietrichson, Captain O. Krefting, the architect, Johan Meyer, Dr. H. Fett, and O. Kolsrud have followed each other's footsteps.

Since the time of Munch the work of restoring and of excavating west of the transept has brought to light the foundations, not only of a three-aisled nave, but of two corner towers flanking the west front; in spite of that, it has not occurred to any of these gentlemen that thereby the interpretation of *Sverre's Saga* by Munch—at any rate, concerning *the* tower—becomes wrong. It is this lack of critique which has caused a cross to be put in the marble floor of the south aisle over " the grave of Jarl Erling."

But they reveal still more their lack of historical knowledge and of ability to appreciate critically the original texts as well as their lack of archæological knowledge and judgment when they take the grave of King Magnus the Good and the plinth of Olaf Kyrre's " cathedral " as conclusive evidence in support of Höyen's fallacy.

As mentioned, the saga of Magnus the Good in *Morkinskinna* and *Fagrskinna* says: " Var Lik Magnussar Konnungs jardat at Kristkirkju fyrir utan Kör " (" the body of King Magnus was buried in the church outside the chancel "), while *Flateyjarbök*, an historical collective work of the fourteenth century, adds: " enn nu er that fyrir innan Kör fyryr Rumi Erkebiskups " (" but now it is inside the chancel before the episcopal throne "). This saga is found inserted in a saga of Harald Haardraade by the owner of the book in 1498. It is obvious that there is nothing impossible in the fact that this insertion, like the saga of Magnus the Good itself, can be a copy from an older MS., dating possibly from the twelfth or the

thirteenth century. It relates to the arrangement of the chancel in the Gothic period such as it must necessarily have been as a result of architectural conditions due to the rebuilding of Archbishop Eystein in the twelfth century, and right down to the fire of 1531.

Fig. 213 reproduces the plan and profile of the extreme east part of the chancel in front of Gradus Presbyterii, as it was altered during the restoring after the great fire of 1531, when the arcades of the nave were incased in the rough walls, as the drawing shows clearly. The

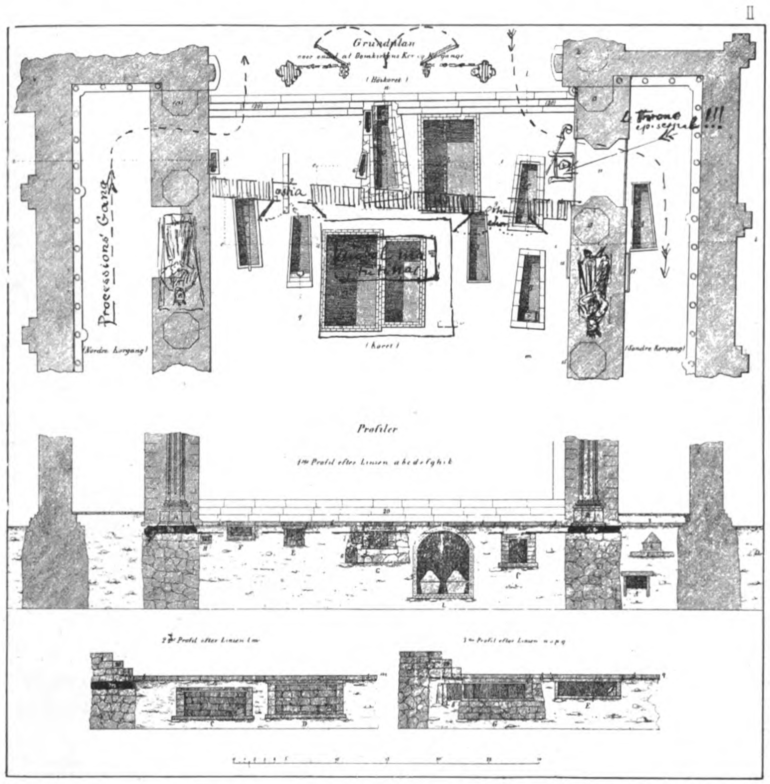


Fig. 213.—Cathedral of Nidaros. Plan and section of east part of the chancel, before restoring.

grave marked A in the south aisle is "the grave of Jarl Erling." The single-aisled church of Olaf Kyrre is supposed to have been included within the walls of the nave. The grave marked G with its foot just near the stair is the supposed grave of Magnus the Good, which has become included in the chancel, through Archbishop Eystein having made the nave of Olaf Kyrre into the chancel. This conclusion has seemed obvious to Captain Krefting,* ignorant of history; but, as it remained uncontradicted, the statement was taken up again by the lecturer on church history, Oluf Kolsrud: "Langhuset var sjölve kyrkja; det som var langhus i Olav-Kyrre-kyrkja, vart no kor saman med tverhuset; *soleis kann soga segja*

* *Annuary of the Society of Antiquaries*, 1866, p. 19.

at Magnus den Gode vart gravlagt utanfyre koren i den gamla Kristkyrkja, men no ligg han innanfyre koren framanfyre erkebiskopstolen." * ("The nave was the church itself; what was the nave in the church of Olaf Kyrre became now the chancel with transept; *the saga can say therefore that Magnus the Good was buried outside the chancel of the old Christ church, but that now he lies within the chancel in front of the Archbishop's throne."*)

According to our experts at home, then, the episcopal throne must have stood, either on the south side—as in fig. 213, where we have written "le trône épiscopal"—or just opposite on the north side. As it must have been understood from our account of the ritual demands of a

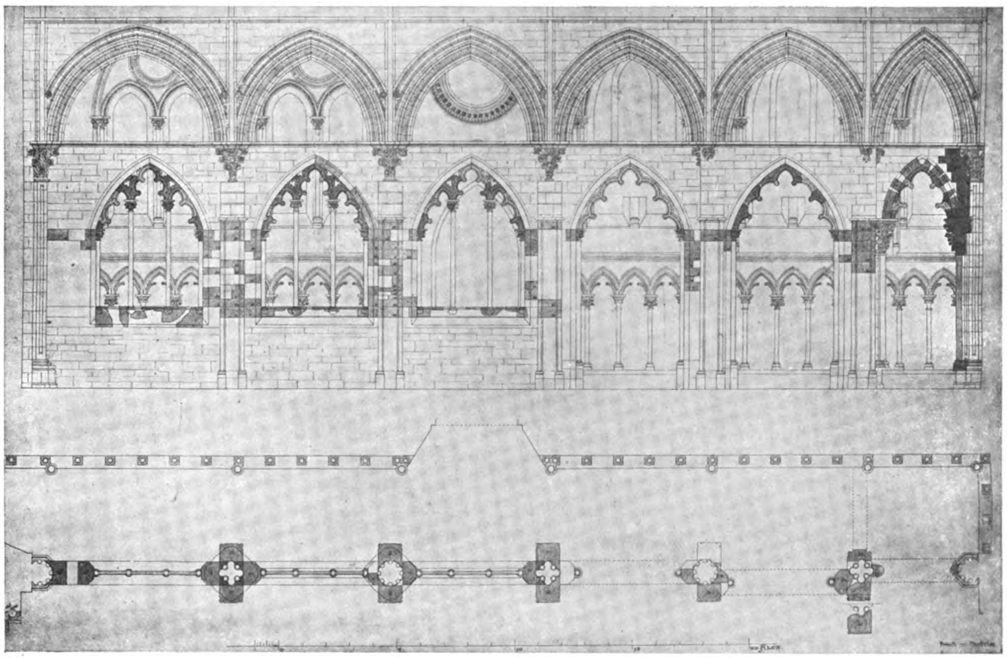


Fig. 214.—Cathedral of Nidaros. Reconstructed choir screen.

cathedral, in connection with the plan of the cathedrals of Dalby and of Chartres, the episcopal throne has *never* stood on such a spot, and in Nidaros it would have been impossible on account of the architectural conditions, for instance, because the side-aisles did not continue directly to the ambulatory round the chancel, as shown on the plan. A secular visitor or a far-travelling pilgrim would have had to go first into the central aisle in order to reach the ambulatory round the closed chancel of the high altar. According to Kolsrud, the Archbishop's throne would have stood, so to speak, in the middle of the procession-path and outside the chancel. It should be unnecessary to say that the same requirements which dictated the arrangement of the chancel in all other cathedrals of Europe must also have been decisive in the cathedral

* Oluf Kolsrud, *Olafskyrka i Throndhjem* (Oslo 1914). Joh. Meyer, *Domkirken i Throndhjem*, p. 80 (Throndhjem 1914), tries to be critical and mentions another grave, which must undoubtedly be an Archbishop's because they found in it a Bishop's staff!

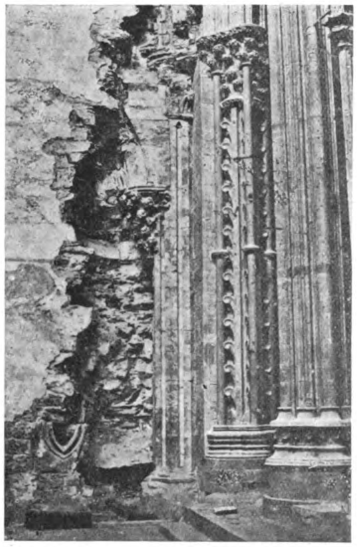


Fig. 215.—Cathedral of Nidaros. Remains bonded in the wall of the choir screen.

of Nidaros. The building itself gives irrefutable evidence that this was the case. It will be sufficient in this respect to give the reconstruction of the screen (fig. 214) which the late architect, Alf Hoflund, assisting in the restoring work, has undertaken and based upon the standing parts and the fragments found. His reconstruction represents this screen as it is supposed to have been built in order to strengthen the thin pillars which, after the fire of 1328, were very much weakened. The drawing shows the northern arcade. The darkest parts indicate the standing remains and the fragments found. In the three western bays we see the screen, in the fourth was Ostium Presbyterii, but not with the full opening, such as it is given in the drawing. The fifth bay is left open. This had been closed, necessarily, while the sixth, as on the drawing, was open on account of the entrance to the retro-choir. A proof that the arrangement was such is found in the pillar still standing against the east wall of the chancel (fig. 215). After the mentioned fire, a slender, detached marble column was raised and stands to-day, supporting a richly blossoming capital (fig. 215) The arch sprung from this, going over to a corresponding capital and column at the next pillar—as it is perfectly correctly reconstructed by the architect. It can be seen that in the reconstruction of the arch, he has found, besides the east half of it, which stood bonded into the thick wall from the time of the Reformation, almost all its remaining part. Without such an opening the mentioned slender column would never have been attached to the pilaster on the east wall of the chancel. From the mentioned first detached pillar, which receives the arch over the passage, there was necessarily a screen of stone or of open metal tracery across the central aisle, whether there stood a matutinal altar or not, in the chancel thus closed. The architectural conditions seem to point, even conclusively in the plan, to the necessity of such an altar, of which the church itself gives also an evidence. The central arch and the two smaller arches on each side of the one leading to the retro-choir, have still, the former, hooks for the hinges of the grated doors; the latter has marks of the grating fixtures. The retro-choir was thus shut out from the procession-path part right across the nave, between the retro-choir and the choir. In the previous fig. 213 we have indicated both the screen and the grating with the grated doors of the retro-choir.

It is not necessary, after these indisputable proofs of the church itself, to give any reason why the episcopal throne at the time referred to in *Flateyjarbök*, or supposed to be referred to, did not and could not stand right opposite the mentioned grave near the steps leading to the retro-choir, but that, in accordance with the arrangement in all Catholic churches of the thirteenth, fourteenth, and fifteenth centuries, it stood farthest up in the presbytery, which in Nidaros began at the fourth bay of the arcade from the east, as the architect Hoflund has indicated rightly. As we shall see later, the Archbishop did not only have here Ostium Presbyterii on his right, but he had also the episcopal entrance porch and the episcopal palace conveniently near. As the distribution was in the Gothic period, so it must necessarily have been in the Norman time on account of the architectural conditions, of which we shall give an account later.

In addition to our researches, there is a circumstance which attracted the attention of Nicolaysen, the antiquary, although he did not thereby feel bound to protest against the criticless use of a grave, as proof or as base for the chronology of an edifice. When examined by experts, the skeleton found in the grave showed that the individual had had a stiff ankle, from which Nicolaysen drew the reasonable conclusion that the skeleton in the grave "could hardly have been King Magnus, because from the saga there is little reason to believe that he had a stiff ankle." *

Apart from the improbability that a man with a stiff ankle could have been the great warrior King Magnus was, according to the sagas, still, to give a chance to this unreasonable claim, we shall argue the possibility of the grave being his. We can imagine that, to save his church from destruction after the fire of 1432, Archbishop Aslak Bolt had to be satisfied with rebuilding the plain walls between the pillars—as we see in fig. 213, and that, as a result, he was obliged to give up the matutinal altar and the screen, and thereby also the procession-path, and that, as entrance to the presbytery which was removed towards east, he kept an Ostium Presbyterii in the south screen and placed the episcopal throne just over on the north side. In this case, also, the addition of 1498 in *Flateyjarbök* could have said that the grave, which before was lying outside, was now inside the choir in front of the episcopal throne. Now, as this *always* stood in the presbytery, it can be understood that the "choir" mentioned in the two sagas means in this case the right side of the said part of the chancel reserved to the priests.

Moreover, as the rough walls, with a distribution of the choir as shown in fig. 213, could *certainly never* have been built before the fire of 1531, and after the Reformation, it is obvious that the account of *Fagrskinna* and *Morkinskinna* can only refer to the distribution of the choir or the conditions of the chancel, as it stood after the Gothic rebuilding of Archbishop Eystein at the end of the twelfth century and right down to the fire of 1531.

We have demonstrated how this distribution was, by our supplementary and correcting notes to fig. 214. We have seen that the presbytery stood on the place between the three western bays in that part of the chancel, and it follows that the Archbishop had his throne farthest up on the south side at the third detached pillar from the east.

In the mentioned sagas we have an authentic, positive and irrefutably decisive evidence, which was sufficient for the enlightened readers of the Middle Ages, but is too scanty for the Protestantism of our time, that, in the cathedral of Nidaros, a removal of the presbytery and of the choir towards west took place—just as it had been the case in all the other European cathedrals, and the reason of which we have given above.

If we add to this acknowledgment the point to which Nicolaysen has drawn attention, it will be proved conclusively that this grave is not the grave of Magnus the Good and that the science of these gentlemen has "shed tears on the wrong tomb"—just as it proves that the addition in *Morkinskinna*, instead of supporting it, speaks decisively, on the contrary, of the deep ignorance of Höyen's theory.

It does not fare any better with the archæological point which Captain *Krefting*,† and with him all the mentioned scientists, have indicated in connection with the graves of "Magnus the Good" and of "Jarl Erling"; that is, the chamfered coping over the foundations, which they have presented as "the plinth" of the edifice of Olaf Kyrre. In the plan (fig. 213) we see this "plinth" in the door frame of the south wall, marked by figure 11. In the profiles we have emphasised it by making it quite black. It would be waste of time to contradict Captain Krefting's worthless and—to use a lenient term—absolutely self-contradictory claim.

* *Annuary of the Society of Antiquaries* (1866), p. 19. † Ibid. p. 17, etc., *Annuary of* 1872, p. 77, etc.

It will be sufficient to refer to fig. 216*a*,* which is Krefting's supplement to the "proof" (fig. 213). The plinth is indicated by figure 3, while the stone indicated by 4 (in fig. 216*b*) is supposed to be "a well-preserved fragment of wall from the choir arch of the church of Olaf Kyrre." The last-named may possibly be a fragment of voussoir belonging to the retro-choir of the older cathedral. But, when comparing this figure, especially the plan of 216*a*, with the profile of fig. 213, anyone with any judgment will immediately recognise that, in reality, this "plinth" could never have been what it claims to be, but only the chamfered coping of the inner foundation walls, upon which rose the pillars of the nave and of the chancel as well as the screen between the choir and the retro-choir, shown in figs. 216*a* and 216*b*. As it will be demonstrated in a following plan, the "plinth" appears again in the nave, where one can see that it continues from the pillars of the central tower towards west under the arcade of the pillars of the nave—that is to say, far outside the "cathedral" of Höyen!

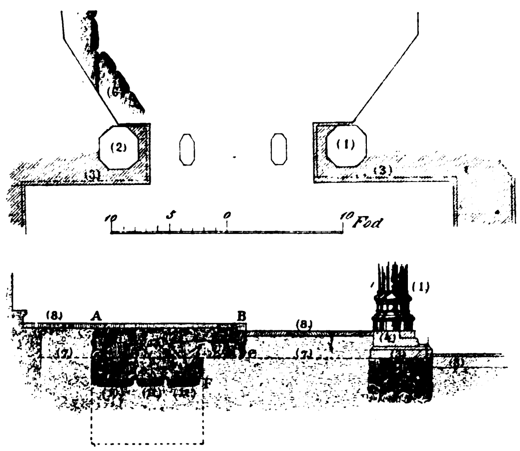


Fig. 216 *a* and *b*.—Cathedral of Nidaros. The supposed "plinth" of Olaf Kyrre's cathedral.

In short, the twin proofs, *Flateyjarbök* and the "plinth" on the excavated foundations, relate the opposite of what the "learned" gentlemen have insisted upon, namely, that the cathedral of Olaf Kyrre did not have a small plan with a single aisle, smaller than the church it was to supersede and smaller than the small fylke churches, but that it was a three-aisled cruciform cathedral distributed throughout according to the ritual demands of the Catholic Church—what our readers will come to acknowledge entirely afterwards.

Without daring to deny the historical, constructive, and artistic possibility of such a plan, nor that the remaining transept dates also perhaps from Olaf Kyrre's time, the above-mentioned architect, Johan Meyer (now Professor at the Technical High School of Norway and chairman of the official Supervising Committee for the restoration of the cathedral) has suggested a new and third point to the two already mentioned in the row of proofs, as a "conclusive" evidence concerning the "cathedral" of Höyen, and which he thinks deserves to be generally known.† We will fulfil his wishes.

In a polemic with H. M. Schirmer, junior, who adopted and defended the original and correct theory of P. A. Munch, Johan Meyer used as argument a quotation from an old Norwegian saga: Koll of Agder, mentioned in a previous chapter, is said to have advised his son, the pretender, to build a cathedral to Magnus, the saint of the Orkneys, "more beautiful than any in these countries" (Orkneys and Norway), therefore the cathedral of St. Magnus in Kirkevaag, which was actually built and by Koll himself half a century after the death of Olaf Kyrre, would have been larger and more splendid than the King's work in Nidaros.‡

* *Annuary of the Society of Antiquaries*, 1872, p. 7. This plinth is also adopted by Johan Meyer, *Domkirken Trondhjem*, p. 13, and by Olaf Kolsrud, c.l., p. 8.

† Johan Meyer, *Domkirken i Trondhjem*, p. 10 (Trondhjem, 1914).

‡ Johan Meyer, ibid., p. 13.

The argument has become an involuntary proof of the wrong use of quotations, on the part of a professor at the Technical School, and of his friend and colleague, the late Professor L. Dietrichson; it shows ignorance of the language used in our ancient civilisation. It is the latter who, in his work, *Monumenta Orcadica*, written in collaboration with Meyer, states in three different places * that Koll from Agder mentions "these countries." If, instead of using an English translation (!), Dietrichson had consulted the work of Munch on this point, as he usually did, he would have found that the right meaning is "in that country" † (we have translated it previously "on the Orkneys"); in the original there is "i thvi landi" ‡ (literally, "in this country"), which the professors have taken to be the plural form, and with them also Dr. H. Fett, at present State Antiquary, in his book on *Norges Kirker i Middelalderen*,§ and Kolsrud, now consulted as an authority on church history, during the restoring work, in his book *Olavskyrkja i Trondhjem*.‖ We mention this as a specimen of the knowledge with which the theory of Höyen is supported, and with it the chronology of the cathedral of Nidaros and other Norwegian medieval churches.¶

The cathedral of St. Magnus in Kirkjuvaag in the Orkneys was by no means more beautiful than the one of Olaf Kyrre in Nidaros; we know its origin well enough to be able to say this. Its political purpose is mentioned above; we shall only say here that it was executed as soon as the promise of the founder could be realised—that is to say, as soon as he became earl of the islands; and the work was conducted by his father, the Norwegian chief, Koll Kalasson, whose cleverness was well known and who was a friend of Olaf Kyrre's son, King Magnus Barfot, and of his grandson, King Sigurd Jorsalafare (the Crusader). From the *Orkneyinga Saga* we know that the son of Koll, Jarl Ragnvald, used to travel in his youth to Oslo, Tunsberg, and Bjorgvin, and that in the summer of 1127 he visited Nidaros, where the cathedral was finished, already before the death of Olaf Kyrre (1093). The father being less famous, the saga does not give his full biography, but, as a member of the King's Court and Council, Koll had probably—although it is not mentioned in the saga—followed either King Magnus or King Sigurd to Nidaros; being a rich landowner and interested, as we saw, in religious matters, it is equally certain that he visited the tomb of the Martyr-King, towards which both he and his wife must have felt a special attraction apart from the usual religious feelings which drove every Northman to Nidaros, because his wife, being a sister of the Martyr-Earl of the Orkneys, was descended from the brother of Olaf the Saint. Professor P. A. Munch, who had himself visited the cathedral of St. Magnus, remarks, on the work of this Norwegian nobleman Koll Kalasson, that it "gives a marvellous proof of the insight into architecture

* Pp. 101, 140, and 168.

† P. A. Munch, *Det norske Folks Historie*, vol. ii, p. 447.

‡ *Orkneyinga Saga*, chap. lxii, ed. G. Vigfusson; *Flateyjarbök*, vol. ii, p. 455.

§ P. 21. ‖ P. 10.

¶ The following curiosity, among others, is an instance of the learning of Mr. Johan Meyer. It is found in his above-mentioned book, *Domkirken i Trondhjem*, p. 107, and followed by a note, p. 108. On one of the many tombstones of the church he found one, which, in his eyes contained an important proof concerning the history of the building of the church, and he remarks: "It is startling to find that small Gothic lettering occurs so early in Norway (probably in 1316). According to the state antiquary, Hildebrand, they are very rare in Sweden before 1380. But the Bible of Aslak Bolt, of which a richly illustrated page is reproduced in Scheibler's *History of Printing and of Journals* [sic], is done in that lettering. It is possible to place it towards 1300, judging from the illustrations, as well as from the character of the initials. It is probable that it had already belonged to one of our Archbishops, and the inscription on the tombstone of the stone-cutter, Mathias, might have been done in imitation of it." It may be excusable that an historian of architecture should not recognise the Bible of Aslak Bolt when he sees it, or that he should not know that this elegant Parisian MS. from the middle of the thirteenth century was acquired by the Archbishop in his youth, when he held an office in the King's Privy Council, as testified by a note in his hand at the end of the book; but a professor at a technical college ought to be acquainted with the fact that the transition from capitals to small letters took place much earlier in books than in lapidary writing. Truly it exhibits a sad picture of intellectual level and academic learning at the present day in Norway, when amateurs of this type are officially put up as authorities.

possessed by a distinguished Norwegian layman of those times." * The saga expresses itself in the following words on the work of Koll †: "Ok eigi miklu sidarr var markadr Grundvollr till Magnuskirkju ok aflat Smida till, ok för svá mikit fram Verkinu á thrim Misserum att minna gekk á fjörum edr fimm thadan ifrá. Kollr var thar mestr Tillannadarmadr Smidarinnar ok hafdi mest Forsogn á." ("And not much later the foundations of the church of St. Magnus were marked and artisans engaged, and the work progressed so well in three and half years that it progressed less in four or five later on. Koll planned the construction and had most to say in the lead"—meaning that he was the leader.)

Here we have a document showing that the cathedral of the Orkneys, dating from 1136, is executed by two Norwegians of high rank and that one of them, the son, had certainly, and the father surely, seen personally the cathedral of Nidaros; the political advice of the father to his son, to build a stone cathedral to the memory of his mother's brother, shows clearly his familiarity with the work of Olaf Kyrre.

Moreover, we get to know that they engaged artisans ("Smida")—that is to say, architects and stone-masons. In connection with this, it can be useful to remember that the great profane and religious building activity which developed in the eleventh century in Nidaros, Bjorgvin, Oslo, etc., was continued no less zealously in the beginning of the twelfth by King Eystein, grandson of Olaf Kyrre and son of King Magnus Barfot, as the saga relates specially, giving also an accurate account of the time. In the years 1107–11 King Eystein caused several edifices to be carried out, of which two churches are prominent: the church of St. Nicholas in the royal palace of Nidaros, which Snorre describes as being extremely richly ornamented and artistically decorated,‡ and the church of St. Michael of the Benedictine monastery of St. Michael in Bjorgvin, which is called a fine building. The first has disappeared without leaving traces; but Nicolaysen has excavated the foundation-walls of the second, and found remains of great importance both for our cathedrals and for the chronology of the buildings.§

The attached figures (217–24), which represent a selection of the few architectural and ornamental parts remaining from the church of St. Michael in Bjorgvin, give a convincing picture of the advanced development of Norwegian architecture during that period. We direct the attention to the frieze in zigzag of fig. 217. On the crypt, which had an apsidal termination towards east, there has been a vault, the webs of which have been supported by cross-ribs, springing from eight detached pillars and twelve pilasters on the walls, as shown by the plan in fig. 219. The reproduced bases and pieces of capitals having a resemblance with the Ionic Order belong to these pillars.

The result of this research of Nicolaysen, which is *especially important* for European archæology, thus convinces us that an elaborate architectural problem has been solved in Norway in the years 1107–1109.

Stone had been used for more than seventy years when Olaf Kyrre began to build his cathedral, and for about 130 years, when Koll Kalasson and his son Jarl Ragnvald of Agder began the foundation of the cathedral, which was to have no equal on the Orkney Islands.

During that time numerous architects and stone-masons were trained, who obviously stood ready, whenever rose the question of building a large church.

It goes without saying that Koll had obtained his artisans from home. It would be impossible to explain the similarity of the two cathedrals without personal knowledge of the

* *Det norske Folks Hist.*, vol. ii, p. 754.

† *Orkneyinga Saga*, ed. G. Vigfusson, p. 132; and *Flateyjarbök*, vol. ii, p. 463.

‡ "Han lét ok gera i Nidarösi i Konungsgardi Nikoláskirkju, ok var that Hüs allmjök vandat at Skurdum ok allri Smid" (*Heimskringla*, ed. Unger, p. 671).

§ *Annuary of the Society of Antiquaries*, 1860, p. 59, etc.

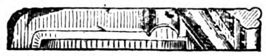


Fig. 217.—Church of St. Michael, Bjorgvin. Piece of a frieze in zig-zag.

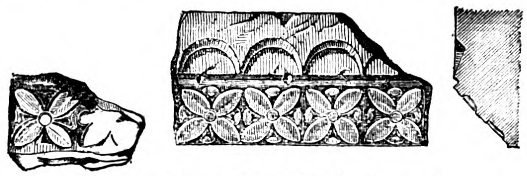


Fig. 218.—Church of St. Michael. Ornament.

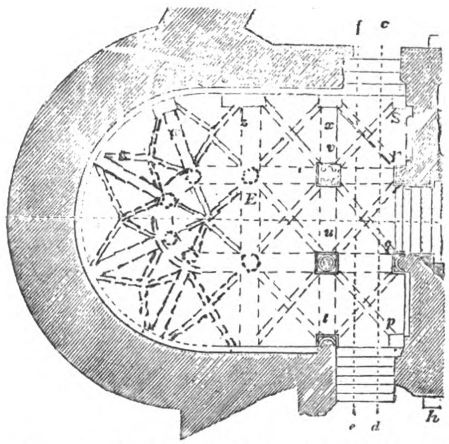


Fig. 219.—Church of St. Michael, Bjorgvin. System of vault in the crypt.

Fig. 220. — Church of St. Michael, Bjorgvin. Ionic-like capital from the crypt.

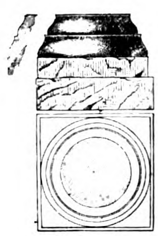


Fig. 224.

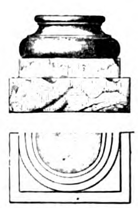


Fig. 223.

Fig. 222.

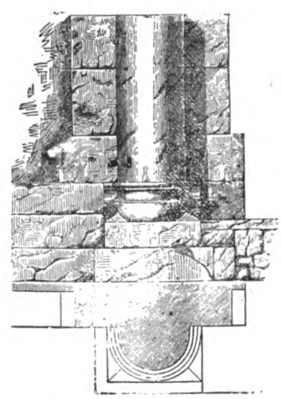


Fig. 221.

Figs. 221–224.—Church of St. Michael, Bjorgvin. Bases of the crypt.

work of Olaf Kyrre on the part of the designer, Koll of Agder, and without the co-operation of the master-builder and artisans from Nidaros. The cathedral of St. Magnus appears to be entirely a conscious copy of the older mother cathedral, suitable to the need of a smaller province; its plan, the chapels of its transepts on two stories near the gables, its elevation, and its early ornamentation, different from the English, not to speak of the undeveloped Scotch, are facts which have struck all those who have studied this cathedral.

even to the vault of the aisles planned from the beginning of the foundation, with webs resting upon cross-ribs—an early occurrence for which the master-builder had had an example at home, in the cathedral of Nidaros and in the church of St. Michael of Bjorgvin.

In fig. 235 the cathedral of Kirkjuvaag is reproduced after the original plan of *Koll*. The part which is entirely black shows what remains of his work, those in hatchings and in dots indicate two different terminations to the now disappeared chancel. We have done the same for the termination towards west. As no traces are found of any strengthening in the foundations for the pillar farthest west, we have pushed the west tower outside the church, as the only reasonable solution. Whatever the termination has been in the east, it will be admitted, by comparison with Olaf Kyrre's cathedral in fig. 231, that there is here a resemblance which can only be the result of slavish copying—slavish, because even what is abnormal in Nidaros has been imitated, that is, the unnecessarily open space between the chapels of the transept and the aisle of the chancel, and for which there was no logical reason in Kirkjuvaag, where Koll had a free site to build on—in opposition to Nidaros, where the aisles had not a regular width on account of older existing buildings which had to be respected. The reason that the chancel is shorter in the cathedral of St. Magnus is because they had no need for a larger one.

What is quoted here of the saga of Orkneyinga shows that, instead of being a "conclusive" proof of the theory of Höyen, it gives, on the contrary, a valuable evidence that the three-aisled European pilgrim cathedral of Olaf Kyrre served as model for the plan of the cathedral of St. Magnus.

It is upon this Höyen-Munch theory that the whole chronology of Norwegian church architecture is built, as we have mentioned it.

In support of this chronology, the same authorities * have found in P. A. Munch a statement which, were it not set in a vague, hypothetical form, would be a match for the "rendering" of the saga of Orkneyinga, by Professor Dietrichson. Munch supposes in his history,† but with a following reservation, that the cathedral of St. Halvard in Oslo "must have been built under Sigurd Jorsalafare, because it is said that when he died he was buried in that church."

To this supposition we can answer that the six Kings who, we are told, were buried in the cathedral of Nidaros, besides Olaf Kyrre, could be thought just for the same reason to have built this cathedral, or that King Sverre built the cathedral of Bjorgvin where he was laid to rest.

Of the two brothers, reigning simultaneously, Eystein and Sigurd Jorsalafare, it can be said with *full knowledge* that neither of them has built any cathedral. On account of the mutual jealousy between them, the saga reports very carefully all that can characterise the reign of these two opposite personalities—especially their foundation of churches. Eystein built five churches, which are all named,‡ while Sigurd built one, also named.§

In the account of the burial of King Sigurd in the church of St. Halvard (1130) this cathedral is mentioned just as in the account on the elder brother's burial, King Eystein, in the cathedral of Nidaros, this is also mentioned, both as well-known facts. The earliest Bishop of Oslo was already dead in 1122. Therefore, if the cathedral of Oslo was not built by Olaf Kyrre, it can only have been raised by the founder of the town, Harald Haardraade—which we think very likely; but not by any of the later Kings.

If we now compare the cathedral of Kirkjuvaag (fig. 235) with the cathedrals of Oslo (fig.

* Meyer, *Mon. Orcadica*, p. 168; Fett, *Norges Kirker i Middelalderen*, p. 19; Kolsrud, *Olavskyrkja*, p. 10.
† *Det norske Folks Hist.*, vol. ii, p. 614.
‡ *Heimskringla*, ed. Unger, pp. 671 and 683. § Ibid., p. 701.

233), of Hamar (fig. 236), and of Stavanger (fig. 234), its size coincides with these, while, on the contrary, its plan differs from theirs through its greater likeness with the mother cathedral of Nidaros.

This likeness, as well as the above-mentioned advice of Koll to his son, show completely that in every point he had taken as models the politic and the church of Olaf Kyrre, which is after all, all the more natural as it was in the interest of a Norwegian subject to tie his emigrated compatriots to the mother-country through building an obvious copy of the cathedral dedicated to their patron—a copy which would serve to underline and to strengthen the relationship of his son with the dynasty of its royal founder and the legitimacy of his earldom.

A survey of the plans of the cathedrals of Dalby, Lund, Oslo, and Stavanger and the above-given account of the necessary requirements of a pilgrim church, not to say a pilgrim cathedral, will save us from wasting more time in contradicting the obviously unreasonable supposition, not to say the historical impossibility, even the ridiculous claim, that Olaf Kyrre should have built a cathedral in his capital to the national patron saint and founder of the dynasty as humble as a country church and even far poorer than the pilgrim church it was meant to supersede.

But, before we begin the special examination of the cathedral of Olaf Kyrre, there is still another point of historical interest to take into consideration: this is the Norwegian church architecture and organisation and its supposed dependence of England. In connection with our earlier demonstration, showing that, for more than two generations before Olaf Kyrre, important churches and civil buildings with superior ornamentations had been built, we must mention the fact that the families of princes and chiefs still kept their friendship and relations with the emigrants settled in Normandy.* This excludes the theory that a country with an ancient artistic culture, and with an easy opportunity of transferring its perfect mastery of wood carving to stone should have been obliged, in order to build the cathedral of Nidaros, to borrow church builders from England, where Anglo-Saxon architecture gives just the evidence of artistic poverty corresponding to the lack of folk-lore of the race.

The attempts to show the particular influence of the Anglo-Saxon church on the newly founded organisation of the Norwegian church, as Professor Conrad Maurer, the most thorough expert on old Norwegian civil and ecclesiastical law, has proved it, are the result of insufficient knowledge on the part of our church historians with their Protestant education, concerning the ecclesiastical conditions in the Middle Ages. One forgets that the apparent relationship between the Norwegian and Anglo-Saxon churches can be carried back to Ecclesia Catholica Romana, which was common to all western countries at the time, as was religious architecture.

One forgets also too easily that the whole of North England, the shires north of Ely, Oxford, Gloucester, Hereford, and Bangor, had had already for a long time a dense Norwegian population, forming principally the upper class. Contemporary lists of persons and of places at that time, in the North of England, are to a great extent full of

* In connection with this, it can be useful to give the previously mentioned statement of J. A. Worsaae in *Den danske Erobring av Normandie og England*, p. 352, note 3: "An Italian historian tells us, that a Norman count from those parts [Bessin and Cotentin], who was at the Italian Court in the eleventh century, declared that he could not speak French (Hugonis Falcandi, *Hist. Sicula, Muratori Script. rer. Ital.*, vol. vii, p. 322). Until about that time 'th' in Normandy was still pronounced in the old Scandinavian manner like a semi-s sound. For this reason they wrote 'tethaurus' for thesaurus, 'Othmund' for Osmund, etc. According to the Scandinavian manner, again, the Normans did not pronounce two nn's: Varenne became 'Warethne,' Brionne 'Briothne' and 'Briosne,' as stated by Leopold Delisle and Auguste le Prevost." Upon this interesting explanation of Worsaae it can be remarked that the indicated peculiarity of the Normans' pronunciation is not "old Scandinavian," nor even "Scandinavian"—that is to say, Danish, or Swedish, but that it is *Norse, specifically West Norwegian pronunciation*, which existed and is still met with in the Norwegian districts of Vestthelemarken, Stavanger, and northward over the *whole* of Western Norway, as well as on the Faröe Islands, Iceland; it was also used on the Orkney Islands and in Shetland. The Normans had thus, as the sagas tell us, their origin in the west of Norway.

Norwegian names; Norwegians and Englishmen appear to have understood one another's language without difficulty. The rapid growth, before mentioned, of the veneration of St. Olaf in those same regions in England coincides with the fact that, while Englishmen helped the Christian Kings of Norway to spread the Christian faith, Norwegians held high ecclesiastical charges in England, partly from choice and partly while they were political refugees.

A mutual influence must have been undoubtedly developed, as it can be noticed in several directions, including architecture. We have a typical example of this exchange taking place at the time of Olaf Kyrre.

A few years after the battles of Stamford Bridge and of Hastings, so decisive for the fate of Norway and England, the new King of England, William the Conqueror, a Norman, sent ambassadors to King Olaf Kyrre in order to inaugurate friendly relations between the two countries. The ambassadors sailed across the North Sea in a Norwegian merchant vessel. A political fugitive had smuggled himself on board; he was a distinguished person, whom William kept as a hostage from the neighbouring province of Lindsey, south of the Humber. In spite of the loud demands of the royal messengers to return him to shore, the Norse captain kept his course, and not only the embassy but the fugitive, who was a well-educated ecclesiastic, and bore the good Norwegian name of Thorgaut, found the most friendly welcome. The King made him a member of his Court, and he remained half a score years with Olaf Kyrre just at the time when the cathedral of Nidaros was being built. We venture to think that Thorgaut, besides the Bishop, had some influence not only upon all decisions concerning the establishment of fixed bishoprics, but possibly also upon the building of the cathedral. After his return to England, probably not without peaceful entreaties from Olaf Kyrre to William, Thorgaut became prior of the monastery attached to the cathedral of Durham, a position he filled between the years 1096–1106, when he removed to St. Andrews in Scotland as Bishop, but only to return to Durham, where he died in 1115. A talented monk of his monastery, named Simeon of Durham, wrote down from the account of Thorgaut, some traits otherwise unknown relating to Olaf Kyrre.*

At the very time when this Thorgaut was prior in Durham, Bishop William of St. Carileph laid the foundations of his cathedral in 1093. We have here a firm support for what we claimed above, that the cathedral of Durham owes its inspiration to the cathedral of Nidaros.

Consequently it is deserving of the greatest attention that Durham cathedral, founded by Bishop William of St. Carileph, the year in which the founder of the cathedral of Nidaros died, is the first of the large cathedrals founded in England in the eleventh century, having a transept so long that it is equal to the length of the nave and chancel, to which is added the width of the transept.

We noticed above that this was the case also with the cathedral of St. Halvard in Oslo founded by Olaf Kyrre. Durham cathedral is the first in England, therefore, which seems to be planned within two equally large squares.

We find this type for the first time in the north of Europe, fully developed in Nidaros cathedral, by Olaf Kyrre, with the central tower placed over the crossing in the middle of the church. Minutoli has already pointed out as not being improbable that this centralising might be due to the influence of the cathedral of St. Sophia in Constantinople, where Norwegians and Normans were as in a second home. We find it in England carried out for the first time at the rebuilding of Canterbury cathedral in 1096 by Prior Ernulph, and in the following century it becomes even a fixed rule for the English cathedrals, only to mention those of Wells, York, and Lincoln. The type was created at Chartres in France, but it

* P. A. Munch, *Det norske Folks Hist*, vol. ii, pp. 395, 410.

spread rapidly on account of its suitability as regards the crowded religious ceremonies held in those large pilgrim cathedrals. It was not only the universality of the Catholic Church which promoted the rapid spread of these Benedictine buildings, but, as we have already reminded before, the conquering expeditions of the Normans created a secular universality, because the higher classes and the princely families in the north of France, in the south of Italy, and in England (besides Russia) were of the same race as those of Denmark, Sweden, and Norway. The new type of church, under the guidance of the Norman Bishops of the order of St. Benedict, struck roots in France, Norway, and England, and nowhere else. The Normans travelled in all the world then known, and had personal knowledge of classic art. For a race having a strong imagination and great ambitions, this became the right type of church. Ever since his Viking time the Norwegian Martyr-King had been famous throughout the world of the Normans, where the fame of his sanctity spread rapidly soon after he had fallen, so that churches were raised in his honour from England to Russia, and from Iceland to Byzance. As no tomb of Apostle existed in the lands occupied by the Normans, they found it all the more natural to resort to the shrine of the saint of their own race, when the need of spiritual consolation or the hope of miraculous recovery put the pilgrim staff in their hand. Already forty years after the death of St. Olaf, Adam of Bremen praises Throndhjem, the already much-visited capital of Norway, and famous for the miracles which took place at the grave of the Saint-King Olaf, where people " throng from the most distant regions " in hope of recovery.

It was just in the following years that the future Prior of Durham lived in Nidaros, and, being of Norwegian descent, he could not have felt like a stranger. He witnessed the raising of the new cathedral in honour of the far-famed Martyr-King. Nothing is more natural than, shortly afterwards, as Prior of the Monastery of Durham, he should make use of his impressions and experiences from the large work just finished in Nidaros, in planning and decorating the new cathedral of his town. The building operations were started in Durham the very year that Olaf Kyrre died, after having, according to the sagas, completed his cathedral. Why should not the monastery of Durham have engaged the master-builders and stone-masons who had just finished their work in Nidaros, where we have seen that the art of stone-cutting was in such a flourishing condition, already before the building of the cathedral of Olaf Kyrre? It is not so much in its plan, as in its rich " Norman " ornamentation, that Durham cathedral has essential features in common with Nidaros, while in its Norman character it represents a great advance over the other English cathedrals of the eleventh century.

Durham was, moreover, the first English cathedral to have cross-ribbed vaulting, the first which was actually planned with such a vault over the nave. Prior Thorgaut had just seen this very form of vault carried out in Nidaros. We do not know if the vault of the nave in the cathedral of Olaf Kyrre was cross-ribbed; it is not likely, but sure traces of the vaulting of the aisles are in existence, and in the chapels of the transept, as well as in the chapter-house, outside the north wall of the chancel, the original cross-ribbed vaults were still standing until 1869, when they were rebuilt by Architect Herm. E. Schirmer, senior.

The Romans used cross-ribs or diagonal ribs to support the cast webs of concrete of their vaults; but the honour of introducing these ribs in the medieval masoned webs seems to rest with the Lombards, as already from the beginning of the eleventh century they had adopted this kind of construction—not because of its superiority, but to economise the cost of wood, their country being poor in that material, the wood of the scaffolding being wasted (the centring of the web). They had learned this art from the Byzantine dome constructions carried on ribs, or groins. Already in 1040 the nave of the church of Sannazzaro Sessia was vaulted in this manner.*

* Arthur Kingsley Porter, *The Construction of Lombard and Gothic Vaults*, pp. 6, 18, 21, etc. (New Haven, 1911)

If the ribbed vault was adopted so rapidly by the Normans in France—although only in the aisles and in smaller spaces—it is not due to the same economical reason as in Lombardy, but undoubtedly to the circumstance that, as ship-builders, they must have immediately discovered the constructive and static advantages of the rib system, principally because of its relationship to the frame in ship-building—an art of which their Norwegian ancestors had been past masters for several centuries.

To experts in ship-building it must be at once obvious that a vault with the sharp forms of a ship's framework must have a greater power of lifting, on account of a lesser horizontal pressure, than a vault built in a semicircle. Therefore this arch appears very early in Norman architecture, without having any relation with the Gothic style—except as far as it holds in itself or rather that it involves the whole system of Gothic construction. Harald Haardraade and the Normans travelling east, had also had the opportunity of seeing the pointed arch in Armenia, where it appeared several centuries before it did in Europe.

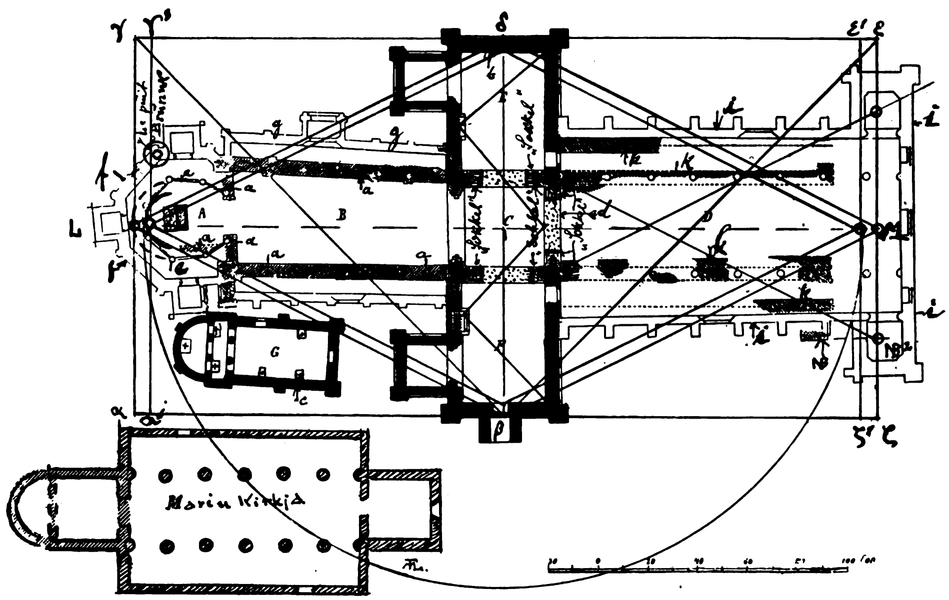


Fig. 244.—Cathedral of Nidaros. The old foundations excavated, after O. Krefting.

Viollet-le-Duc, Ruprich Robert, and Minutoli have found in the Norwegian art of ship-building an explanation of the early developed system of construction of Norman architecture in religious, civil, and military buildings.

It was not from the south only that England obtained her Norman influences. Eirik Blodoks, the son of the first King of united Norway, had already been King in Northumberland, in the first half of the tenth century, whilst the higher class of all the north of England was composed of emigrants from Norway.

The Norman chiefs of William the Conqueror coming from the south met in the north of England the Norwegian chiefs coming from the north, while the southern architecture of his Norman Bishops met, in Northumberland, the architecture of the north.

Another evidence of this influence coming from the north—although not of an architectural but of a religious kind—is the custom, which was only practised in two churches in England, as far as we know, the cathedral of Durham and in the half Norwegian town of York, towards about the eleventh century: the shrine of the saint was carried in a procession out of the church and round the town,* a custom which was not known anywhere else than in Nidaros, where, from the beginning, it was the custom to bear the shrine of St. Olaf round the town, probably because the older churches, including the church of St. Mary of Harald Haardraade, were too small and unable to hold the many pilgrims on St. Olaf's Day, so as

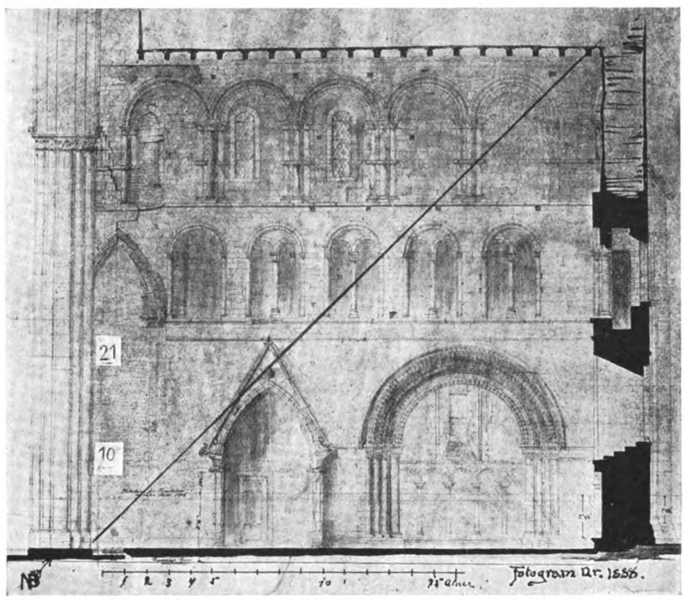


Fig. 245.—Cathedral of Nidaros. Measured drawing of interior of east wall of transept before restoring.

to allow them to approach the shrine, standing as it did in the small single-aisled chancel at the top of the church.

This custom in Durham, exceptional in England until the canonisation of Bishop William, in York after 1154, was undoubtedly taken from Nidaros by Prior Thorgaut.

But later also we can detect signs of influence from the *Gothic* pilgrim cathedral of Nidaros on English architecture, as we shall prove regarding Lincoln cathedral.

Unfortunately it has been absolutely necessary to undertake a kind of sorting in the wilderness of ignorant talk which has been allowed to cover up the remains of the cathedral of Olaf Kyrre. After this we can begin to ascertain its foundations.

In fig. 244 we can see the plan of the cathedral, by Captain O. Krefting, where the

* At York the relics of its Archbishop, St. William (note, died 1154) were contained in a portable shrine; at Durham there was a portable shrine "wherin the bones of the holie man Saint Bede was inshrined, being accustomed to be taiken downe every festival daie, when there was any solempne Procession, and carried with iiij Monncks in tyme of Procession" (Francis Bond, *Introduction to English Church Architecture*, vol. i, p. 99. London, 1913).

excavation of the oldest foundation is made clear in hatchings. We have put in black the standing Norman part, the transept, and what is called the chapter-house. Moreover, we have drawn the plan of the church of St. Mary of Harald Haardraade on the place where, according to the indication of *Flateyjarbök*, everyone agrees that it stood. The diagonal hatching in the drawing of the plan of the cathedral, going from left to right (from the bottom), marks the foundations of the "cathedral" of Höyen. It is over these that the chamfered coping-stone lies, the one which, instead of being an external plinth, shows by its shape that it has stood under interior partition walls. The foundations under this stone are all marked with the small letter *a*.

We have shown, in horizontal hatchings and arrows marked "socle," where this chamfered

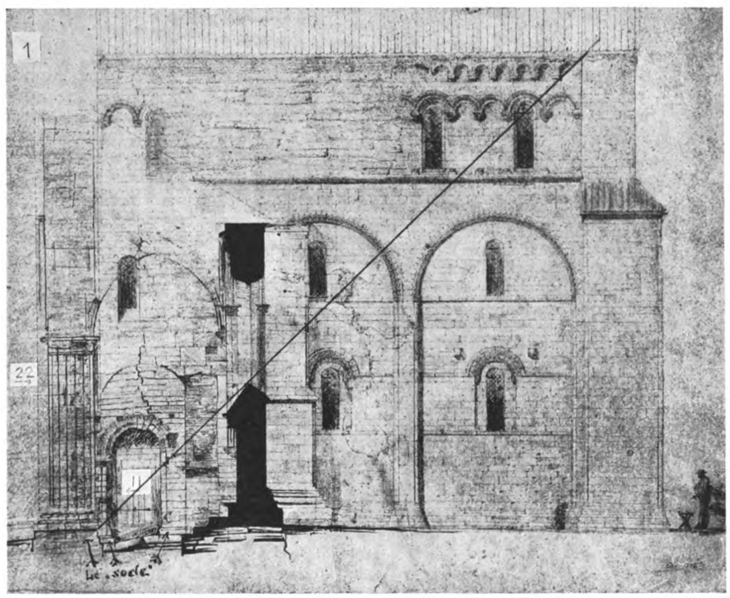


Fig. 246.—Cathedral of Nidaros. Measured drawing of interior of west wall of transept before restoring.

coping is also found outside the "cathedral" of Höyen. The parts with dots between the pillars of the central tower indicate that here also the "socle" was found, serving as base to the plinth of the screen, which stood once round the chancel, as we shall prove now.

With this object in view we give the fig. 245, which reproduces the measured drawing of architect Christie, before the restoring. We see the "socle" beneath the real plinth, under the south-east pillar of the central tower, marked by NB and an arrow.

Furthermore, we give the measured drawing of Christie of the outside surface of the west wall of the south transept (fig. 246). Three arrows marked "le Socle" show that this lies (1) under the plinth of the pilasters of the main arcade against the south-west pillar of the central tower—that is to say, in the nave—and (2) under the dividing wall between the south aisle and the south transept—that is to say, on each side of the porch leading to the transept.

We attract attention, moreover, to the marks left by the outer walls of the Norman aisles

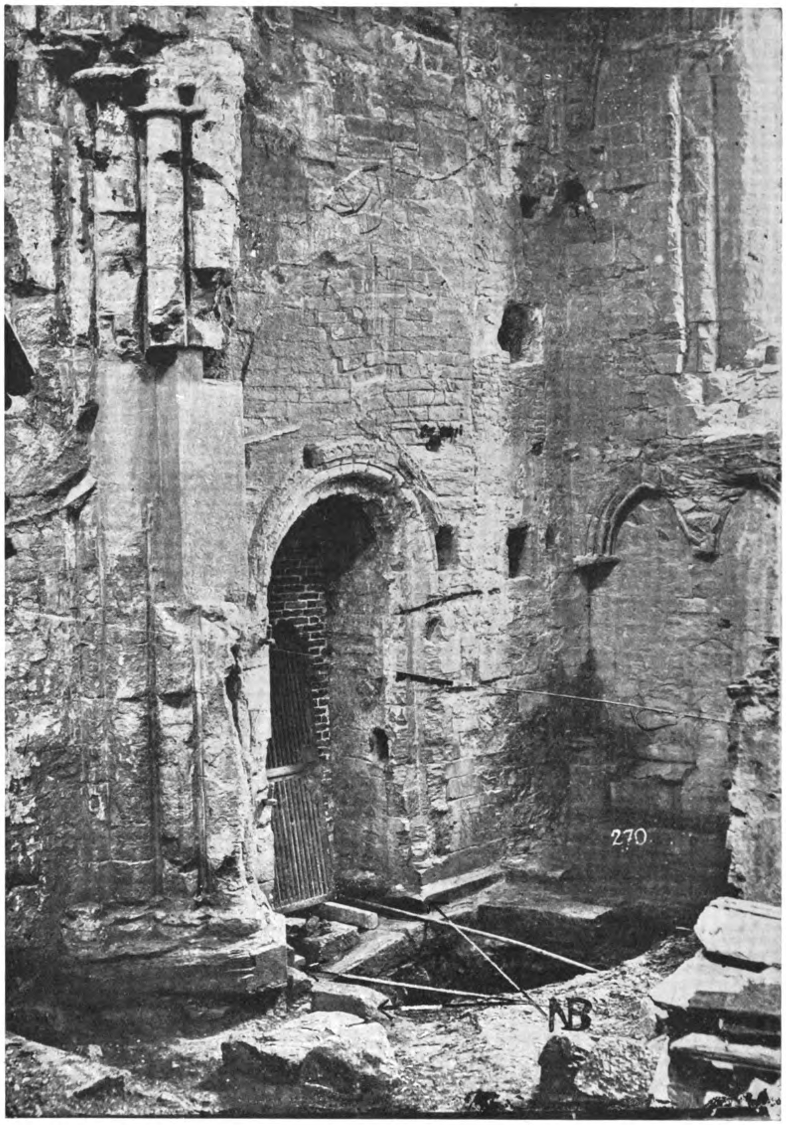


Fig. 247.—Cathedral of Nidaros. South-west corner of south aisle before restoring.

and their vault. We see additionally the not uncommon phenomenon in medieval cathedrals of this type that the threshold of the porch and the arch over it, as well as the masonry, are cracked on account of the great weight of the central tower.

In fig. 247 we reproduce a photograph as supplement to the previous orthogonal drawing. The "socle" marked NB and three arrows under the dividing wall and over the foundations of the arcade appear in perspective with all desirable clearness. The porch is from Olaf Kyrre.

Fig. 248 is the photograph of the drawing of 1908, viz. after the restoring of the places on both sides of the same porch—that is to say, the wall of the transept and the south-west pillar of the tower seen from east. Under this the "socle" is marked with two arrows. Finally, in fig. 249, we show the plan of the parts around the south-west pillar of the tower. As this shows, the chamfered coping continues both towards east and towards north. The case is the same with the north-west pillar of the tower. The stone is now broken, but it has run undoubtedly from the west pillars to the east pillars of the central tower, and it has served as base to the plinth of the screen, which developed between the two west pillars of the tower into a high, chapel-like, vaulted building, the pulpitum, right across the nave, and of important width, because the platform, or the floor above its vault, which is called in English Roodloft—as we said before—had to provide room for the singers and the organ. The chamfered coping-stone had, therefore, extended undoubtedly several ells longer towards west in the nave than the two ells we see at present in its broken condition.

Thus this base shows, not only by its shape, which is equally chamfered on two sides, but by its position under the partition walls still standing, that it could never have been part of an external plinth; the walls under which it has been, and still is, have been partition-walls in the interior distribution of the cathedral, according to the ritualistic requirements.

In relation with this we will now make a special point of the drawing of Captain Krefting (fig. 250), of the foundation-walls under different parts of the church. His opinion is that these walls show such a widely different character that he thinks he can find for certain the signs of four different building periods.

In the oldest period, 1067–93, he reckons: (1) Foundation-wall *a*, which is the supposed foundation of the "cathedral" of Höyen. Here he has drawn in the "socle,"* as it can be seen. (2) In the second period, 1160–79, he reckons the foundation-walls in herring-bone masonry marked *b*, *c*, *d*, and *k*, which he calls Romanesque, and claims to be of the same character as the foundations of French and English churches of the twelfth century. The foundation walls "*b*" and "*d*" lie under the transept, and "*c*" under the "Chapter-house," while "*k*" lies under the nave. (3) In the third period, from 1180 to (latest) 1215, he reckons the foundation-walls "*e*" and "*f*" under the inner and outer walls of the octagonal apse of 1180–88, as well as "*g*" under the wall of the aisle of the chancel—latest 1215. (4) Finally "*i*" under the Gothic wall, west of the transept, represents a fourth period from 1248.

This comfortable and cheap theory of Krefting has obtained the unreserved support of the "authorities" before mentioned.

After having asked unsuccessfully in English, French, and German libraries for books on medieval foundations, it occurred to us to inquire of the distinguished archæologist and historian of architecture, Francis Bond. We sent him an enlarged copy of Krefting's drawings, with accurate quotations of his writings and theory, without making any remarks whatsoever. Mr. Francis Bond was kind enough to answer in a letter dated September 11th, 1914.

"Dear Mr. Lund,

"I have no time to go into the matter fully, without personal inspection *in situ*. As to herring-bone work, it is usually not later than *c.* 1050, e.g. the west front of Le Mans

* In the plan (fig. 244) we have introduced the corresponding letters of the various foundation-walls.

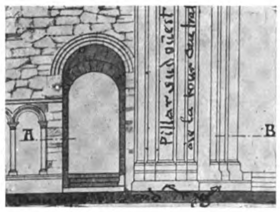


Fig. 248.—Cathedral of Nidaros. West wall of south transept and south-west pillar of tower.

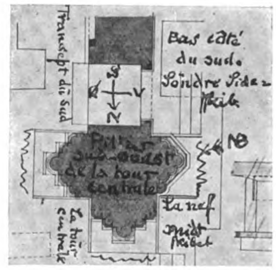


Fig. 249.—Cathedral of Nidaros. Plan of part with south-west pillar of tower.

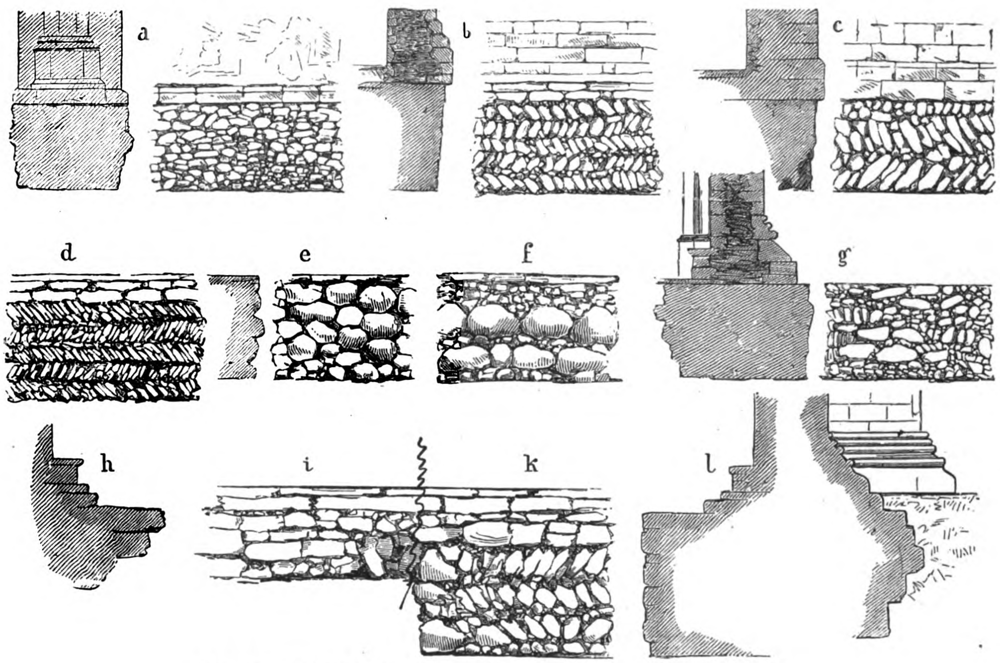


Fig. 250.—Cathedral of Nidaros. Foundation masonry, after O. Krefting.

Cathedral and the apse of Cérisy-le-Forêt are both of the first half of the eleventh century. But both methods of *appareil* just went on simultaneously, so that one can hardly have any certainty as to the chronological sequence of your foundations. I know no book on medieval foundations."

In the eleventh edition of *Encyclopædia Britannica* we find on herring-bone wall: "Herring-bone, a term in architecture, applied to alternate courses of bricks or stones, which are

laid diagonally with binding courses above and below: this is said to give a better bond to the wall, especially when the stone employed is stratified, such as Stonefieldstone, and too thin to be laid in horizontal courses. Although it is only occasionally found in modern buildings, it was a type of construction constantly employed in Roman, Byzantine, and Romanesque work, and in the latter is regarded as a test of a very early date. It is frequently found in the Byzantine walls in Asia Minor, and in Byzantine churches was employed decoratively to give variety to the surface. Sometimes the diagonal courses are reversed one above the other. Examples in France exist in the churches at Querqueville in Normandy and St. Christophe at Suèvres (Loire-et-Cher), both dating from the tenth century, and in England herring-bone masonry is found in the walls of castles, such as at Guildford, Colchester and Tamworth." (Vol. xiii, p. 391*b*.)

This article of *Encyclopædia Britannica*, which accounts mostly for masonry above the ground, holds, however, an important technical information to the effect that herring-bone work was used when the stone was too thin to be used in horizontal courses. A collective glance at the various walls in fig. 250 will convince us at once that in Nidaros the shape and the suitability of the stone have determined the type of masonry, and, if we divide these walls according to the suitability of the stone, we get a quite different and more instructive grouping than the one of Captain Krefting. It can be seen:

(1) That walls "*b*," "*c*," and "*d*" are "herring-bone walls" of one kind of material, small flat stones.
(2) That walls "*g*" and "*k*" are also herring-bone, but of several kinds of material, small flat stones set diagonally, between larger and thicker ones.
(3) That wall "*e*" is of one kind of material, larger round stones.
(4) That wall "*f*" consists of large round stones with small ones to fill in.
(5) That wall "*i*" consists of evenly laid rough stones in horizontal courses, and
(6) That wall "*a*" consists of one kind of material, small round stones.

Among the walls of his third period, which he sets between 1180 and latest 1215, Krefting considers, as mentioned, wall "*g*" to be the latest. But, judging from its character, this is a mixed herring-bone wall, and thus according to his remarkable theory, it should rather represent a transition "style" from his unmixed herring-bone wall dating from the twelfth century to walls "*e*" and "*f*," without herring-bone "style," which he dates from 1180–88. It is obvious that chronological conclusions are not arrived at so easily.

We know with certainty that wall "*i*" under the west front, and its side-tower, are the latest of all—that is, from 1248; walls "*f*" and "*e*" are older. But wall "*a*," which is, however, most related in every way to the last-named "*f*" and "*e*," and which is claimed by Krefting as being the oldest, is the only right supposition.

The foundation-wall under the aisle of the chancel, called "*g*," is an excellent example of how the size and shape of the stone has determined the type of wall. This appears still more clearly in fig. 251, which reproduces the photograph of the foundation under the east and south walls of the southern transept. The larger stones are laid horizontally, while the small flat ones are placed diagonally in herring-bone formation.

To an experienced archæologist the walls marked *b*, *c*, *d*, *g*, and *k* will appear generally to be the oldest. As the foundation-wall "*c*" is situated under the Chapter-house, *b*, *d*, *g*, and *k* were the foundation-walls of the cathedral.

Over the foundations "*a*" of the wall of the central aisle of the chancel, under, and between the west pillars of the central tower and two ells along the foundation-wall "*k*" under the arcade of the nave, there is still found, as we demonstrated, the mentioned chamfered coping. This shows that all these foundations, including "*b*," under the south wall of the southern transept, belong to the cathedral of Olaf Kyrre. Quite distinct from these

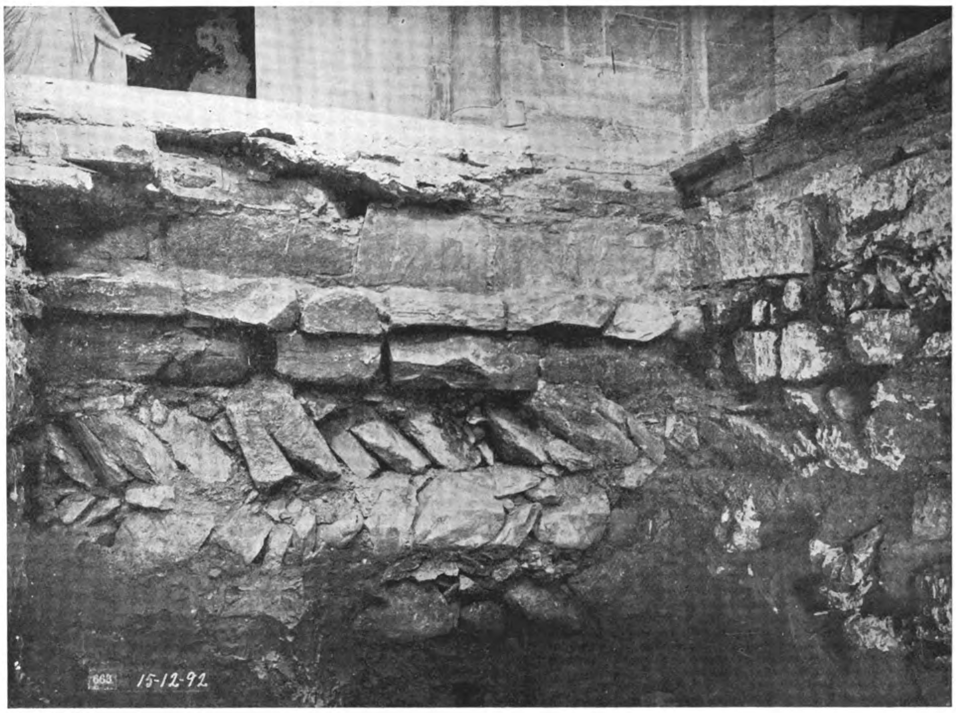


Fig. 251.—Cathedral of Nidaros. Foundation masonry under east and south walls of transept.

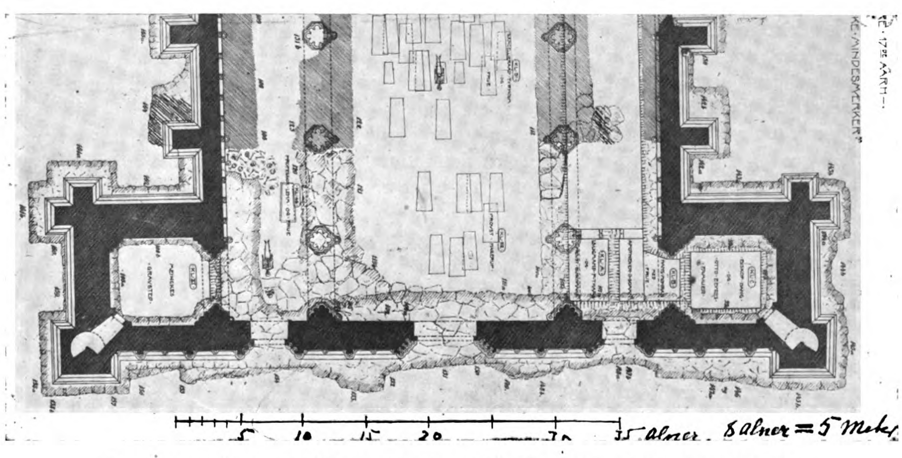


Fig. 252.—Cathedral of Nidaros. Remains of Olaf Kyrre's foundations farthest west.

are "*e*," "*f*," and "*i*," under the wall of the aisles of the nave; they are distinctly the work of Archbishop Eystein, 1161–88.

In fig. 252 we see how the foundations of the cathedral of Olaf Kyrre have been broken off and replaced by others in 1248, when the front was removed towards west; this was carried out by Archbishop Sigurd, as we shall see, according to the plan of Archbishop Eystein.

We have thus witnessed that the difference claimed to exist between the various foundation-walls is no difference of essentials, but that it is due to the different stones being used with great dexterity—that is to say, with complete technical regard of the material; when there is an essential difference it is just found in a wall, built at a later date, and consequently well known.

We can now begin to investigate how far there is, in the plan also, an architectonic relationship between these seemingly so different foundations; in other words, if they can have been built at the same period, according to the rules of religious architecture, such as we found these to be the determining agent in a cathedral of pilgrimage after the type of Chartres. From the lines of 45° with the horizontal plan, in figs. 245 and 246, it will be already noticed, as regards the elevation, that the side-walls of the transept are designed *ad quadratum*. From the diagonals drawn in fig. 244 it will be seen that this applies also to the plan of the transept, which contains two squares on each side of its transverse axis = the longitudinal axis of the church, L M. From this already appears the system, according to which the cathedral of Olaf Kyrre has been planned. It will be remembered that from the system of Notre-Dame in Paris we were able to show its original plan. We will now satisfy ourselves as regards Nidaros cathedral. It will be seen that on both sides of the longitudinal axis $\beta\ \delta$ of the transept, we have drawn the squares $\beta\ \delta\ \gamma\ a$ and $\beta\ \delta\ \epsilon\ \zeta$. In these squares, two squares are indicated by diagonals on the inner length of the transept. Between the sides $\gamma\ a$ and $\gamma^1\ a^1$ as well as $\epsilon^1\ \zeta^1$ and $\epsilon\ \zeta$, the cathedral obtains its west and east extremities. The excavated foundations, both in east and west, prove that this is not a mere theoretical supposition.

We shall first study the east extremity. As the plan shows, the foundation of the north side-wall of the retro-choir is determined and marked *a*. If we continue this choir as a rectangle with an apsidal termination—as indicated on the drawing—it will be seen that the summit of the semicircle of the apse falls precisely between the two squares $\gamma\, a$ and $\gamma^1 a^1$. Thanks to the remaining foundations, we can assert with certainty that the east half of the cathedral has been designed in a large square on the inner and outer length of the transept. The case is the same with the west half. As it can be noticed in fig. 252, the original foundations of the west front were taken away to make room for the new front; but sufficient remains, however, to enable us to determine with certainty, not only the length of the west front, but its width also. Similar to what we did for the plans of the cathedrals of Rouen, Lincoln, and Wells, we draw in fig. 244 diagonals according to the angle of 63° 26′ from the axes of the western pillars of the central tower. It will be seen, for instance, that the diagonal from the south-west pillar intersects the side $\epsilon\ \zeta$ of the square marked NB¹. The perpendicular drawn in strokes, from this point on the side of the square, coincides with the outer line of the remaining foundation of a projecting corner tower, marked NB; the width of the front is given thereby. The foundation-wall of the front itself must have stood between the sides $\epsilon^1 \zeta^1$ and $\epsilon\,\zeta$ of the square. We shall find later, in the parts of the cathedral still standing, an indirect proof that this has been the case. In this cursory analysis we have already been able to come to the conclusion that the cathedral of Olaf Kyrre has been designed inside two large squares on the length of the transept; by transferring now this result to a larger plan we shall understand the separate parts of the design and the reason

of their peculiarities. Here (fig. 253) the foundations of the cathedral of Olaf Kyrre are in horizontal hatchings, parallel to the transverse axis $\beta\,\delta$. The width of the aisle of the nave is marked on the west wall of the transept and shown by traces on the vault. The inner surface of the side-walls of the aisles is marked by a broken red line. The outer surface is drawn with black broken lines. It will be seen that these walls lie precisely inside the walls of the Gothic aisles, rebuilt later and still standing at present. The still existing walls of the aisles of the chancel are standing on the same place as the walls of Olaf Kyrre, and coincide therefore with these on the plan.

The starting-point of the following analysis is obviously indicated in the unanimous accounts of the sagas as to the King's first resting-place, and the well which sprang near it, and which, from the very first chapel built here, and during all the successive changes, have been the fixed points from which the cathedral has developed. The high altar on the grave is marked in the plan by a Maltese cross, and the holy well by two circles and the denomination "le puit" and the Norse word "Brunnr."

To begin with the high altar, the two diagonals of the half-square drawn in black and broken lines show that the west wall of the transept lies *ad quadratum* on the distance from this to the high altar towards east. If we now take the well, we can see that the broken red lines from here, running parallel to the longitudinal axis of the church, coincide with the inner surface of the wall of the south aisle of the nave. It will be seen, furthermore, that the thick red line marked NB, passing through the centre of the well, according to an angle of 45°, and from there to point A on the inner wall of the north aisle of the chancel, is deflected to point C in the south-west corner of the chancel, in order to end at point B in the corresponding corner of the transept. The whole chancel east of the transept lies, therefore, inside two squares, marked by the two diagonals of 45°, C A "le puit." The two diagonals of 45°, D, E, N, show that the nave contains two squares west of the transept.

Moreover, it will be noticed from the diagonals after 63° 26′, marked M, F, G, and G, H, L, that west and east of the transverse axis $\beta\ \delta$ of the plan through the central point G, there are two squares on the outer width of the nave; that is to say, four squares altogether in the whole length. Finally the reader will be able to see for himself. with the help of compasses, that the distance from the line $\zeta\ \epsilon$ of the front to the well corresponds to the sum of the side of three squares on the total width of the front, as this is determined by the diagonals of 63° 26′ from the west pillars of the central tower and as these diagonals coincide with the remaining foundations marked NB[2], outside the north aisle. It is not possible to form an idea of the termination of the plan towards west from this remaining small piece of foundation, but, when considered together with the already explained geometrical system of the cathedral, it suggests that the design was terminated by powerful towers strongly projecting on the sides.

Concerning the east termination, one can only make suppositions, as the foundation here also is disturbed by later work. The plan of the retro-choir is put in dotted lines, like the west front. The well where the pilgrims had controlled access, as mentioned, is indicated built in with the retro-choir, as it is still to-day.

We have indicated the position of the pillars of the central aisle, over the foundations of Olaf Kyrre, according to an intercolumniation of a little more than seven ells. This distance is dictated by the central aisle, by the remaining traces of the vault of the side-aisle; in the choir, where the distance is different, it is reckoned according to the traces of both Ostia Presbyterii found on the chamfered coping, the so-called "plinth."

In our comparison of the cathedral of the Orkneys with the cathedral of Nidaros, we said that Koll had also copied the abnormal and unnecessary space between the chapels of the transept and the aisles of the chancel—a condition which in Nidaros, as we said, had its

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reason in the fact that the architect did not have an entirely free site to build on, but that the original chapel had to be respected, this giving rise to various peculiarities. In order to better explain these, it will be preferable first to consider the Chapter-house. This building had been used in that capacity on account of its situation, but its entire plan shows that it was not intended for that purpose, as we shall demonstrate more fully later.

In France and in Germany, but rarely in England, it was usual to build the Chapter-house as a church, fitted with an altar. The number and position of the doors were determined according to whether the priests of the cathedral were regular or members of an order, *viz.* living together in a monastery according to monastic rules, or secular, *viz.* priests who could live together in a common house at pleasure, or each in his own house, without monastic rules. In the first case the Chapter-house had two entrances, one from the church and one from the cloister; in the other case, it had only one entrance, from the church.

But in Nidaros, where the priests of the cathedral were secular, we have not only the peculiarity of the Chapter-house having two porches opening both outside, one to south and one to west; but, judging from the still existing aumbries, it had altogether eight altars. Among these the north altar in the small transept indicated, is undisturbed. These two points prove that the Chapter-house must have been originally built as a church proper—independent of the cathedral.

The plan of the site in fig. 224 illustrates the position of this building in relation to the church of St. Mary of Harald Haardraade (1047–66). As we mentioned before, we know from the sagas that this church had a porch on the north—that is to say, towards the town, which it must have had naturally; and this was also the case with the cathedral, while the Chapter-house had its porches south and west. Herman M. Schirmer sees in this circumstance an evidence that it originally had an open situation towards south, and he adds that in the remaining Chapter-house we have still standing the chapel which was first built on the spot where the King was buried the first year after his death, being of opinion that the account which Snorre gives of the high altar standing on that very place is due to the priests adulterating tradition to advantage the cathedral.*

Although, from the architectural side, as we shall see, there is nothing against the supposition of Schirmer, still we must maintain decidedly with Professor Storm † that the reverence of the Middle Ages, and consequently its accuracy concerning holy places, does not agree with such carelessness, or with a clerical fraud, which would originally rest with Thorgaut and Olaf Kyrre.

It is a custom as improper as it is dangerous to do like our modern critics of historical manuscripts who correct historical documents when these do not agree with their opinions or their misunderstandings. Snorre Sturlason obtained his knowledge in Nidaros, 180 years after the chapel was built, and H. M. Schirmer obtained his 680 years after him. Consequently we have more confidence in Snorre, and we let him speak: "Thar á Melnum, sem Olafr Konungr hafdi i Jördu legit, kám upp fagr Brunnr ok féngu Menn Böt Meina sinna af thvi Vatni. Var thar veittr Umbünadr, ok hefir that Vatn verit jafnan sidan vandliga vardveitt. Kapella var fyrst ger, ok thar sett Altarit, sem verit hafdi Leidit Konungsins. En nú stendr i theim Stad Kristkirkja. Lét Eysteinn Erkebiskup thar setja Háaltarit i theim sama Stad, sem Leidit hafdi verit Konungsins, thá er hann reisti thetta hit mikla Musteri, er nu stendr; hafdi ok verit i theim Stad Altarit i fornu Kristkirkju." ‡ ("On the high sand-hill where King Olaf had been laid in the earth, there came up a fair well and men obtained cure from this water.

* H. M. Schirmer, *Kristkirken i Nidaros*, p. 70, etc.; also the article on "Skrudhuset" in the *Annuary of the Society of Antiquaries*, 1901, p. 60, etc.

† His statement is referred to by Schirmer in *Kristkirken i Nidaros*, p. 70.

‡ *Heimskringla*, ed. Unger, p. 508.

The well was built over and since then the water has been generally taken care of. A chapel was first made and the altar put there, where the King's resting-place [viz. grave] had been, but now Kristkirkju [Christ Church] stands on that place. Archbishop Eystein caused the high altar to be sat upon the place, which had been the King's resting-place; then he raised the great minster, which stands now; on this place the altar had also been in the old Christ Church.") It will be seen that the place of the King's grave is the fixed point in all three buildings which Snorre speaks of: the chapel, the cathedral of Olaf Kyrre, and the cathedral of Archbishop Eystein.

Without inquiring what is really meant by the word "Kapella," it has been concluded straight away that it meant a small chapel, a little oratory, where there could have been an altar, but where no regular Mass was said; that is to say, no celebration and no burial. In the meantime, Fritzner* has given in his *Ordbog over det gamle norske Sprog* (*Dictionary of the old Norwegian language*) an exhaustive interpretation of the meaning of the word in old Norwegian. In Norway and in Iceland "Kapella" is seen to have been used for "all churches that are not fylke churches, but are dependent of a fylke church. It seems then that "Kapella" is used by Snorre as the opposite of principal church, or in this special case as the opposite of the cathedral built afterwards. The "Kapella" of Magnus Olafson had stood forty years when it had to give way for the cathedral of Olaf Kyrre. But this lapse of time is long enough to have allowed the chapel, under the ever-increasing visits of pilgrims, to grow rich with many endowments and altars. In the ecclesiastical law instituted by St. Olaf it is said: "Nu eru Kirkjur allar adrar, er Gerd skall upphalda: Fiördungskirkjur ok Ottungskirkjur, Heradrskirkjur ok Högendiskirkjur, theim Kirkjum ollum skall upphalda ok eigi Tupt eyda." † ("Now here are the churches which are bound to be kept up: the churches of Fjerding and of Otting, the churches of Herad, and Annexe churches or private chapels; all these churches must be kept up and their site must not be laid waste.") A law corresponding accurately to it is contained in *Frostathingsretten*, vol. ii, chaps. vii and xiii. The latter fixes the fine imposed on the owner of a private chapel who does not get it rebuilt at once if damaged; as one can see these are the usual rules of the Roman Church for the protection of churches, spiritually and economically—that is to say, to protect institutions already founded.

It will thus be seen that when King Olaf Kyrre caused the older "chapel" to be pulled down he was bound, according to the Church law, not to allow it to disappear or to let its finances be absorbed in the cathedral newly founded by him, which it must also have been the interest of the Bishop to prevent. It must be taken into consideration that it would have caused a loss of income to the cathedral if there had been no house of worship on the holy place of pilgrimage during the pulling down of the original chapel, and while building the cathedral, at any rate, as regards the chancel. Therefore, according to the Church laws, and for economic reasons, the chancel must necessarily have been the first part of the cathedral to be built so as to replace the chapel before it was pulled down, and have its altars removed into the new place. In this way only can the existence of the many altars and of the two free exits in the "Chapter-house" be explained.

The old "Kapella" of Snorre has thus been an entirely independent church, just like the other parish churches of the town. Even long after the creation of the chapter in 1152 documents relating to the cathedral bear witness to the fact that an ecclesiastical fund was attached to the "Kapella," while this had no part itself in the enjoyment of legacies and funds of the cathedral, but, similar to other parish churches of the town, it had to defray

* Art. "Kapella," 2nd edition (Christiania, 1886–96).

† "Gulathingsret," chap. xii; *Norges gamle Love*, vol. i, p. 8 (Christiania, 1846).

its own expenses from its own legacies and moneys. When Archbishop Jorund (1287-1309) took away from the members of the chapter part of their incomes, they produced evidences of their rights. The claim was, that out of the provisions which *all Archbishops*, some Kings and Bishops had willed to Artidir, "or masses for their soul," there should be given each year to each "Stüka," viz. altar-chapel, besides money, some pounds of wax—"ütan till Mariustüka skall Artidakerti faázt sem einni hverri Kirkju i Bönum üt," viz. "but to the chapel of Mary the candles must be given like to any other church in the town." * It means, then, that all the deceased Archbishops, some Kings, and some Bishops in their wills had left to the cathedral and each of the parish churches in the town, as well as to the Lady Chapel, a fixed sum for salaries and expenses connected with saying masses for their soul on the anniversary of their death or several times during the year.

It is obvious that this "Mariustüka" does not mean an endowment in compensation of the church of St. Mary of Harald Haardraade pulled down by Archbishop Eystein, because this was rebuilt and with its endowments transferred to the Benedictine monastery in Elgesætr.† The district of the church of St. Mary obtained also its compensation in the new church of St. Mary, dating from that time. From this evidence it can be concluded with absolute certainty that the chapel rebuilt by Olaf Kyrre to replace the original one became the Chapter-house already as soon as the chapter was established, and that its altars with their funds have become in consequence tied to the cathedral as an independent institution. This is already apparent in the pointed expression in the evidence mentioned, "all the Archbishops," and in the fact that the members of the chapter had had that right from immemorial time—"alla æfi, svá at that er or Manna Minni." The correctness of our opinion is supported by a letter dating from 1229, written to King Hákon Magnusson, where it is announced that the members of the chapter were being deprived of their rights, those mentioned in the evidence produced before—which Archbishop Jon had given them, and which the three previous Kings had confirmed, that is to say, his brother Eirik, his father Magnus Lagaböter, and his grandfather Hákon Hákonson the Elder (1217–63). As no other Archbishop Jon had existed before that King than the first Archbishop from 1152–57, it is obvious that the chapel had existed before the chapter was instituted, in 1152. The Chapter-house, with its church plan and its many altars, cannot have been built, then, by any other than Olaf Kyrre. As the chief result of this inquiry, we can thus decide that this is the original chapel built on the King's grave, and which appears in the transactions of 1293 under the name of "Mariustüka," moreover that this chapel had been dedicated to the Virgin Mary, a fact which was not known before.

The documents mentioned support thus the statement of Snorre that the principal altar of the chapel stood on the same spot as the main altar of the later cathedral—in other words, that the chapel has been the nucleus from which the later construction was developed. As we know now that the word "Kapella" of old Norwegian terminology does not mean an oratory, having no priest, and as we have seen also that the chapel in compensation had many altars, and that, besides the mentioned evidence, we have knowledge of the chapel of the Virgin having stood, in relation to the cathedral, like the other parish churches of the town, we can conclude that the original chapel on the King's grave possessed these characteristics—as institution and as building. As a logical consequence of this conclusion, it

* *Diplomatarium Norwegicum III*, Nos. 32, 34, and 35, year 1293.

† On this point also, Professor *Johan Meyer* has given out his unimpeachable opinion. We leave it to the reader, if he wishes it, to plunge into the thoughts and meanings of Mr. Meyer, and to this end we refer him to his book *Domkirken i Trondhjem*, p. 26, etc. We do not consider ourselves personally obliged to sacrifice time to it, as his given "facts" are quite as unreliable, his reasoning as inexperienced, and his arguments of the same airy texture as those which we have previously been obliged to deal with.

follows that we have in this the original building, the chapel of Magnus Olafson, built over the King's grave; this is the little parish church which, in his complete ignorance of medieval conditions, Höyen has made into the "cathedral" of Olaf Kyrre. Later, we shall see that they tried to preserve this original chapel from motives of reverence. It is from this, that the plan of the cathedral has developed and obtained the characteristics that are abnormal. The unusual narrowness of the aisles is, then, directly caused by the chosen nucleus, because this had to be fitted into the design of a pilgrim cathedral.

It is the original length of the chapel, and not its width, which has caused the narrow aisles. If, besides the two fixed points—the altar and the well—the architect had only been limited by the east width of the original chapel, but free otherwise, the plan would have been different. He would then have been able to give the aisles the regular width. We have indicated this in the plan in two ways, by the two semicircles, the larger having as radius the axial distance between the east corners of the chapel, and the smallest, the axial distance between the west corners. The total width of the chancel is then given by the diameters of these semicircles. With the systematic use of the two squares system, with two squares on this width, it is obvious that the west wall of the chancel, viz. the east pillars of the central tower, would have reached a good deal farther west, as they would have come outside the site of the chapel. The red diagonals drawn at an angle of 45°, from the well to point A and deflected from here to point C, show, however, that the chancel is carefully designed inside a rectangle composed of two squares between the given limit, the well in the east and the front wall of the chapel in the west. Inside the frame thus given the architect has been able to retain the original chapel as the nucleus of the plan of the cathedral. The chancel obtained its abnormal traits from this, its great obliqueness, and the aisles their narrowness.

The architect has developed his plan towards west with the same perfect mastery of the rules of religious architecture. The width of the transept is determined by the original width of the chapel, by means of the square of the central tower, and the length of the transept—that is, the position of the gables, by the intersecting points of the transverse axis β δ of the central tower with the diagonals of the half-square from the given point L at the summit of the semicircle of the apse. From this he has obtained the square β, L, δ, on the length of the transept, east of the transverse axis and also the equally large square δ, M, β, west of it. Here, also, he has been limited by the width of the original chapel, and his task was to design the nave inside two squares—as marked by the diagonals D, E, N. For this reason, the aisles of the nave are abnormally narrow. He could not have had a normal width without coming outside the side ζ ϵ of the square, although the site would have allowed it.

Finally, we must find the reason of the abnormally open space between the aisles of the chancel and the chapels of the transept. As it is proved from our analysis, the transept is not, as regarded until now, unusually long, but quite according to the rule. Nor is the placing of the chapels in any way abnormal; they have been determined by the rules governing the normal width of the aisles, as shown by the semicircles explained above; and the depth towards east represents accurately the normal width of an aisle, if this had been planned, like in Durham, where the aisles also are narrow, but where the open space between the chapel and the aisle of the chancel has been filled up when carrying out the aisle. Therefore what is abnormal in Nidaros is not the length of the transept, nor the placing of the chapels, but it is the open space which is produced by the abnormally narrow aisles. It is this trait which has been copied slavishly, without any reasonable cause, in the cathedral of the Orkneys.

We must remark, finally, that through his conscious arrangement, the architect has succeeded in getting the length of the cathedral into four squares on the exterior width of the nave, two squares on each side of the transverse axis β δ of the plan; it will be noticed that

in this manner he has succeeded in carrying out, in the large pilgrim cathedral, the rules of the Chartres type within the given conditions.

In support of our opinion that the original chapel on the King's grave was preserved reverently as nucleus of the plan of the cathedral, and also to explain the original distribution of the chancel, to which we promised above to come back, we shall now attract attention on what appears from the accounts of the sagas concerning royal burials and the excavations which took place during the restoring; that the chancel of the original chapel has *never* been really a chancel, viz. the presbytery, in the cathedral, but that it *has been kept as a separate chapel on the original resting-place of the King*.

In addition to the saint "on the High Altar," we have complete knowledge of the following royal burials:

(1) Magnus the Good (died 1047), "fyrir utan Kör, enn nu i Kör fyrir framan Rumi Erkebiskups." ("Buried outside the chancel, and now buried in the chancel in front of the episcopal throne.")
(2) Olaf Kyrre (died 1093) buried "in the church."
(3) Haakon Magnusson (died 1095), buried "in the church."
(4) Olaf Magnusson (died 1115), buried "in the church."
(5) Eystein Magnusson (died 1122), buried "in the church."
(6) Haakon Sigurd Herdabreidi (died 1162); he was laid in the "stone wall south of the chancel" by King Sverre, after the Gothic rebuilding of Archbishop Eystein.*
(7) Guthorm Sigurdsson (died 1204), buried "in the church."
(8) Inge Baardsson (died 1217), buried in "the stone wall on the south side of the chancel."
(9) Cecilia (died about 1185), King Sverre's sister and mother of King Inge Baardsson, buried on the north side of the chancel.
(10) Baard Guthormsson (died 1194) her husband, buried on the north side of the chancel.

In addition, twenty-seven of the thirty-one Bishops and Archbishops of the cathedral, from Olaf Kyrre to the Reformation, have been buried in the chancel. Four of these only have been found, but not identified, however.

As shown in the plan, the chancel of the original chapel—that is, the retro-choir of Olaf Kyrre's cathedral, used later as feretorium—was evidently quite small. But, if the Höyen-Munch theory were correct, one would have a right to think that some of the first five named Kings at least, especially the founder of the cathedral, Olaf Kyrre, would have been buried in the chancel; the excavations carried out, however, have proved that in the part claimed to be the original chancel *no trace whatever of medieval graves has been found, not a single one*.

This fact proves irrefutably that the original chancel of the chapel was kept purely as a chapel on the original grave of the King, and had been included in the new plan of the cathedral as *Feretorium*, where no other burials could take place so as not to inconvenience the pilgrims who, on vigil nights, were standing round the shrine. The Middle Ages have not thought any of the followers of St. Olaf on the throne worthy to be buried near his resting-place. He lies alone in his chapel: "thar stendr hátt i Höfudkirkju Olafs Skrin yfir Altari," as it is said in *Noregs Konungatal*, written about 1190.

On account of this the chancel of the chapel was used already from the beginning as feretorium, and probably the high altar stood there, but it was only used for the great church ceremonies; therefore the presbytery must have been lower down than Gradus Presbyterii. The narratives preserved, which give accounts of the miracles occurring in the church from its

* Snorre, p. 786.

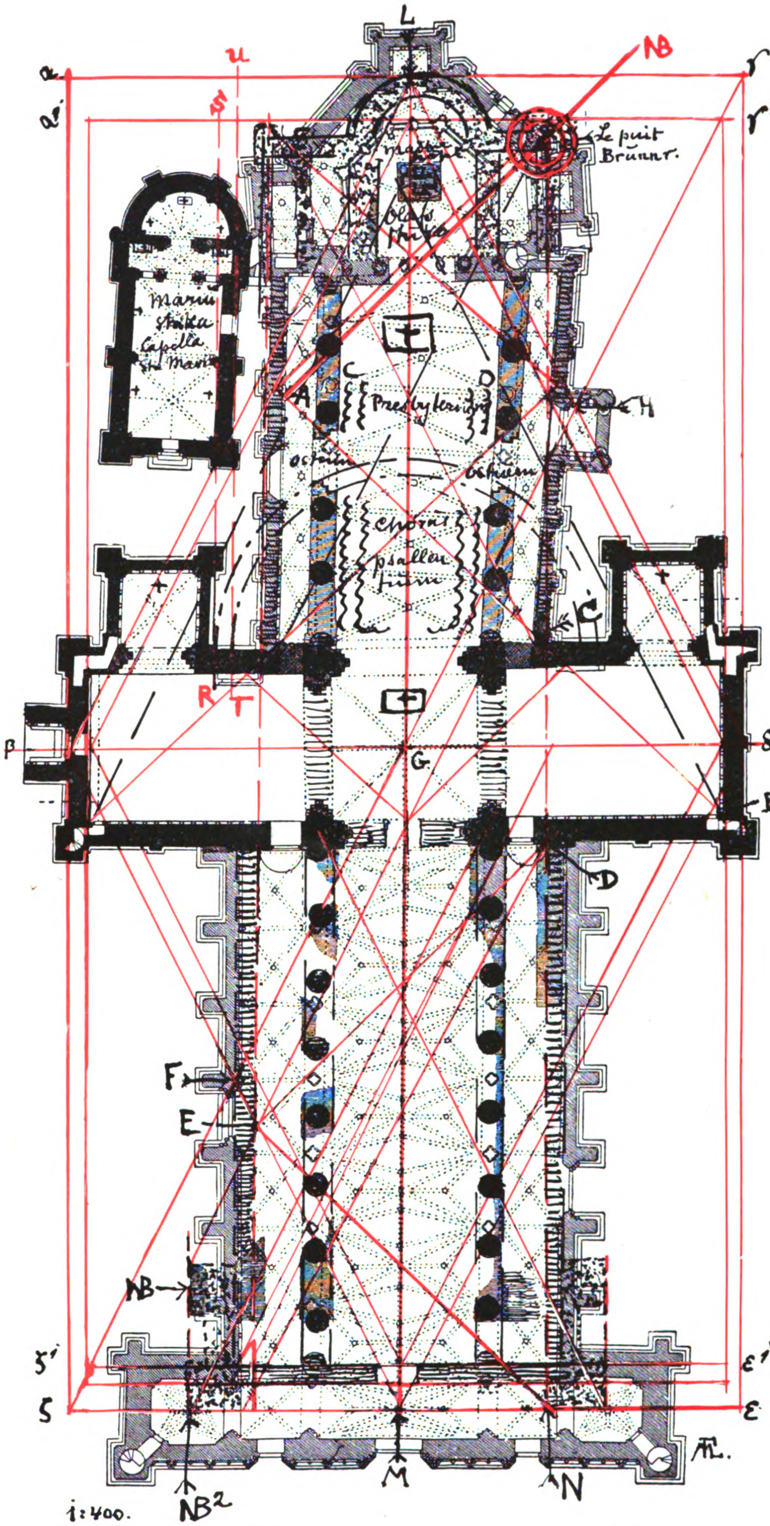


Fig. 253.—Cathedral of Nidaros. Analysis of plan of Olaf Kyrre's cathedral.

first existence, are sufficient proof to all who can read an ancient document that this must have been the case. We shall only name the collection of legends in *Passio et Miracula Beati Olaui*. Archbishop Eystein mentions there, personally, some events taking place in the cathedral until his flight in 1180. We are told in one tale after another how the crippled pilgrims and their relatives stood day and night near the relics of the saint. We cannot quote every case. We can only give a single one, which seems to have happened towards about 1159–80, or possibly during the time when Eystein was member of the chapter. He relates the story of a boy from Rendalen who was lame and quite deficient mentally ("Simul namque membrorum omnium paralisi percussus est, et mentis inualitudine"). After many vain attempts to regain health, his parents and friends went with the apparently soulless body to the church of St. Olaf, where the cripple and his followers, who attended him continually, stood in the church from Michaelmas to St. Olaf's Day, day and night in prayer and entreaties, only leaving the church in turn, when the needs of the body had to be attended to.*

In this document we have the undisputed evidence of Archbishop Eystein himself for the fact that before 1180—that is, before the time when, according to the received but wrong theories, Archbishop Eystein is supposed to have rebuilt the chancel—there was a feretorium in the cathedral. It is unnecessary to explain that there would never have been enough room for such a long or even short stay near the saint's shrine if this had not stood in his own chapel—outside the presbytery. This is supported also by the documents of the cathedral itself, when, in the enumeration of the various altars, "Olafsstuka" is mentioned, viz. the chapel of St. Olaf. This chapel seems to have had its own treasurer to look after its valuables; it must be this chapel which is designated under the name of "öfri Kirkja"—the upper church—when, in the document of 1293 quoted before, mention is made of the various offices which, according to ancient custom, could not be disposed of without the advice and consent of the members of the chapter.† A mention is equally made in the same document of 1293 of the right of the members of the chapter, from the very first Archbishop, to partake in the appointment of the officials of the "stukur" of the cathedral—that is to say, of the various chapels projecting from the cathedral.

This expression only is sufficient to show convincingly that the cathedral of Olaf Kyrre was a quite otherwise complicated building than the parish church of Höyen, in the plan of which such projecting chapels cannot be reconciled.

If we compare now the result which we have obtained above in our analysis of the supposed grave of Magnus the Good with the information imparted by the legends in *Passio et Miracula Beati Olaui*, we can conclude once for all that the presbytery of Olaf Kyrre's cathedral, neither before nor after the establishment of the Archbishop's see, could have been in Olafsstuka. We have seen that the presbytery, the "Kör" of *Flateyjarbök*, was removed towards west. It must then have stood originally between the steps of the chancel—Gradus Presbyterii—leading up to the retro-choir, viz. Olafsstuka, or öfri Kirkja, and between the two Ostia Presbyterii, of which, as shown in the plan (fig. 253), there are marks found in the supposed "socle."

As we have already emphasised before, the cathedral had from its earliest time a college

* "Hic cum per multa viarum discrimina a parentibus et amicis, quasi corpus exanime, ad ecclesiam beati Olaui duceretur, laudabili deuocione et mira constancia eorum, qui eum duxerant, a festo beati Michaelis, usque ad perceptam gratiam" (viz. on the day of St. Olaf when the miracle happened), "nisi cum necessitatibus corporis obsequerentur, die noctuque in ecclesia servatus est" (*Passio et Miracula Beati Olaui*, by F. Metcalfe, p. 106. Oxford, 1881). As it is difficult to imagine that the sick could live in the church, then not heated, during the whole of the severe winter, from September 29 to July 29 of the following year, "a festo" in the MS. must mean "revelatione" beati Michaelis, which would make the stay in the church last from May 8 till July 29, which the climatic condition of Nidaros does not make impossible.

† *Dipl. Norv.*, vol. iii, Nos. 34 and 35.

council demanding a large presbytery; we have an evidence of this in the mentioned collection of Latin legends in a narrative telling of the conditions in King Eystein Magnusson's time (1103–22) and where the "Seniores" * of the cathedral are mentioned.

It is evident that the way to "Olafsstuka" was arranged such that it went outside the presbytery, below Gradus Presbyterii, but obviously it is difficult now to indicate this passage. The remaining door at the east extremity of the north aisle of the chancel indicates that there was once an extension of this for the use of the pilgrims; there has been likewise a corresponding issue from the south aisle. Another account in the same collection of narratives previous to 1150 witnesses to the existence of these aisles in the original cathedral. There we hear of a girl who had been a cripple from childhood, with distorted feet; on St. Olaf's Day some holy priests carried her in an Amminiculum (a carrying-chair, something like a high baby-chair), and took her "ante ostium chori," where she was cured when the holy relics were lifted to be carried in the procession.† The shrine was not taken down, as Bishop Bang and Professor Gustav Storm have arbitrarily translated.‡ On account of the arrangement of the chancel as demonstrated above, the shrine was carried from the retro-choir (öfro Kirkju) or feretory, down to the choir, viz. the presbytery, and put on a trestle standing on the floor and then lifted (*leuaretur*); this verb could not have been used if this scene had been enacted in the retro-choir—Olafsstuka—where the shrine stood upon its pedestal higher than a man, at the back of the altar, in which case the Latin expression for "been taken down" would surely have been used. Neither did the girl lie on the floor, as Bishop Bang translates it, but, according to the custom of the time, she was carried through the women's side on the north aisle of the chancel and sat in her chair, ante ostium chori, viz. ante ostium presbyterii, because "chorus" in this case, as in *Flateyjarbök*, means undoubtedly the presbytery, and not chorus psallentium, the lower choir, where the shrine obviously had not and could not have been taken.

The reader will find the result of this far-reaching examination in the plan (fig. 253), where we have shown the distribution of the chancel in the original cathedral of Olaf Kyrre until the establishment of the Archbishop's see.

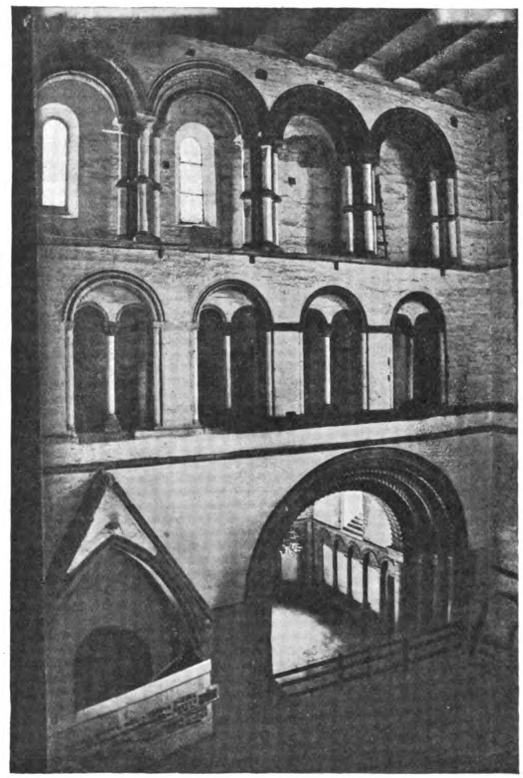


Fig. 254.—Cathedral of Nidaros. East wall of south transept, photograph.

* Page 102.

† *Passio et Miracula Beati Olaui*, p. 95: "Hec piorum amminiculo ante ostium chori deuecta est, et cum sanctissimum corpus ad processionem leuaretur," etc.

‡ *Historisk Tidsskrift*, third column, vol. i, p. 274 (Kristiania, 1890).

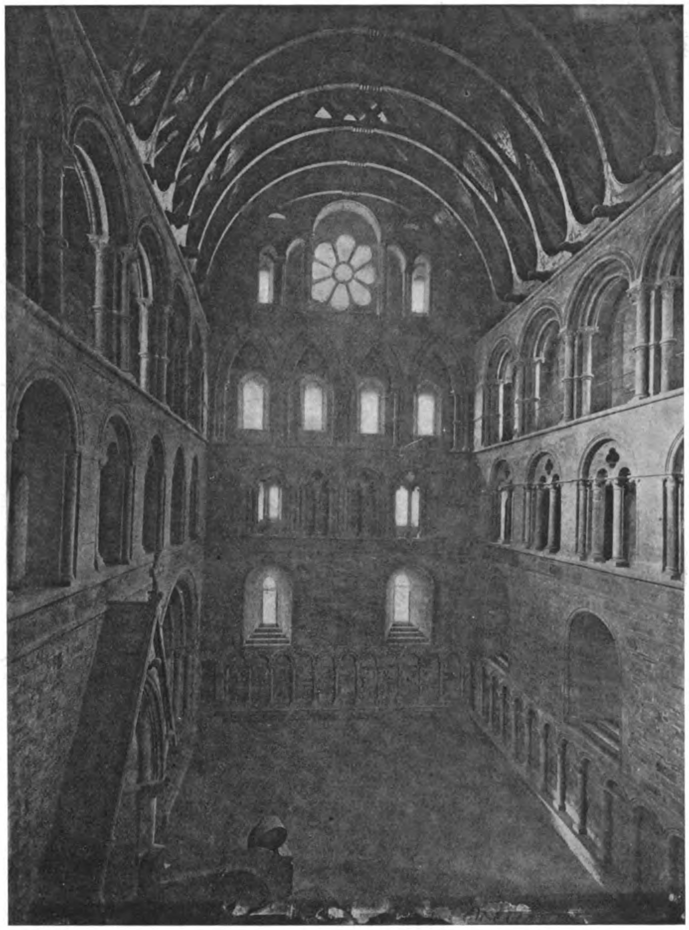


Fig. 255.—Cathedral of Nidaros. South transept restored.

In the chancel of the original chapel, on the grave, the high altar on that spot, is marked with the words "maître-autel," and the chapel itself with the name "Olafsstuka," the part reserved for the Bishop and his Seniores "presbyterium," and the lower choir "chorus psallentium." We can say that in Olaf Kyrre's cathedral—as a result of architectural conditions and of the existence of older buildings—there was, from its first foundation, a distribution which, as far as we know, occurs later only in France and in England.

The two lower stories of the transept of this cathedral stand for the most part in their original condition.

We reproduce here, as a supplement of the measured drawing of fig. 245, a photograph of the south transept previous to the restoring. We see here the triforium quite unchanged

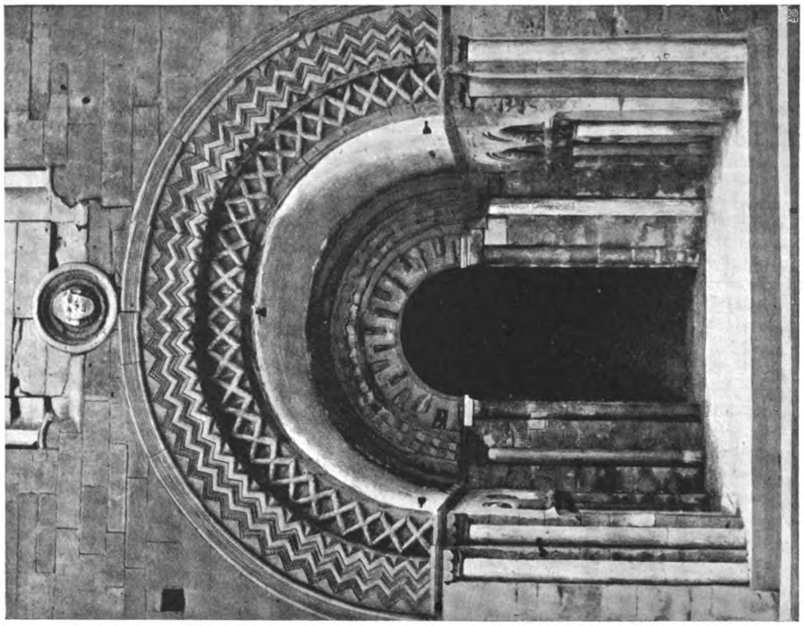


Fig. 257.—Cathedral of Nidaros. Door of north transept with porch.

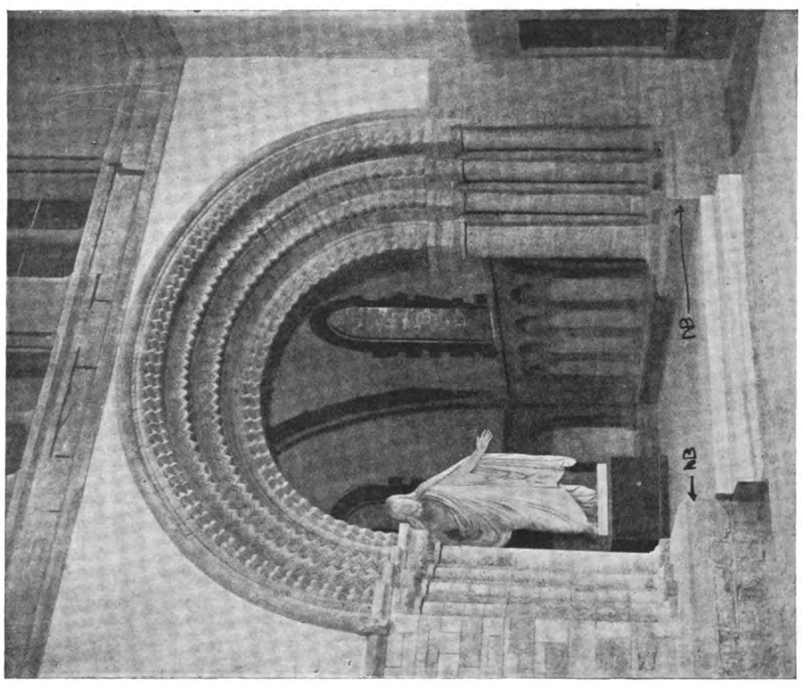


Fig. 256.—Cathedral of Nidaros. Chapel in south transept.

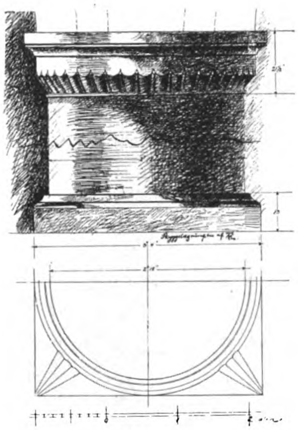


Fig. 258a

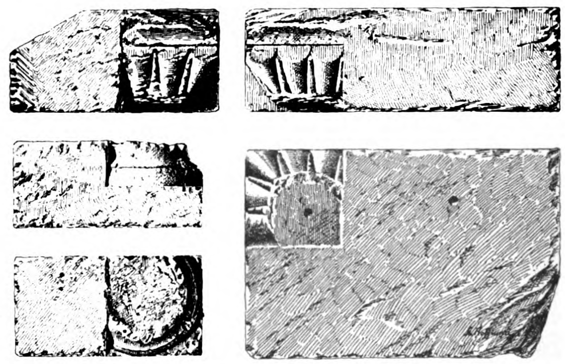


Fig. 259.—Cathedral of Nidaros. Corner capital, with base from the cathedral of Olaf Kyrre.

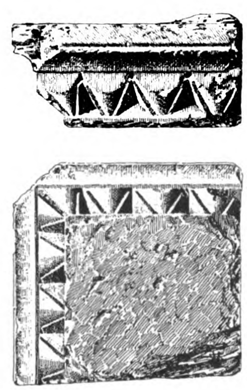


Fig. 260.—Cathedral of Nidaros. Rectangular corner capital from the cathedral of Olaf Kyrre.

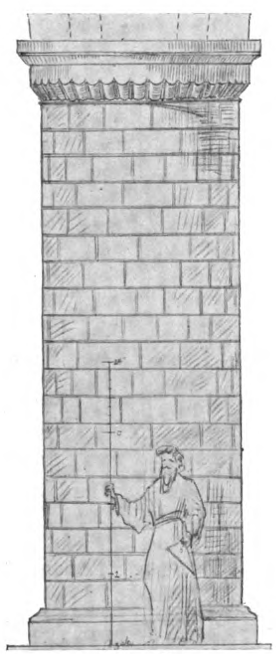


Fig. 258b

Figs. 258 *a* and *b*.—Cathedral of Nidaros. Pillar from the cathedral of Olaf Kyrre.

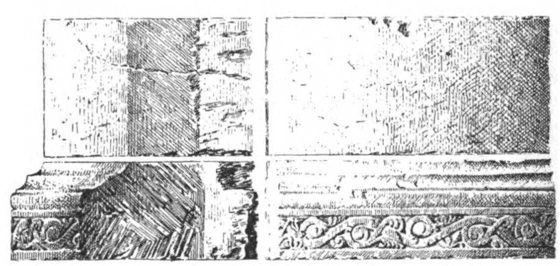


Fig. 261.—Cathedral of Nidaros. Piece of a smaller column, from the cathedral of Olaf Kyrre.

Fig. 262

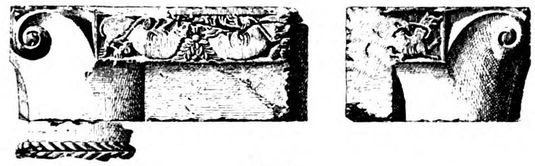


Fig. 264

Fig. 265

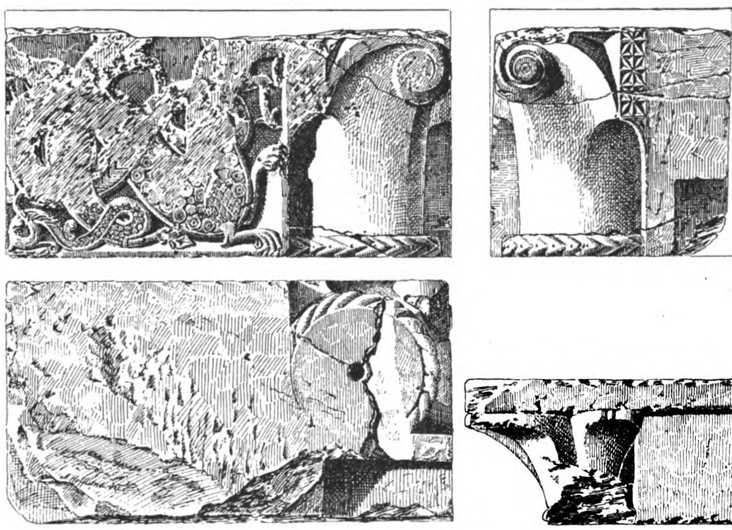


Fig. 263

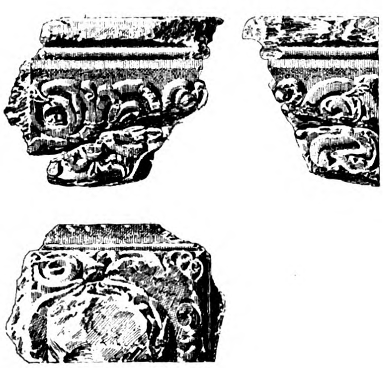


Fig. 267

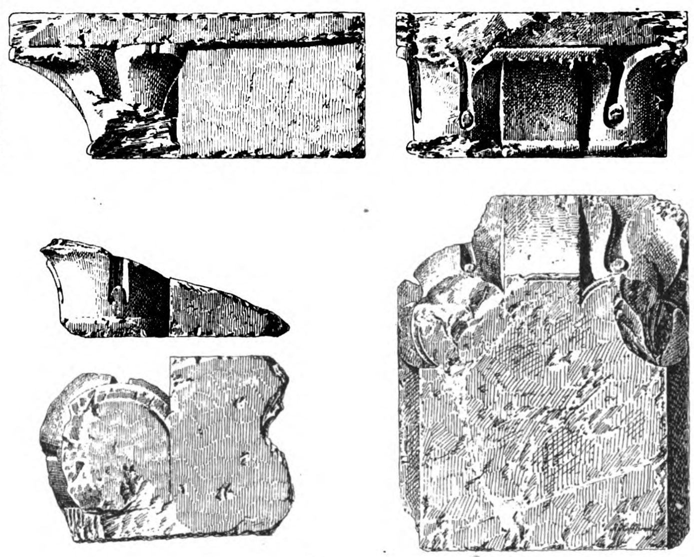


Fig. 266

Figs. 262–267.—Cathedral of Nidaros. Capitals from the cathedral of Olaf Kyrre.

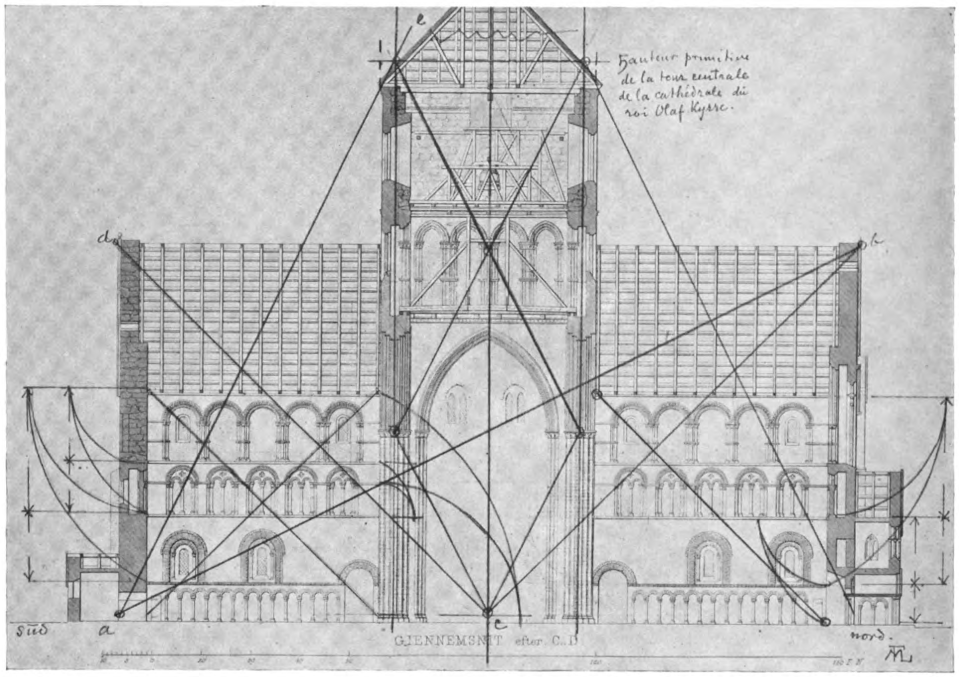


Fig. 268.—Cathedral of Nidaros. Longitudinal section of transept.

since the time of Olaf Kyrre (fig. 254). Furthermore, we reproduce, in fig. 255, the photograph of the interior of the south transept under its restored aspect. The gable-wall, including the triforium, which had been destroyed in the seventeenth century on account of the collapse of the spire, was entirely newly built by the architect Christie. The timber roof construction is new, copied from other medieval timber constructions in other churches of Thröndelagen. The clerestory and triforium of the west wall were modernised by Archbishop Eystein. When carrying out this work, stones from the cathedral of Olaf Kyrre were used. Some of these, which have great historical value, are reproduced lower. It will be seen that, except for the clerestory, the transept has kept its very ancient character.

We reproduce equally in fig. 256 a photograph of the chapel of the south transept. Its vault is carried by large cross-ribs, meeting together at the top without any marked key-stone—as it was always the case in old cross-rib vaulting.

The ribs and the archivault over the entrance are richly ornamented with the zigzag pattern. Before the restoration the floor of the transept was on the same level as the floor of the chapel, but it was lowered to be on a level with the floor of the central tower, which had sunk on account of its weight. It will be noticed from the arrows marked NB that the chamfered "socle" is also to be found here.

Finally, in the northern porch (fig. 257), with its "Forhall," "Anddyre" (as they called it in the old days), we get a good impression of the ornamental character of Olaf Kyrre's cathedral.

Except for a part of the east gable of the chancel, no standing remains are left of the chancel and nave of Olaf Kyrre; but, among the collection of loose stones from this cathedral, there are architectural fragments which, judging from their character and from the spot on which they were found, must have belonged undoubtedly to the part which has disappeared.

Figs. 258 *a* and *b* reproduce the plinth and capital, as well as the reconstruction of the mighty pillars of the arcade of the nave. The diameter of the pillar is 2 ells 18 inches, and the pillar above the plinth had, according to the system, a height of 8 ells to the upper edge of the capital. Low down on the plan will be noticed the filling ornament in the corner, the "griffe," in a simple form, characteristic of the eleventh century; related to this is the corner ornament on the plinth of the contemporary cathedral of Stavanger (see figs. 144*b* and 144*c*).

Fig. 259 reproduces a corner capital with base, and fig. 260 a rectangular corner capital, or a springer stone for a small arch belonging to a secondary part of the construction, as the height from the lower edge of the capital to the upper edge of the abacus is only six inches. It will be noticed that the abacus on the two small capitals in figs. 259 and 260 has the same profile as the abaci on the capitals of the large pillars.

Fig. 261 shows part of a small column of 20 inches diameter with ornamented plinth. Figs. 262–7 reproduce capitals, all strong and firm in line. The corner capital of fig. 267 has been richly ornamented. The shapes in figs. 262 and 266 are particularly interesting. The latter is found again in the chapter-house. But all these forms of capitals point to relations with Normandy, where the same can be seen in churches from the eleventh century, especially in the district of Calvados, round Caen; but in Norway. on account of the plastic steatite and of the ancient wood-carving art, these shapes possess a firmer line, and the carving is finer and sharper than in Normandy.

Our task does not allow us to give a demonstration of the interesting resemblance between the oldest decorative art of Normandy and of the country of its origin. We must proceed to our main object: the demonstration of the geometrical system of Olaf Kyrre's cathedral.

We have already done it as regards the plan and occasionally touched upon it for the elevation. Fig. 268 reproduces the longitudinal section of the transept from the measured drawing done previous to the restoration by Herm. E. Schirmer, senior, Plate II in the previously mentioned book of plates in collaboration with P. A. Munch. The analysis carried out does not require further comment; in connection with figs. 245 and 246, which reproduce the south transept, we have shown here, in the longitudinal section, by a line of 45°, that the height of the wall in the north transept is also determined *ad quadratum* on the inner length. It appears from the two diagonals *c d* and *c b*, according to an angle of 45°, coming from the intersecting point *c*, between the line of the plinth and the axis of the central tower, that the height of the gables and of the ridge are determined *ad quadratum* on the half-length of the transept, from which it follows that the longitudinal section is designed inside two squares, one on each side of the central axis. There is an interesting play between the height of the tower and the length, because the former is determined by three squares on the axial distance between the walls of the tower, and by two squares, for instance, on the length of the south transept, from the outer wall to the axis of the wall of the tower, as shown by diagonal *a e*. It is clear that this gives also the height of the original Norman pyramid of the tower.

The constructions of the sectio aurea carried out here do not need any explanation; they show indisputably that this mode of proportioning has been used for the distribution of the several parts of the building.

* * *

We mentioned above that it was on Nidaros cathedral that we discovered the geometrical system of religious architecture used in the past.

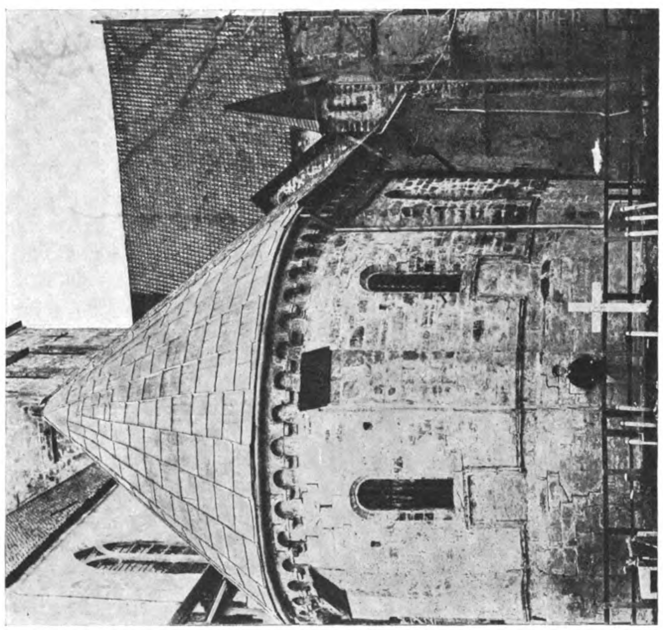


Fig. 269.—Cathedral of Nidaros. Chapter-house seen from N.E.

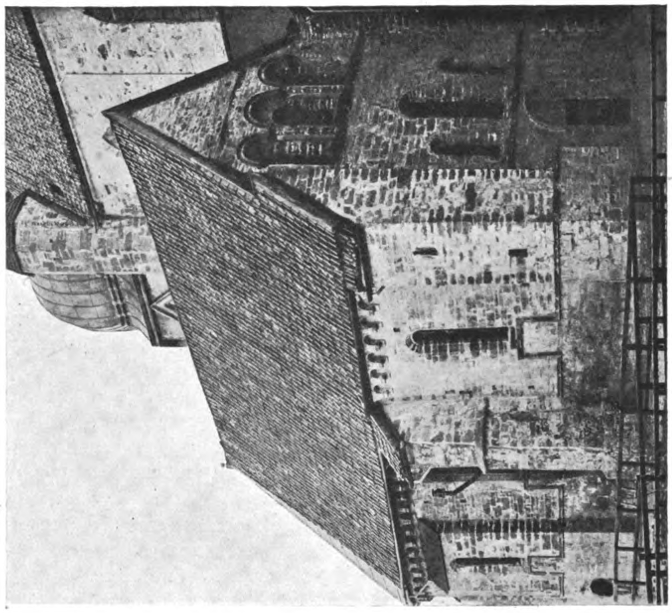


Fig. 270.—Cathedral of Nidaros. Chapter-house seen from N.W.

Just as it revealed itself in the plan, so it can be seen in the elevation of the transept, clear and simple, and so indisputable that, if nothing else than this piece of architecture remained from the temples of the past, no one with understanding could deny that it had been used, once the conditions are known.

This result, which has been already given in our introductory analysis of Olaf Kyrre's cathedral, will be further supported and enlarged upon in the analysis of the later rebuilding of the cathedral, in Gothic style, by Archbishop Eystein.*

Before we turn to the Gothic cathedral we must examine closer the *chapter-house*, the ancient appearance of which can be seen in two photographs taken previous to the restoration (fig. 269 from north-east, and fig. 270 from north-west).

Fig. 271 reproduces the plan, the transverse section, and the longitudinal section of the chapter-house, after the measured drawing of Herm. E. Schirmer previous to his restoration in 1869. The severely systematic design of the building is seen so clearly and so convincingly as soon as we glance at the drawings that we think it sufficient to indicate only the principal features.

The circle and the diagonal A B, according to 45°, drawn in the transverse section, show that the interior is drawn *ad quadratum*. Where the same line, on its way, intersects the line of the outer wall we get the level of the cornice. The square standing on its point shows that the height of the capital is determined *ad quadratum* on the half-width of the interior. The same result is arrived at in the analysis of the longitudinal section, which shows that the length of the interior is designed inside three squares on the interior width. Therefore all three plans are designed *ad quadratum*. The windows of the south walls, which were filled in long ago, are drawn in accurately, as they are in the north wall. It will be seen that their placing is not arbitrary, but accurately and geometrically determined according to the same rule as the principal lines of the interior.

As regards the exterior (in the absence of a drawing we use the longitudinal section) the diagonal A B from the line of the exterior plinth to point B, and the diagonal of the half-square from there to point C on the plinth show that the level of the ridge is determined *ad quadratum* on the length of the nave—that is to say, precisely in the same way as in figs. 127, 130, and 132. The diagonal E D in strokes, and the fully drawn parallel show that the height of the summit of the gable is not arbitrarily chosen either: see diagonal C D of the half-square in the transverse section.

It has been thought that the ring, or amulet-like string found just in the middle of the pilasters which carry the ribs of the vault, and the pointed shape of the arches—represented a transition style from the twelfth century. This is quite a misunderstanding. The so-called "bague," or annulet, has a function, as we know, having made its appearance in the twelfth century, when they began to use thin columns made of one stone ("monoliths"); sometimes they dared not use them to their full height, but cut them in the middle, or, according to a harmonious division, often the sectio aurea, and in the line of division they introduced a stone which was bonded either to the nucleus of the cluster of columns or to the wall, in which manner the thin column was attached. In plan, this stone was also made in the circular shape of the

* An English architect, *Mr. W. D. Caröe*, of London, in a letter to the architect and leader of the work of restoration of the cathedral, Mr. Nordhagen (who produced this letter to support his campaign against our work), has thought fit to ridicule our work before it was developed and published. He said, among other things, that he could show at least ten systems in each church of Christendom. The two other "experts," employed by Mr. Nordhagen, the Danish architect and professor, *Mr. Martin Nyrop*, and the Swedish architect, *Mr. Ragnar Ostberg*, after being consulted, have also sent contributions to the daily press ridiculing the "system." These gentlemen have only succeeded in proving that they do not understand it, as *they* speak of the introduction of "system lines," valueless in themselves, in buildings already carried out, while *our* work is on the discovered *common system*, unknown until now, and which was used to design *all* religious architecture, in classic and medieval times.

18

column; it was beautifully moulded and sometimes also ornamented. On account of its likeness to a ring which surrounds the shaft of a column, or to the hoop of a mast (the hoop of wood which holds the fore-and-aft sail to the mast), the French have given it the name of "bague," and the English the name of "annulet." But in the chapter-house of Nidaros the supposed annulet has no such function, and it is not an annulet. Figs. 269 and 270 show that outside, on a height with the springer of the arch of the west door, there is a string-course running round the whole building and draws up under the window-sill. It is taken up again inside, and, as it would be illogical to break the string-course, it runs across the columns, which, it must be noticed, do not consist of one stone only, but, like the walls at the back, are built of small stones. This string-course, which is in no way related to an annulet, has only the appearance of it.

As regards the arches of the vault, their resemblance to the pointed arch is only apparent also. The vault has the character of the very oldest rib-vaulting. The webs are exceptionally thick, and they are carried by big diagonal ribs, running to the summit without any keystone. Neither is the shape of the arch "ogival," or pointed, because it has an "apex," as it is curiously called—that is to say, that its lines run outside or above the line of the arch formed by the semicircle. Like the oldest rib-vaults, this one is also very much domed—which means that it has the umbrella shape of the cupola, because the arch of the cross-ribs or diagonal ribs has been drawn in the arc of a semicircle, as shown in the plan and transverse section.

The radius of the diagonal ribs is marked in the plan with the large letter R. The inner semicircle represents thus, as shown by the transfer to the transverse section, the intrados of the diagonal ribs and the outer semicircle represents their extrados. Now, to make room for the windows, the wall-arches, and with them also the transverse arches, are lifted somewhat in the following manner: in each bay of the vault, both in the longitudinal section and in the plan, a circle is drawn with a diameter = the distance between the axis of the pilasters marked 1 and 2. In the plan and transverse section the half-diameter is marked R. With the radius of this circle, marked R^1, the wall-arches and the transverse arches are drawn in the plan, the longitudinal and the transverse sections, the centres are only moved a little to the side of the centre of the fundamental circle. Obviously, room for the windows could have been found by lifting a circular arch of the vault upwards, and placing its centre as much higher as the centres of the broken arch of the semicircle were moved to the side, namely, to point S—marked in the plan and in the longitudinal section; but the stability of the wall would have been doubly endangered thereby, firstly by the semicircular shape of the transverse arches and secondly because the lateral thrust of these would have been lifted higher up. But, on the other hand, by removing the centre of the semicircle a little to the side, the wise constructor has obtained a sharper form of rib upon which he could rest his heavy webs. This kind of arch appears in the early high vault of the nave in Durham, which is no "ogival," or pointed arch vault, nor has it anything to do with "Gothic" style, but is entirely Romanesque work, carried out by self-reliant men experienced in construction.

The case is the same with the vault of the small "transept." This obtains its pointed shape from the height, which had to correspond to the height of the nave and of the chancel, and also from the technical reason that in this narrow space it was difficult to build up the wooden centring of the arches, whereas in this steep arch it was possible to put one stone upon another without support.

Therefore in its construction the chapter-house is quite Norman in every way. It is possible that there may have been attempts made to modernise it here and there, when the chapel was taken as chapter-house; but it is certain, in any case, that its ancient and Byzantine character is preserved.

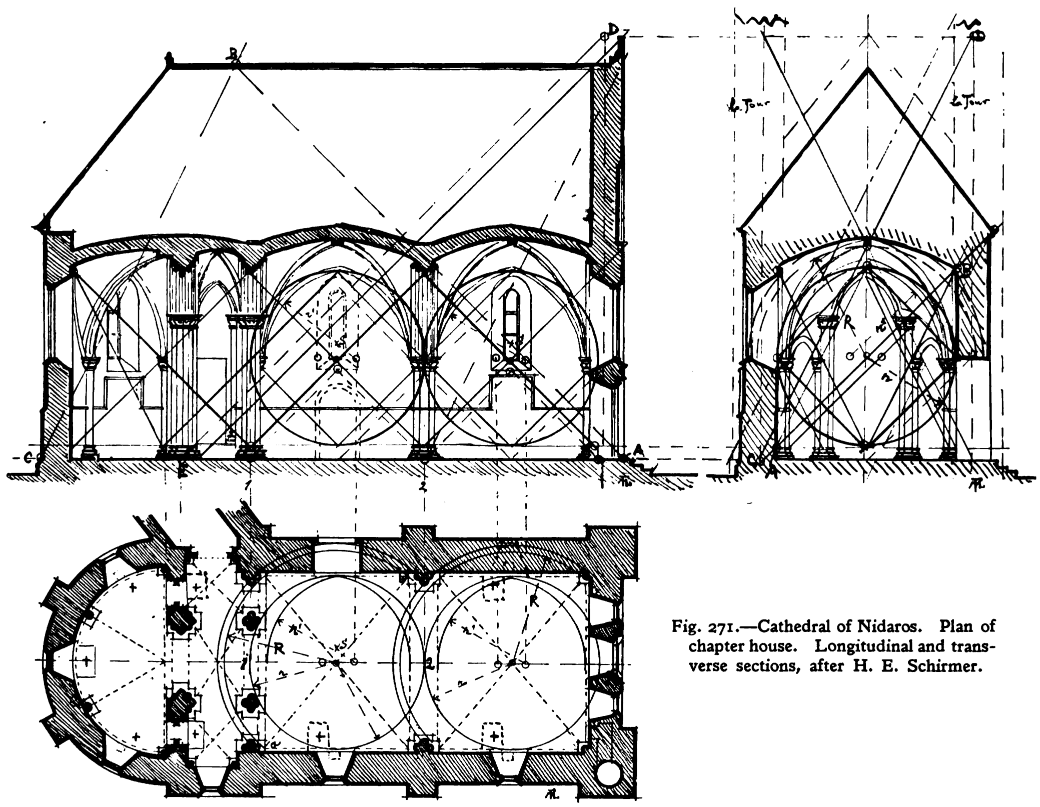


Fig. 271.—Cathedral of Nidaros. Plan of chapter house. Longitudinal and transverse sections, after H. E. Schirmer.

The chapter-house of Nidaros cathedral is one of the most interesting monuments in the history of European architecture, and it is an evidence of how early Norwegians had understood to realise impressions and lessons learned in their travels round the whole of the then civilised world, and to use them into their ancient art of construction; it also bears witness to the fact that only the best material and work were used in the cathedral of the national saint.

It must be admitted that this small religious building is strictly according to the rules of religious temple architecture, as it is the case also with the cathedral, in plan and in elevation.

After having thus indicated the size and character of the cathedral of Olaf Kyrre, we must now turn to Plate XVII. If we compare them we see that this cathedral is somewhat larger than the one of Lund, but that it is small, however, in comparison with the large cathedrals of Chartres, Canterbury, and Durham. It takes its place, notwithstanding, as a perfect cathedral, worthy to be an international pilgrim cathedral, made to suit the conditions of a smaller country. This cathedral of Olaf Kyrre must be considered as having stood unaltered since its completion—previous to 1093 until the establishment of the Archbishop's see in 1152. On the occasion of this new dignity the work of modernising it was undertaken. The clerestory of the transept is a clear evidence of it.

We have seen from the documents mentioned that the original chapel was attached to the cathedral as an independent institution. Seven of its eight altars had to be removed into the cathedral, which necessitated a change in their order and in their arrangement. We have a proof of this in the inscriptions of 1161 preserved in the chapels of the transept, where it says that in that year the altars with the names of the given saints were consecrated. It seems as if the altars of the chapel were put together on account of the lack of room in the cathedral. Anyhow, it is worth noticing that the number of the altars removed is the same as the number of the saints to whom the newly consecrated altars were dedicated. Naturally those of the cathedral itself had to be reconsecrated, having been moved during the rebuilding.

These alterations, which seem to have been completed and consecrated in the third year of the second Archbishop, were to serve as introduction to other more extensive building operations.

In his journey to Rome, to receive his palladium, Archbishop Eystein had probably travelled through Paris to greet Abbot *Suger* of St. Denis, and there he may have received impressions of the Gothic rebuilding of its famous cloister and pilgrim church.

His time of office is divided in two periods, 1159 to 1180, when he had to escape to England, where he became Abbot of the Benedictine monastery of Bury St. Edmunds, and from 1183 after his reconciliation with King Sverre, until his death in 1188. During the whole of these twenty-nine years this powerful prelate and superior man was occupied in rebuilding his cathedral in Gothic style—a work which was only completed after 1248, when Archbishop Sigurd "removed the foundations of the cathedral as much west as they now stand," as said in the saga of Haakon Haakonson.

In the following analysis of the Gothic cathedral we shall bring evidence to prove that all its Gothic parts, the preserved remains of the Gothic chancel and nave, are built according to a complete and previously made plan of Archbishop Eystein and his architect—a plan strictly carried out until the completion of the work, although some details may have undergone the development shown by the preserved fragments.

CHAPTER XVII

THE GOTHIC CATHEDRAL OF ARCHBISHOP EYSTEIN AND ITS GEOMETRICAL SYSTEM

AFTER the previous exhaustive demonstration, showing that a geometrical system has been at the base of the proportioning, in classic and in medieval religious architecture, and after having proved that this system was used in the cathedral of Olaf Kyrre also, it might have been sufficient to show one drawing after another, of the existing Gothic parts of Nidaros cathedral in plan and in elevation, to prove that this system has been the guiding factor at every point in the proportioning of the church and its architectural development. This ought to be obvious, as it is through the very discovery of the system of this cathedral, that we have succeeded in proving its general use all the way back to the oldest Greek temple architecture. But, as our demonstration is also to be a justification of the proposal which we made of reconstructing and completing the cathedral, and as it is the first time in five or six hundred years that a geometrical hegemonikon is to be used in building, and, moreover, as a new edifice will grow out of this system, it will be right and at the same time interesting to carry out our analysis of Nidaros cathedral in the same manner and to follow the same line which led to our discovery,—out of which the final scheme of reconstruction developed of its own accord from the remains which are preserved. From this we shall see how the scheme carried out according to this geometrical system agrees with the existing remains and corresponds with the principal points of the preserved portions; at the same time we shall see how the entire re-erected church comes within the frame of the system *ad quadratum*, as given in the previous demonstrations. It should be noticed that we did not seek this result, but that it came into existence through the logical use of the law of proportion found in the building itself.

As already remarked above, the creation of the Archbishop's see caused alterations and extensions in the cathedral of Olaf Kyrre. When dealing with the inscriptions in the chapels of the transept, we gave a proof already, that these alterations were started directly after. But the real rebuilding of the cathedral into Gothic style was, according to the direct saying of Snorre, started and finished—except the west front —by Archbishop Eystein., who took possession of the see in 1159.

It has been a common theory that this rebuilding was only started by the Archbishop in 1183, when he returned from his exile of three years in England. According to this, it was there that he got the idea of his octagonal apse from what is called the "Corona" in Canterbury cathedral. It is incomprehensible how such a theory could have arisen, as that "corona"

—the chapel placed farthest east in the cathedral, at the back of the feretory—neither in style nor from the point of view of the ritual has the slightest relationship with the octagonal building of Eystein, as the illustrations of the corona, given here, show perfectly (see figs. 272 and 273). It has also been the accepted opinion previously that the rebuilding of the cathedral in Gothic times was undertaken without any definite plan, in bits and haphazard, by various masters and at different times—that is to say, in the same loose and accidental way in which the work of restoration of the cathedral has been done in the last ten years.

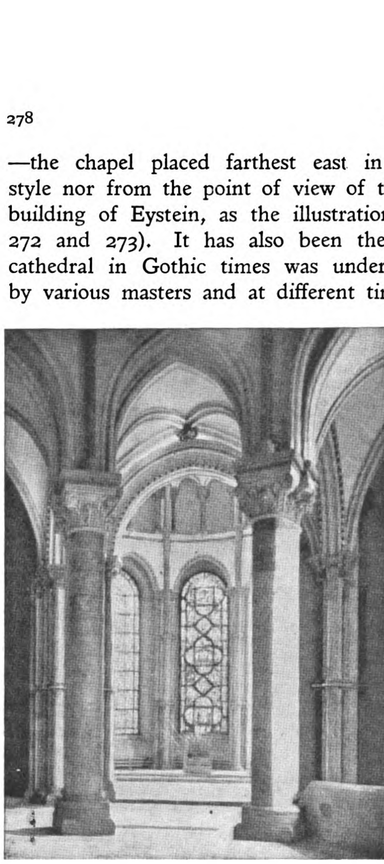


Fig. 272.—Cathedral of Canterbury. The corona.

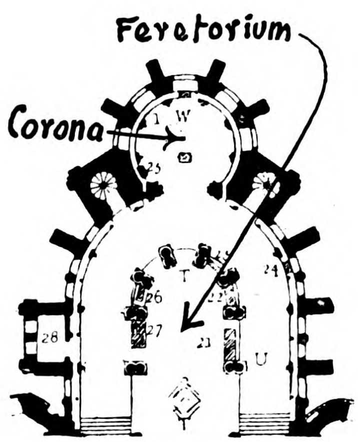


Fig. 273.—Cathedral of Canterbury. Plan of the corona.

After what has been explained before, it must be evident that such a way of reasoning rests only upon the utter ignorance of religious architecture; cathedrals were never extended haphazard, but according to fixed rules, and we shall see that, just as the Norman cathedral of Nidaros was planned in *one* jet by Olaf Kyrre, so the Gothic rebuilding was from *one* complete plan, worked out by Archbishop Eystein, according to the same fixed temple rules which formed the basis of the Chartres type.

In the last chapter we said that the chancel of the original chapel was made into the "feretorium" in Olaf Kyrre's cathedral, and that on each side of this, there must have stood other chapels, but without any connecting ambulatory (see fig. 263).

It is obvious that Archbishop Eystein must have been forced by sheer necessity to correct this and obtain a procession-path round the shrine of the saint. We have here the starting-point of the new plans of Eystein. Moreover, we have maintained, during the analysis of Olaf Kyrre's cathedral, that this had been designed with due regard to the requirements of a cathedral of pilgrimage and given a chancel sufficiently long to accommodate a numerous assembly of priests during the times of great church festivals. Consequently Archbishop Eystein did not require to extend his chancel farther east. Moreover, it must be remembered that an extension towards east—similar to what was done in contemporary continental cathedrals—could not be done here, on account of the two holy places which were fixed points: the resting place of the King and the well; these determined for Eystein, just as it did for Olaf Kyrre before him, the natural and unsurpassable barrier or termination of the cathedral in the east. For that reason, no extension worth mentioning could be thought of in that direction. His problem was then limited to the planning of the ambulatory with adjoining chapels, as part of the plan of rebuilding, in Gothic style, Olaf Kyrre's cathedral. The still preserved return of the cornice of the

south-east corner of the tower is a complete evidence that the reconstruction of the transept was included in the plan.

* * *

In the plan of Olaf Kyrre's cathedral (fig. 244) the outline of the extended chancel of Archbishop Eystein is drawn. His arrangements to obtain the necessary ambulatory are very obvious. In the same illustration the outline of his scheme for the extension of the nave towards west is also drawn.

How strictly the principle *ad quadratum* is carried through in this planning, is shown by the actual plan of the building of Eystein (fig. 275); the diagonals of the half-square show that the west front is removed only so far towards west as he found it necessary to extend the chancel with ambulatory and chapel towards east: in other words, the plan is still solved within two large squares, one on each side of the transverse axis through the centre of the central tower. On account of the removal of the front towards west, he has been able, as regards the nave, to correct the previously mentioned narrowness of the aisles in the Norman plan, because the increased length of the nave gave an opportunity of establishing regular proportions between its width and the width of the aisles, while fulfilling the conditions of two squares in the nave.

As concerns the chancel, the chapter-house has stood in the way of a corresponding extension in the width of the aisles; this is why he used the same foundation for the walls of the aisles as in the church of Olaf Kyrre. The principle is also applied to the planning of the flanking towers of the west front.

During the examination of Lincoln cathedral (fig. 80), of Rouen (fig. 82), and of Wells (fig. 89), we saw that the placing of the side towers of these churches was determined by the diagonal lines of the half-square, starting from the pillars of the central tower. It will be remembered, from the previous chapter, that we demonstrated the employment of this principle already in the considerably older cathedral of Olaf Kyrre. From the plan of Archbishop Eystein (fig. 275) it will be seen that the diagonals from the pillars of the central tower determine the position of the flanking towers—that is to say, the width of the front on the west side of the large western square. It has been clearly Archbishop Eystein's architectural aim, with the help of these projecting corner towers, to give the impression of the powerful front of a five-aisled cathedral to this three-aisled one. As shown by the number introduced in the plan facing the front, it is accurately divided into six according to the principle of a five-aisled church.

The preserved medieval remains of the wall of Nidaros cathedral are given in the attached bird's-eye view (fig. 274). There remains of the Norman cathedral of Olaf Kyrre, as it can be seen, the *transept* and the *chapter-house*, north of the chancel.

Of the Gothic cathedral of Archbishop Eystein the entire *octagon* remains. It is hidden in the drawing behind the east gable wall of the chancel, also preserved. As regards the corner towers, there are in this gable-wall some remains of the Norman cathedral. Furthermore, of the *chancel*, there remains the side-walls of the aisles.

The side-walls of the nave have disappeared, but remains of them are found on the west of the eastern gable of the chancel and on the eastern side of the central tower. In fig. 215 we have already shown the remaining parts of the lower story on the north side. Fig. 276 shows the condition on the south towards the east gable-wall. In the triforium and the clerestory some parts remain similarly (see fig. 274). The division of the stories to the top of the wall can be accurately determined thereby. The wall-arches of the nave and of the aisles, and the springers of the vault, are also preserved. The original columns of the main arcades of the

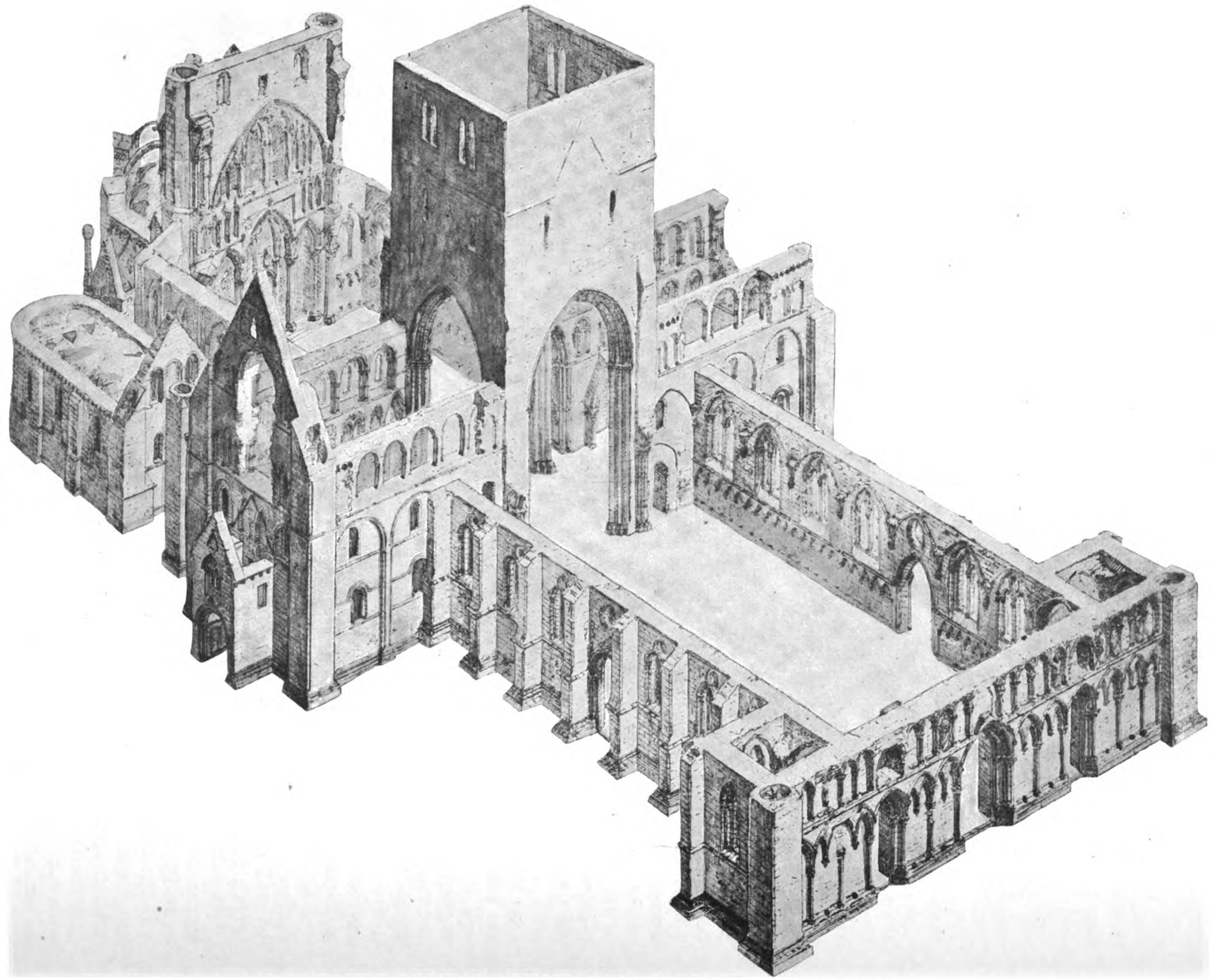


Fig. 274.—Cathedral of Nidaros. Bird's-eye of the remaining medieval part, after J. Mathiesen.

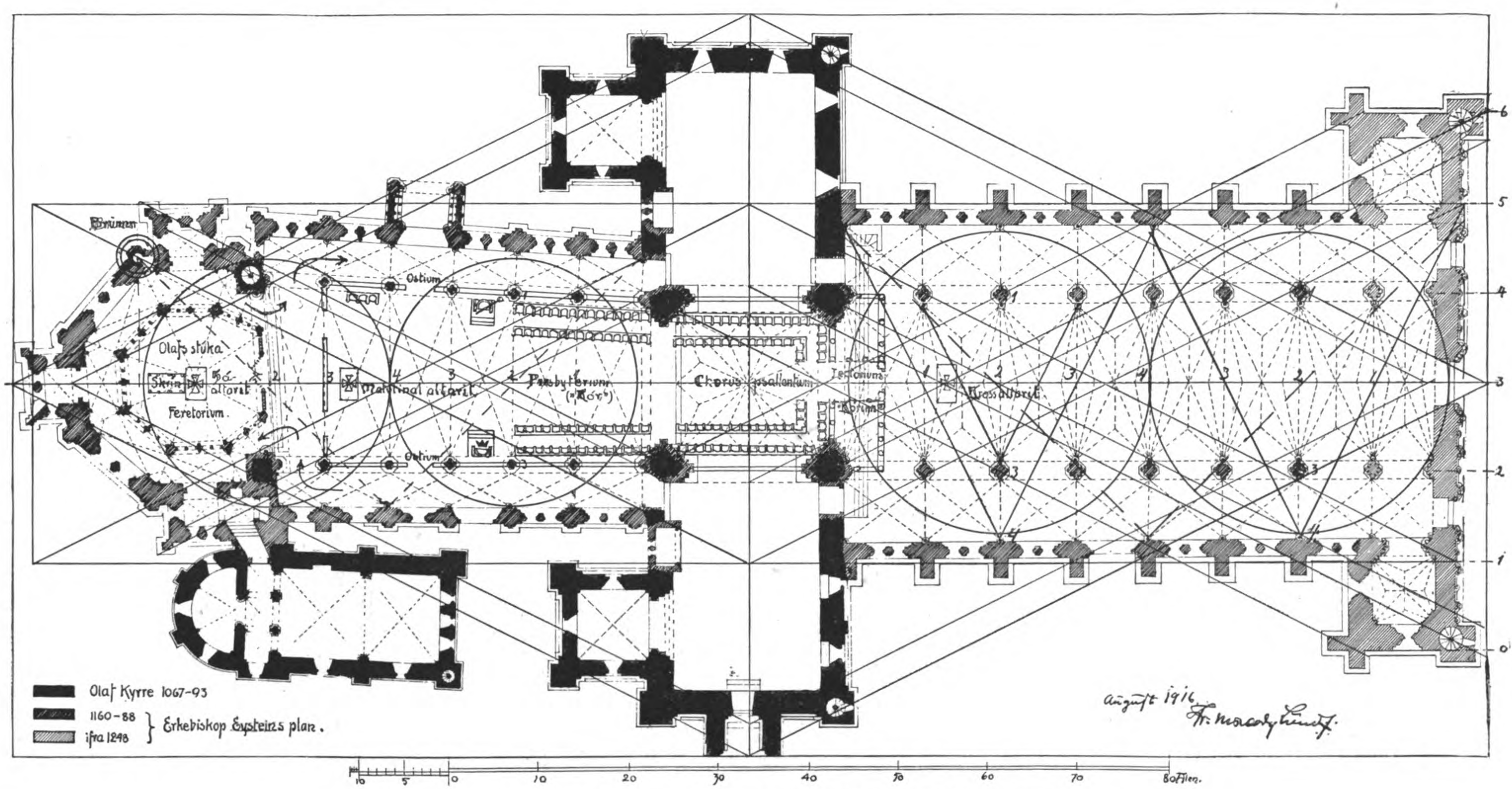


Fig. 275.—Cathedral of Nidaros. Plan of the cathedral of Archbishop Eystein.

supporting walls, which were built in the sixteenth century, have been also found (see fig. 277).

By means of these preserved parts bonded in the walls themselves, and by authentic finds of loose stones, used as building material in the before-mentioned supporting walls, perfect and sure indications were available for a correct reconstruction of the chancel, wherefore the chancel rebuilt by Christie, as far as concerns the interior, can be considered as being the original one. But, in his anxiety regarding stability, he allowed himself to alter the transverse section of the pillars found. The original ones consisted of a nucleus of steatite, with and without detached light marble columns alternately (see fig. 277).

This regrettable alteration consists in having discarded the marble columns and increased accordingly the central nucleus of steatite, besides increasing the collective transverse section of five inches.

As regards the exterior, he has not entirely reconstructed the cornice satisfactorily through not allowing the wall of the nave to continue upwards above it—as it can be seen by the remaining portion. But more of this later.

Of the *central tower* which was part of the Norman design but rebuilt in Gothic times, there remain the pillars and the walls to the top of the vault of the lantern, consequently including the clerestory, while the belfry stage has disappeared. The vault itself is gone, but the springers remain. The clerestory was partly rebuilt in the sixteenth century.

Finally, as regards the *nave*, there remain the walls of the aisles to the cornice with the decorated frieze included, and the west front with its two flanking towers to the same height. We refer to the previous figure 247, reproducing the interior of the ruins of the nave.

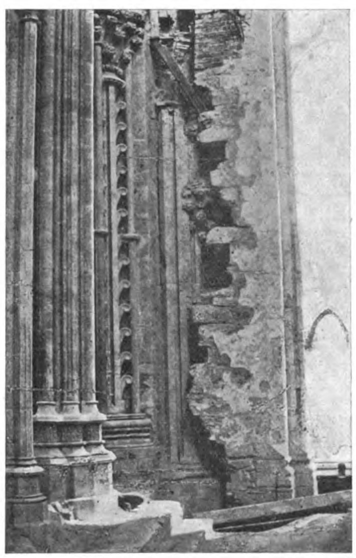


Fig. 276.—Cathedral of Nidaros. Remains bonded in the wall on south choir-screen.

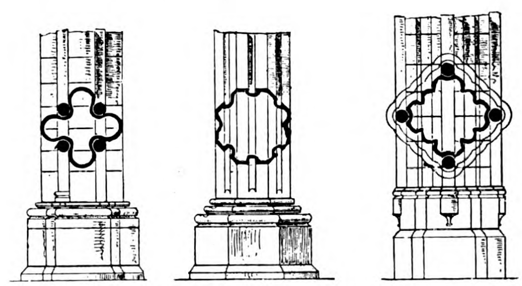


Fig. 277.—Cathedral of Nidaros. Original pillars of main arcades of the chancel.

As for the side-walls of the nave there exist only fragments on the inner side of the front and on the west side of the central tower, whereby the height of the capitals on the main arcade is ascertained.

During the work of restoration, while pulling down the supporting walls built later, and in the town and surrounding districts, a number of architectural fragments were found, such as mouldings, pieces of arches and columns, parts of highly decorated wall-panels, window traceries, etc. All these stones were collected by Christie, with love and reverence, and used in his reconstruction of the two lower stories of the side-walls, with such understanding, that

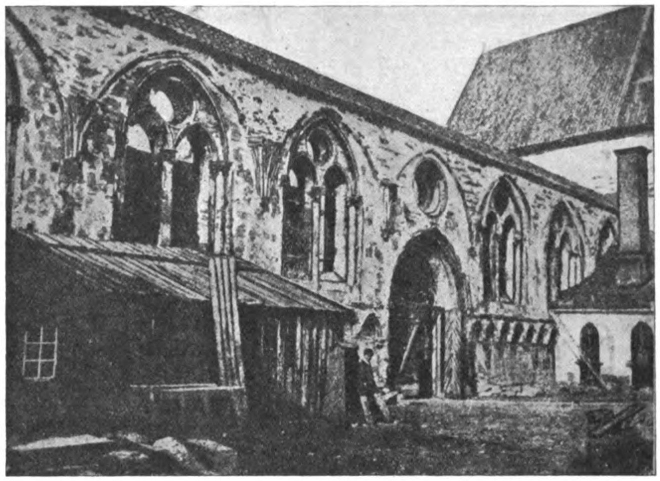


Fig. 278.—Cathedral of Nidaros. Interior of the ruin of the nave.

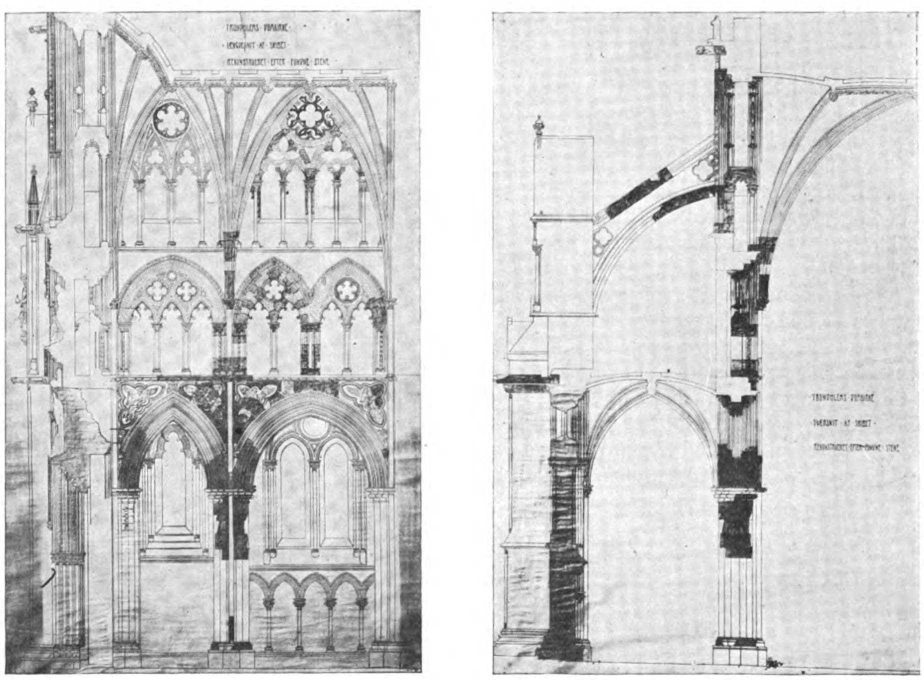


Fig. 279.—Cathedral of Nidaros. Archæological documentation of the architect Christie for his reconstruction of the longitudinal and transverse sections.

up to the floor of the clerestory, they are, so to speak, within an inch in accordance with the original height; we shall furnish proof of this later, when going through our analyses. As regards the clerestory the finds were not so rich, consisting of rose tracery with fragments of columns belonging to the windows of the clerestory, and as these finds formed the only fast point for the reconstruction of this part, it turned out entirely wrong and prevented a correct solution of the west front.

Fig. 279 reproduces the archæological documentation of Christie for his reconstruction of the side-walls of the nave. The stones found are shaded.

* * *

After having convinced ourselves that the plan of the church is strictly carried out according to the principle *ad quadratum*, we shall furnish proof that this is also the case with all the existing parts of the cathedral. The octagon is the oldest. With the exception of a few later repairs, such as to the lower arcade and to the triforium, occurring after various fires, it is in the main preserved from the time of Archbishop Eystein. The ambulatory and the chapels are entirely preserved. It is thus one of the oldest Gothic buildings in Europe, as it is proved to date from before 1179.

The oldest parts of the octagon, the ambulatory with chapels, look so ancient in their Romanesque construction and the Byzantine character of their ornamentation that one could be tempted to believe them remains from the Norman cathedral of Olaf Kyrre, were it not for the direct statement of Snorre to the contrary. We refer to fig. 280.

The plan of the octagon is older in character than the clerestory of the transept rebuilt by Archbishop Eystein. If these parts of the octagon are looked upon from their external appearance and their pointed arches and considered Gothic, then they represent the oldest remains of Gothic style in Europe.

It is beyond our task to go more closely into this matter affecting the earliest existence of this building. It must be sufficient for the present to indicate the valuable chronological fact that, according to a medieval inscription, which still existed in the eighteenth century, or, at any rate, was referred to in the seventeenth, the chancel of the church of Saurshaug in Thröndelagen was consecrated by Archbishop Eystein in 1184, after its reconstruction (see Gerh. Schöning, *Reise gjennem en Deel af Norge*, 1773–5, vol. i, p. 236).

We reproduce here in fig. 281, as an example of the ornamentation in this church, a capital which shows absolutely the same characteristics of style as the ornamentation of the octagon. This must have existed in the church of Saurshaug before Archbishop Eystein came back from his exile in the summer of 1183—that is to say, before he could have had time to think even of the preliminary work of building the octagon, which, according to the prevalent theory, was started after his return.

It will be understood, by what has been said, that this in itself is sufficient to upset the received chronology of the building process under Archbishop Eystein.

We give three different analyses of the transverse section of this octagon, beginning with fig. 282.

After introducing the line of base A and B, and the axes of the walls, the geometrical auxiliary construction is carried out exactly as in the earlier analyses of other churches. It will be seen how the height of the wall, the division of the vault and of the stories fall exactly within this construction and according to the proportion of 1 : 2. It will be noticed how the webs of the vault were remade during later repairs, and given a horizontal groin, while the existing original wall-arches show that the webs rose from the top of the windows to the keystone, in the shape of an umbrella vault, as in early vaulting.

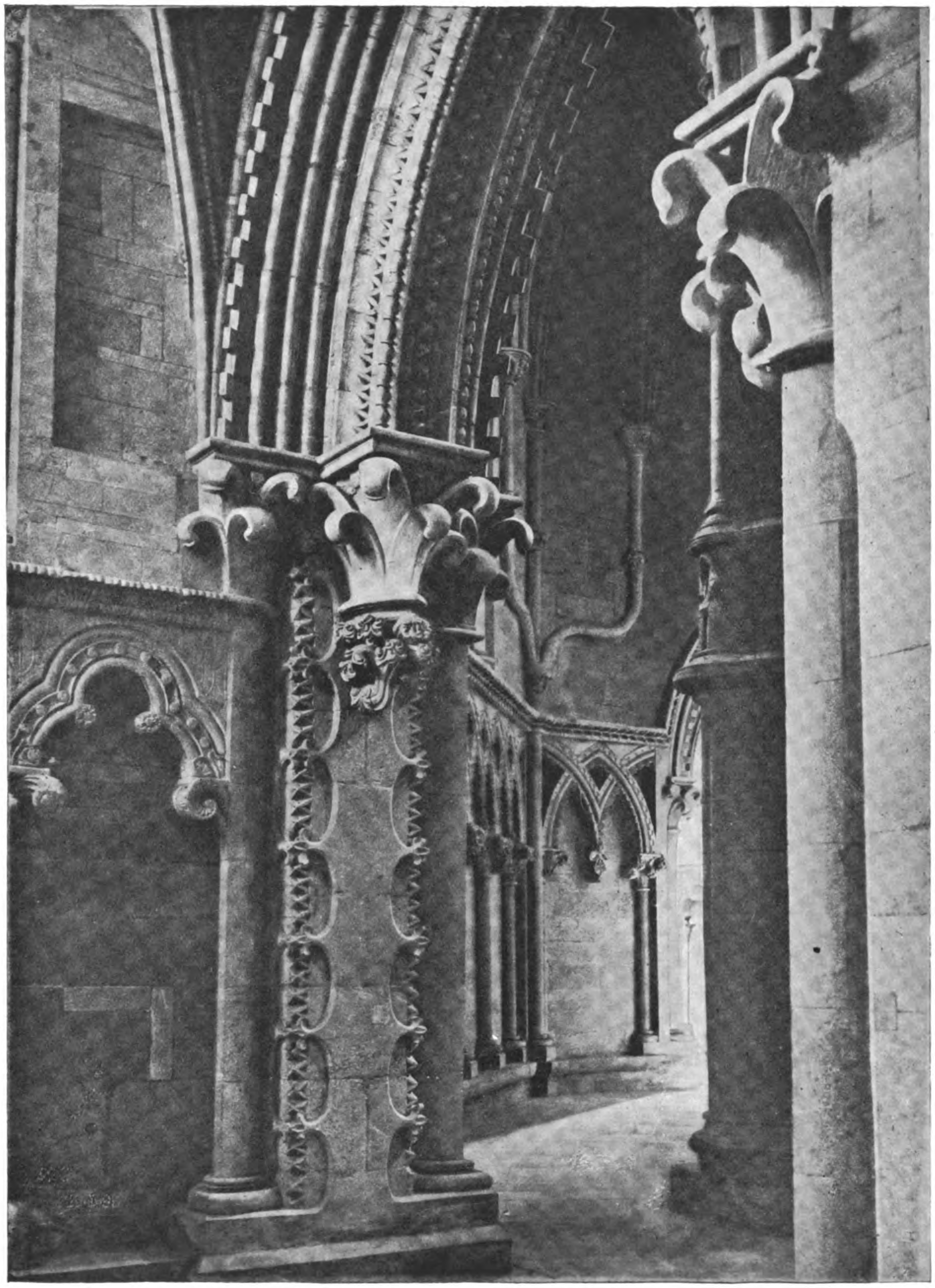


Fig. 280.—Cathedral of Nidaros. Interior of ambulatory.

In Analysis II, fig. 283, there are lines drawn according to the angle of 63° 26′. As it can be seen by the two diagonals starting from point *a*, the whole distribution is determined by this angle, the lines clearly passing through all the chief points, and corresponding with one another in a complete interplay, further shown by other corresponding lines, drawn in the central aisle and in the chapels.

Analysis III (fig. 284) shows finally that the proportioning thus produced, as well for the main as for the smaller divisions, falls within the proportion of the sectio aurea. Eight of these proportions are given in the drawing, in arithmetical equation.

Our analysis includes, moreover, the preserved east gable-wall of the chancel, with the open-work or pierced wall between this and the octagon (see fig. 285). We give, as basis of our examination, the measured drawing of Christie of this pierced wall (fig. 286, Pl. XVIII), with Analysis I of the gable-wall. All this wall was preserved as it stands until level E F. Christie has only reconstructed and added to the part over this level.

The proportion between the inner width and height up to the vault of the octagon is 1 : 2 as indicated. The proportion in the central aisle of the chancel is quite different, being as 1 : 1·5.

At first sight this seems a contradiction—as if the architect had not been guided by any

Fig. 281.—Church of Saurshaug. Capital.

geometrical system of proportion, but only by the natural wish to obtain the same height in the chancel as in the octagon, this being an absolute necessity on account of the pierced gable-wall (fig. 285), and because there is an opening between the octagon and the chancel reaching to the top of the vault of the former (see longitudinal section, Pl. XIX, fig. 297). But we shall see that there is nothing accidental in this arrangement, no more here than in other churches previously examined.

It is known that all rebuildings of medieval cathedrals were started from the east. This has also been the case in Nidaros. This rebuilding was done here, as everywhere else, according to a previously settled plan, as already pointed out, based on the system *ad quadratum*. In consequence of this, we shall find that the very difference which exists between the octagon and the chancel as regards the proportion between width and height, is a necessary result of the architect's intelligent use of the system. At the same time, we shall be convinced that the proportion of the octagon is a result of the arrangement of the chancel, and not vice-versa, and that this proportion of 1 : 1·5 is the very result of the principle *ad quadratum* carried out logically.

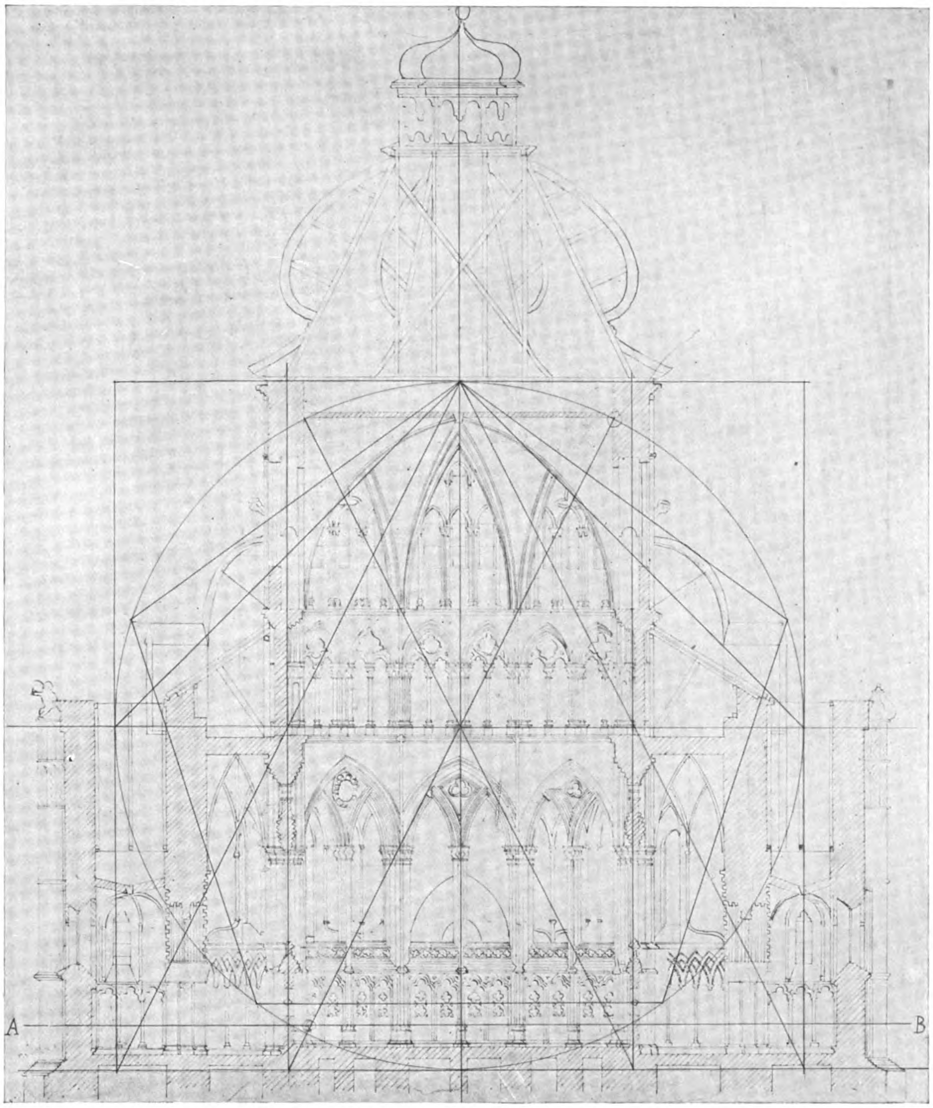


Fig. 282.—Cathedral of Nidaros. Octagon, analysis I.

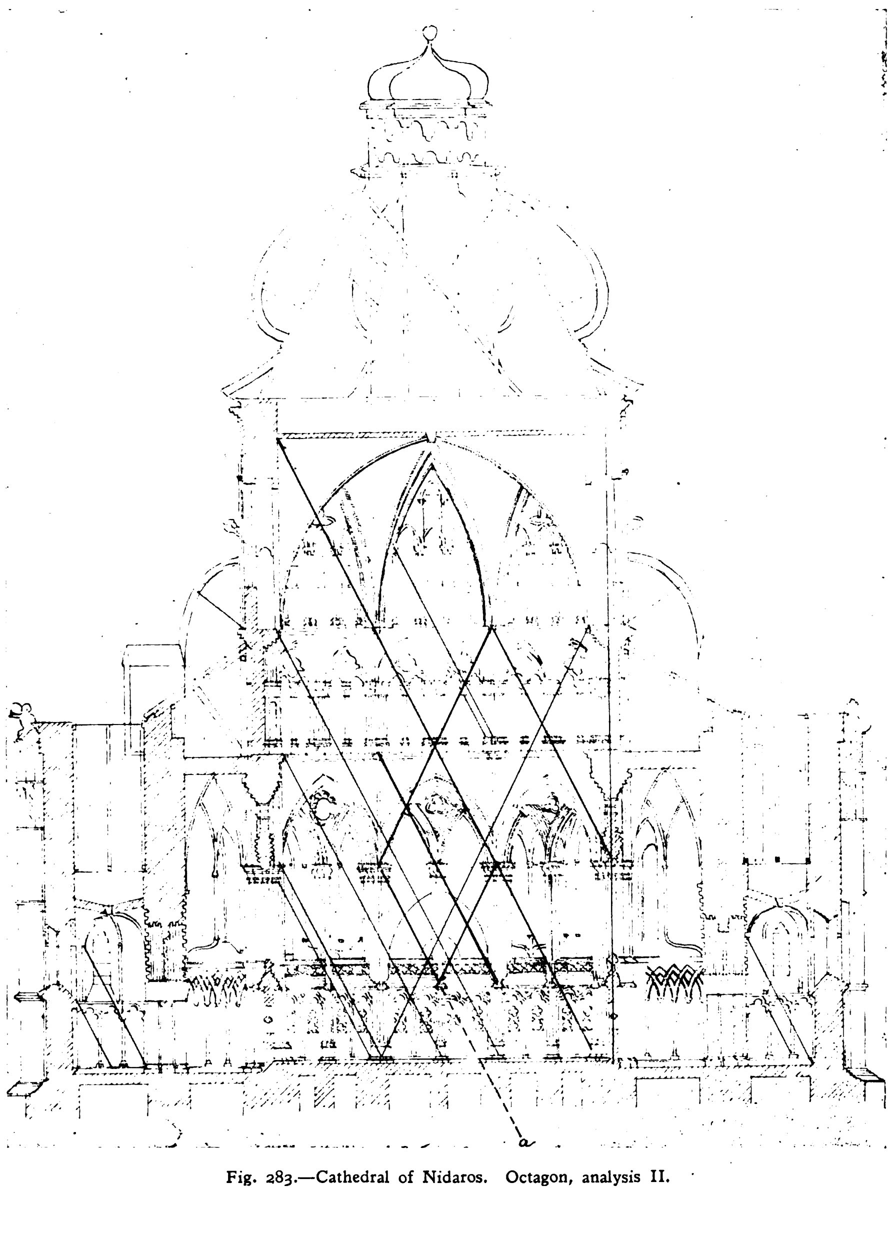


Fig. 283.—Cathedral of Nidaros. Octagon, analysis II.

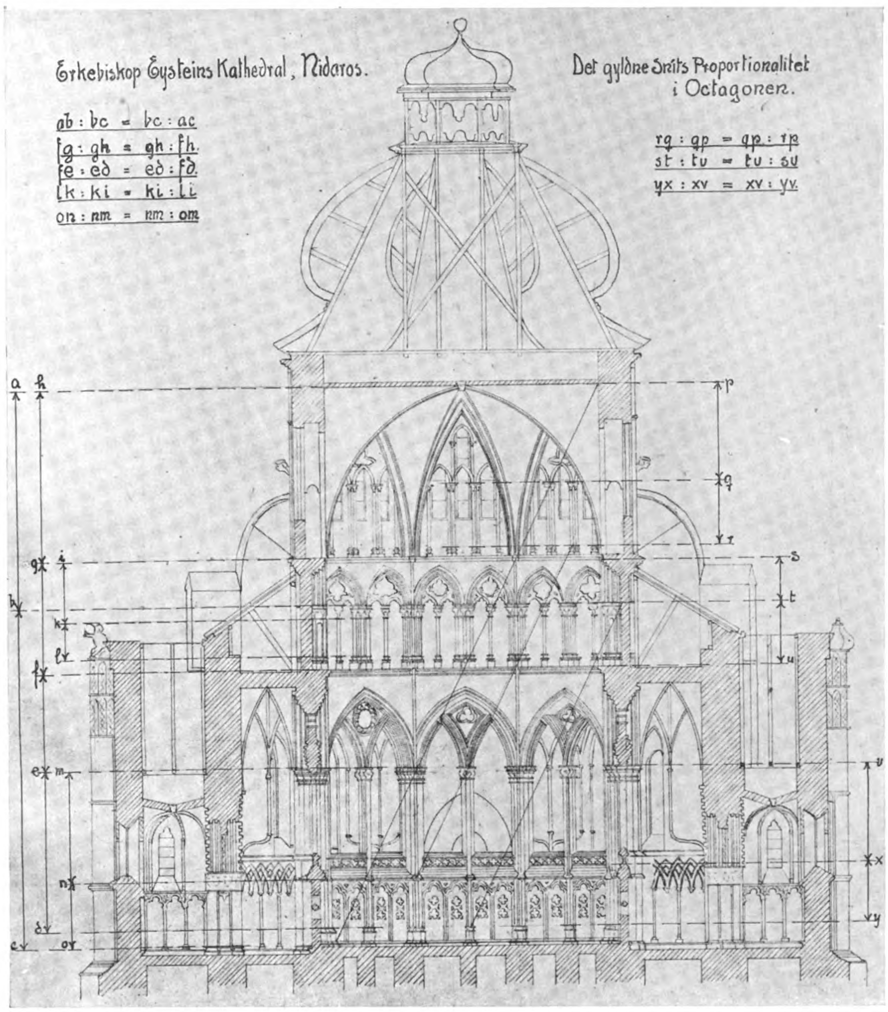


Fig. 284.—Cathedral of Nidaros. Octagon, analysis III.

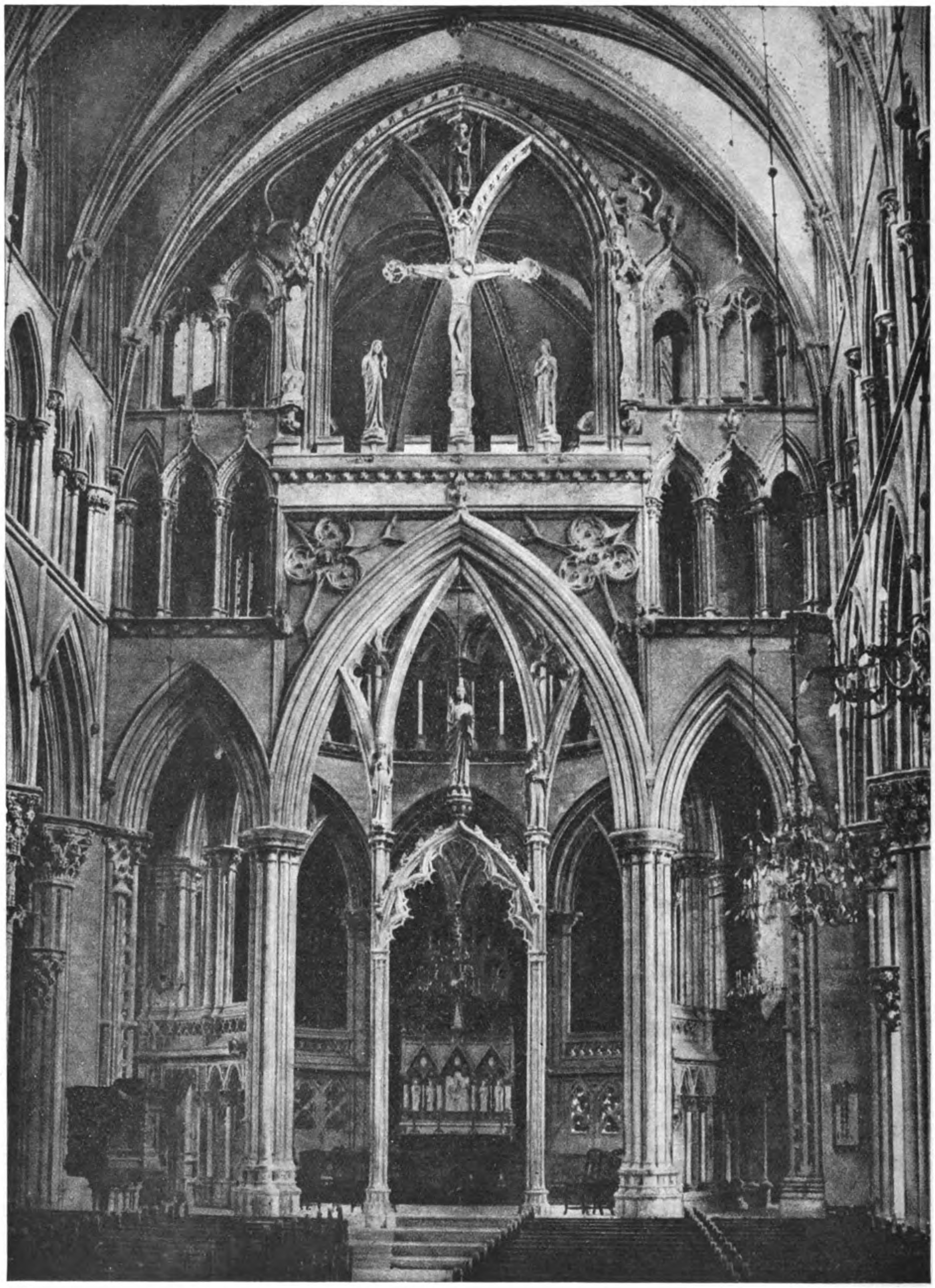


Fig. 285.—Cathedral of Nidaros. Interior of chancel towards the octagon.

Before we start our demonstration we shall first consider the different alternatives of vaulting which must have presented themselves to the architect if he wished to have, at any cost, a proportion of 1 : 2 between the width of the chancel and the height of the vault, see fig. 286, Pl. XVIII, Analysis I, of the pierced gable-wall with alternatives of vaulting.

The first important thing we recognise is that such a proceeding would be meaningless and cause unsolvable constructive conflicts.

The axes of the walls of the chancel are marked G^1 and H^1, and the stroke-line above the plinth of the chancel is marked A^2—B^2.

The verticals G^2 and H^2 represent the axes of the walls of the nave on the west side of the central tower, similarly the horizontal A^1—B^1 represents the line above the plinth of the nave west of the central tower.

Finally, the verticals I and K represent the inner width of the chancel towards the east side of the central tower.

Here we have the base of no less than four alternatives of vaults, which could all be carried out, height in proportion to width, as 2 : 1.

First alternative is represented by the vault level C^1—D^1, which depends upon the axial distance at the gable-wall of the chancel, the lines G^1 H^1 used as radius of the stroke circle, whereby the level of the vault is determined in the usual way according to the system.

Second alternative, represented by the level C^2—D^2, which depends upon the axial distance of the walls of the nave, G^2 H^2, used as radius similar to first alternative.

Third alternative, represented by C^3—D^3, which is dependent in the same manner as second alternative, but the rectangle of the interior is supposed sunk down from the plinth-line A^1 B^1 of the nave to the plinth-line A^2 B^2 of the chancel.

Fourth alternative, represented by the level C^4 D^4, which gives the height of the vault above the line of the plinth A^2—B^2, if the inner width of the chancel at the east wall of the central tower, the vertical lines I and K, are a condition of the proportion of 2 : 1. Finally, the level of the historically certain and real vault is marked C^5—D^5.

This level, as will be seen, lies as much as 11 feet lower than in first alternative, and nearly 5 feet lower than the lowest alternative of all, the fourth one, which depends on the smallest inner width of the chancel near the central tower.

If we try now the height in the fourth alternative we shall be convinced that even this could not be used without breaking the rules of the system *ad quadratum,* which the medieval architect has succeeded in carrying out in spite of the difficulties which existing and binding conditions were putting in his way.

We have already pointed out, when going through the plan, the unavoidable conditions: the tomb of the King, the well towards east, the chapter-house on the north, and finally towards west the width of the original chapel which had determined the central tower of Olaf Kyrre, and which had to form the centre also in the cathedral of Archbishop Eystein.

In the drawing (fig. 286, Pl. XVIII) it will be seen, from the given auxiliary construction having as starting-point the axial distance of the pillars of the nave, the lines G^2—H^2, what width the chancel would have had normally, if the church had been planned, from the beginning, on the basis of this axial distance, without obstacles from older venerable buildings or parts of buildings. The inner surface of the wall of the south aisle would have coincided with the point marked by two rings on the horizontal line A^1—B^1. Instead it is situated 2 ells and 15 inches within. In the same way the inner surface of the standing wall of the northern aisle lies 3 ells farther in than it would have done if it had depended upon the width of the central aisle according to the system.

In fig. 287 (analysis II, Pl. XVIII), the distance of the chapter-house to the chancel is introduced, on the left of the drawing, with the word "Kapel." Both from the plan and

the previous drawing it will be seen that the central aisle of the chancel is considerably broader towards east than towards west—more than 3 ells.

In the present fig. 287 we have introduced the auxiliary construction by starting from the axial distance between the pillars near the gable-wall of the chancel, marked G and H. The inner surface of the south wall of the aisle would in that case have been at the point marked by a small ring, or 3 ells 20 inches outside the real surface, while the inner surface of the wall of the north aisle would have been above 4 ells outside the real inner surface, just at the point where the outer line of the wall of the chapter-house coincides. It will be seen that there is no room here for the wall of the aisle, not any more than for the buttresses which are a necessity in all the first four alternatives of higher vaulting.

According to the system of the three-aisled church, the chancel against the east gable should have had a total inner width of 37 ells 12 inches; this width is reduced in reality by no less than 7 ells 20 inches as a consequence of the aisles not having the normal width.

But this circumstance is not the only determining factor in the height of the vault of the nave. The length of the chancel east-west is also determining. This length, which was given by the original chapel in the plan of Olaf Kyrre, was thus made unalterable, to Archbishop Eystein, by the tower, as we said before. It is 42 ells.

We have seen by what precedes how the principle *ad quadratum* has been, and must be used in all plans in order that its harmonising functions should be in full play. We saw how the principle was systematically carried throughout in Paris and in Cologne, and we saw how in Lincoln and Wells, some faults were made which brought disharmony, under the given conditions of already existing older parts of buildings.

In Nidaros cathedral there are none of these faults.

It could have been tempting for a less independent-minded architect to be regardless of consequences and to try makeshifts by which he could have muddled through and built his vault over the central aisle of the chancel according to the lowest alternative, No. 4, and thereby realised the proportion of 1 : 2. This is probably how an architect of our time would have reasoned, because the extension and the strengthening of the buttresses in this alternative can be considered feasible.

But this expedient would have been one-sided and could only suit the transverse section; it could not be done longitudinally on account of the given length of 42 ells.

The proportion of 1 : 2, as we know, is the result of the square divided into four.

The level above the line of the plinth, according to alternative 4, is 32 ells, and the distance is 42 ells to the central tower; $42 \div 32 = 10$ ells. When this square with a base of 32 ells is divided into four, 4 bays are obtained with a distance of 8 ells between the axes of the columns, and two bays of 5 ells. The result would be 2 double bays with proportion between width and height as 1 : 2, according to the regular proportioning of the church, and one double bay with proportion of 1 : 3·2; or, if each bay is reckoned separately, 4 bays having the proportion of 1 : 4, and 2 bays of 1 : 6·4.

Here also he could have muddled through by means of an "accommodation," giving all the six bays the same axial distance, and thus obtained a proportion of 1 : 5·33 for each bay.

After the preceding analysis of Lincoln and Wells cathedrals, it will be understood that the balance would have been spoilt in any case and that he could not have made it correspond either with the immediately adjoining octagon because the vault of the chancel would have stood a whole five feet above the vault of the former.

The *magister operum* of Nidaros was apparently not like an impulsive modern architect, distributing arbitrarily; he must have been a thinker, and he chose the only correct height for his vault, such as this had to be according to a right use of the harmonising auxiliary principle.

The height of the medieval vault is 29 ells 12 inches above the plinth, while the length of the chancel is 42 ells; $42 \div 29^{\circ}\ 12^{1} = 12{\cdot}12$.

These 12 ells 12 inches give the axial distance of 6 ells 6 inches for every one of the two completing bays, while the axial distance is 7 ells 9 inches for the four regular bays within the square. For the irregular bays, reckoned two together, we get a proportion of 1 : 2·306 against 1 : 2 for the regular ones.

The architect has spread out this small difference on the six bays, to which he has given a somewhat equal axial distance of 7 ells as far as the obliquity of the plan of the chancel allowed him. In this way every double bay has a proportion between the axial distance and the height of 1 : 2·107.

Through this insignificant divergence he has not only succeeded in realising—although approximately—the longitudinal elevation, according to the division of the square into four, but he has also realised the transverse section within an inch *ad quadratum*, on the total inner width of the chancel, marked on the drawing by the line A—B.

It is interesting to notice the geometrical proportion in which the square on this line, A, B, C, D, stands to the larger square on the base *a—b* of the auxiliary construction. We have divided this line into eight parts, four equally large on each side of the central axis, marked o.

We see, by means of the two rings upon the line above the plinths of the chancel, one to the left of A and the other to the right of B, what inner width the chancel should have had if there had been no obstacles from existing buildings.

The disposition is purely scientific. By reason of the given width and length of the chancel, the height of the vault is determined from points A and B on the plinth-line, therefore according to the smaller square A, B, C, D, while the height of the gable-wall to the base of the triangle is determined *ad quadratum* on the larger base *a—b*, therefore the square *a, b, c, d* —this being on account of the planned rebuilding of the nave to the west of the central tower, where the three-aisled church could be realised freely and without obstacle, in its regular width, and to a corresponding height.

This height of the nave to which the exterior of the chancel had to correspond is determined directly by the axial distance between the pillars of the nave on the west of the central tower, marked in fig. 287 by the circle traced in strokes. It will be seen how the diagonal of the half-square to the left through the intersecting point of the horizontal diameter with the periphery, coincides upwards with a trace of original roof still remaining on the west side of the central tower, shown in hatchings, fig. 287. We shall see, moreover, in the analysis of the west front, that the diagonal going downwards also marks off the tangent through the nadir of the stroke circle, the half-square of the west front (fig. 287, marked *x y*). It is with the given height of the nave, thus produced, that the exterior of the chancel has to co-operate in order to obtain the average height of the ridge, for the nave and the chancel. Therefore Archbishop Eystein has been forced, through the given circumstances, to work out both exterior and interior of the chancel according to two different given conditions. We shall develop this statement later.

The vault of the chancel is, as we proved it, literally constructed *ad quadratum*, in a perfectly scientific way, and could not have been done otherwise, if the system was to be used correctly. Through the given conditions, the transverse section of the central aisle is crushingly low, being as 1 : 1·5, as shown by the diagonals of 45°. This is emphasised still more by the much too narrow aisles.

But the architect has avoided the ungraceful effect which breaking the rules of proportions of the church would cause, by his arrangement of the surfaces of the pierced wall of the chancel; this is not done as a matter of "feelings," but in what Viollet-le-Duc would

have called a perfectly "savant" way of proportioning. We saw that, to be able to realise the height of the vault *ad quadratum*, the architect has divided the base $a-b$ of the square of the auxiliary construction into eight parts and succeeded in this way in obtaining the reduced square A—B of the interior, divided into six, as for the planning of a five-aisled church.

If we look at the square divided in this manner it will be seen that the distribution of the gable of the chancel, including the aisles, resembles at once the transverse section of a five-aisled church. The central aisle in such a building rises vigorously upwards to the vault over the portion 1—0—1, like an independent feature slender and high, while the part over the portions 1—2 and 2—3, similar to a system of abutment, supports and emphasises the height of this central part. The crushing inharmonious proportion of 1 : 1·5 of the central aisle is completely neutralised by the manner in which the pierced gable-wall of the chancel is distributed; thanks to this arrangement it rises as slender and elegant as the bays of the side-walls.

We found in the octagon (fig. 283), that the architectural divisions were systematically carried out, by means of the diagonal at an angle of 63° 26′. We found, at the same time, that all the chief points of this distribution correspond mutually, just as in Paris, Vezelay, and Cologne.

It is obvious that the same method was used in the architectural distribution of the gable-wall of the chancel, as shown in fig. 288, Pl. XVIII. Nothing is accidental; everything is sure, firm, and methodical, according to fixed laws of harmony belonging to geometry. The drawing speaks for itself in this case, and needs no explanation. It will also be seen that the distribution thus produced falls within the proportion of the sectio aurea.

As in the classic temples, in Cologne and in Paris, we find in the proportioning of the cathedral of Nidaros, an inner single and "irrational" harmony in the different divisions as regards their relationship to one another. Nothing is "accidental," nothing is "from the depth of subconsciousness," nothing is based upon the vague, uncontrolled and subjectively emotional "feelings" of our modern art jargon. Everything is scientifically developed. This gable-wall which, in its fundamental proportion, was at variance with the fundamental proportion of the church, is, through this ancient method, worked perfectly into the harmony existing in the octagon, to which it can compare in beauty.

The same methodical connection fixed within laws, which we found to exist between the octagon and the chancel, will be also discovered in the following analysis, as the directing agent between the chancel and the nave—that is to say, for the plan west of the transept, both inside and outside.

* * *

The Nave built by Archbishop Eystein

As an introduction to our analysis we have traced the main lines of the geometrical auxiliary construction, in Christie's drawing for the reconstruction of the transverse section of the nave (fig. 289), to show the difference between the height of Christie's vault and the height of the vault according to the system of proportion of the church itself.

Moreover, we have emphasised the remaining original trace of roof upon the west wall of the central tower by an arrow marked NB. We shall speak more fully of this later on.

We reproduce also in fig. 290, Mr. Nordhagen's transverse section, carried out in the office of the architect appointed to the cathedral, in December 1915. In the drawing we have introduced various system constructions, and we have also drawn in various heights of vaulting and of roofing.

As regards the vault of the nave, we cannot help stating that few parts of the work of

restoration have been carried out more arbitrarily, where criticism has been more justified, and at the same time less heeded.

In the drawing, three different heights of vaulting can be seen. The lowest is Christie's, which we have introduced, and which we pronounced as wrong, namely, too low, likewise the height of the wall, directly after the exhibition of the competing drawings, in 1903.

After the death of Christie, an architects' competition took place in 1908 for the reconstruction of the west front. During this competition we protested repeatedly against the low height of the wall and of the vault, as given by the committee, and we indicated the original trace of roof on the central tower, whereby a change was made in the programme, in which the height was increased, but still not sufficiently, which called for further protest on our part, without result, however. Later, under the present leading architect, Mr. Nordhagen, the vault has several times undergone modifications haphazard, sometimes up, sometimes down; while maintaining the height of it against the wall, he has lifted the ridge of the vault up or down, as if it had been an umbrella. It is impossible to know which height he proposes to give it now, from the present documents, as Mr. Nordhagen's latest statements (in the pamphlet, *Trondhjems Domkirke*, October 1915) are not correct; but the position and shape of the vault can be considered to be somewhat near the middle one of the vault sections given in the drawing. But the highest vault indicated is what the proportions of the church demand.

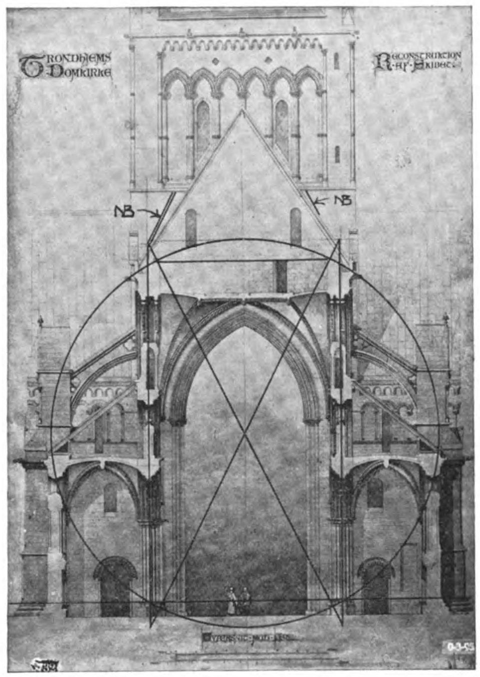


Fig. 289.—Cathedral of Nidaros. Transverse section of the nave, after Christie.

The arch under the central tower is medieval, as it will be remembered from the bird's-eye view: so are the walls of the aisles, with the buttresses and the frieze of the cornice. The columns under the walls of the nave and the vault of the aisles can also be considered as dating from that time. We shall point out further that the thick black lines of the central tower, marked "Gl takspor," show the above-mentioned existing original trace of roof.

The geometrical system of a three-aisled church is also shown in these drawings, starting on the line at the top of the plinth. Besides the height of the vault determined by the system, we must first of all indicate this decisive point: that the old trace of roof coincides *exactly* with the line of construction. The angle at the base of these traces is the same as the one we saw in Cologne and in a number of other churches, namely, 63° 26′; it was this angle, and also the same angle in the fragments of a discovered canopy, which led to our discovery of the system, as explained above.

Under these medieval traces there are two more roof lines drawn on the wall of the central tower; the lowest of these in strokes, marked 1, represents Christie's proposed reconstruction of the roof, because Christie denied absolutely that there was any trace. Our criticism

on that occasion brought about a long and bitter controversy. One can form an idea of the lack of understanding, the self-sufficiency, and the stubbornness of the people to whom the supervision of this great work of restoration in Trondhjem has been entrusted when we explain that it took us five years, from 1903 to 1908, to finally obtain an official acknowledgment of the fact, that this trace found on the wall of the tower is really the trace of the medieval roof.

But even after this, the new leading architect, Mr. Nordhagen, appointed in 1909, has been anxious to show his originality at any cost by raising the roof according to his idea. It is shown in the drawing by the middle line, in strokes and dots, marked II. The reason given for placing the roof as he does reveals so completely a lack of architectural understanding on his part, and on the part of the committee, besides having a strong touch of humour, that it deserves to be mentioned. Among the fragments of loose stones, an angle stone, or "skew corbel," was found comparatively well preserved; this is supposed to have terminated the coping of the gable-wall of the west front. By placing this loose fragment on his drawing board and gravely measuring the angle then formed, Mr. Nordhagen has found that this stone, about 1 ell long, gives an angle of 61° 30′, by which the ridge would fall 1 ell lower than what is distinctly shown by the original trace on the wall, 5 ells long. The chairman of the committee, Mr. Meyer, accepted this difference of angle measured with such accuracy and which a little mortar added in a corner of the horizontal course could disturb. Fig. 291 reproduces Mr. Nordhagen's own illustration of this profoundness.

As a specimen of the scientific capacity of these two trusted architects, we give an extract of the explanation offered on this important question.

In his letter of June 12, 1911, accompanying the design partly accepted later, Mr. Nordhagen writes: "It must be considered as probable that the angle of the first roof of the nave cannot have been much at variance with the trace. My proposal to lower the ridge 2 ells, that is, to reduce the angle of the roof from $63\frac{1}{3}°$ to $61\frac{1}{3}°$, is justified by architectural considerations." The church would have "a more beautiful skyline and a more harmonious distribution of the masses." "The mutual proportion between the nave and the chancel becomes natural" (!) "On the whole, it must be strongly recommended to consider if my proposed lowering of the ridge ought not to be accepted." "In connection with this, it may be of interest to state that a careful measuring is being made of the stone which was the finish of the coping on the north side of the gable of the west front. This angle appears to be $61\frac{1}{2}°$. It corresponds, therefore, very closely (!) with my proposed angle of roof $61\frac{1}{3}°$, the difference being only 10′. The angle of the trace is $63\frac{1}{3}°$, and therefore it does not correspond with the line of the "skew corbel," except by a difference of 1° 50′." The meaning of this contradictory statement is that the trace differs 1° 50′ from the angle of the skew corbel according to the measuring of Mr. Nordhagen.

While Mr. Solberg, an architect and member of the committee opposed strongly the idea of giving up the sure indication of the trace, the chairman, Mr. Meyer, accepted it of course, at once, with the following explanation:

"It seems to me somewhat risky to go away from the trace of the old roof, when it is as clear as in the present case, even in such a small degree as proposed by the architect. According to what he says himself, an angle stone from the coping of the main gable, the angle of which can be measured very exactly, shows that the west gable must have had a height lying somewhere between that given in the present design and the one indicated by the trace. [Therefore the chairman of the committee has not personally examined the stone.] If a modification towards this average height does not damage the esthetic effect (!), what I do not believe, *I will recommend that the height of the roof should be fixed in accordance with what the old coping stone* (*that is, the stone found*) *indicates* [!!!] In this way the ridge will be situated

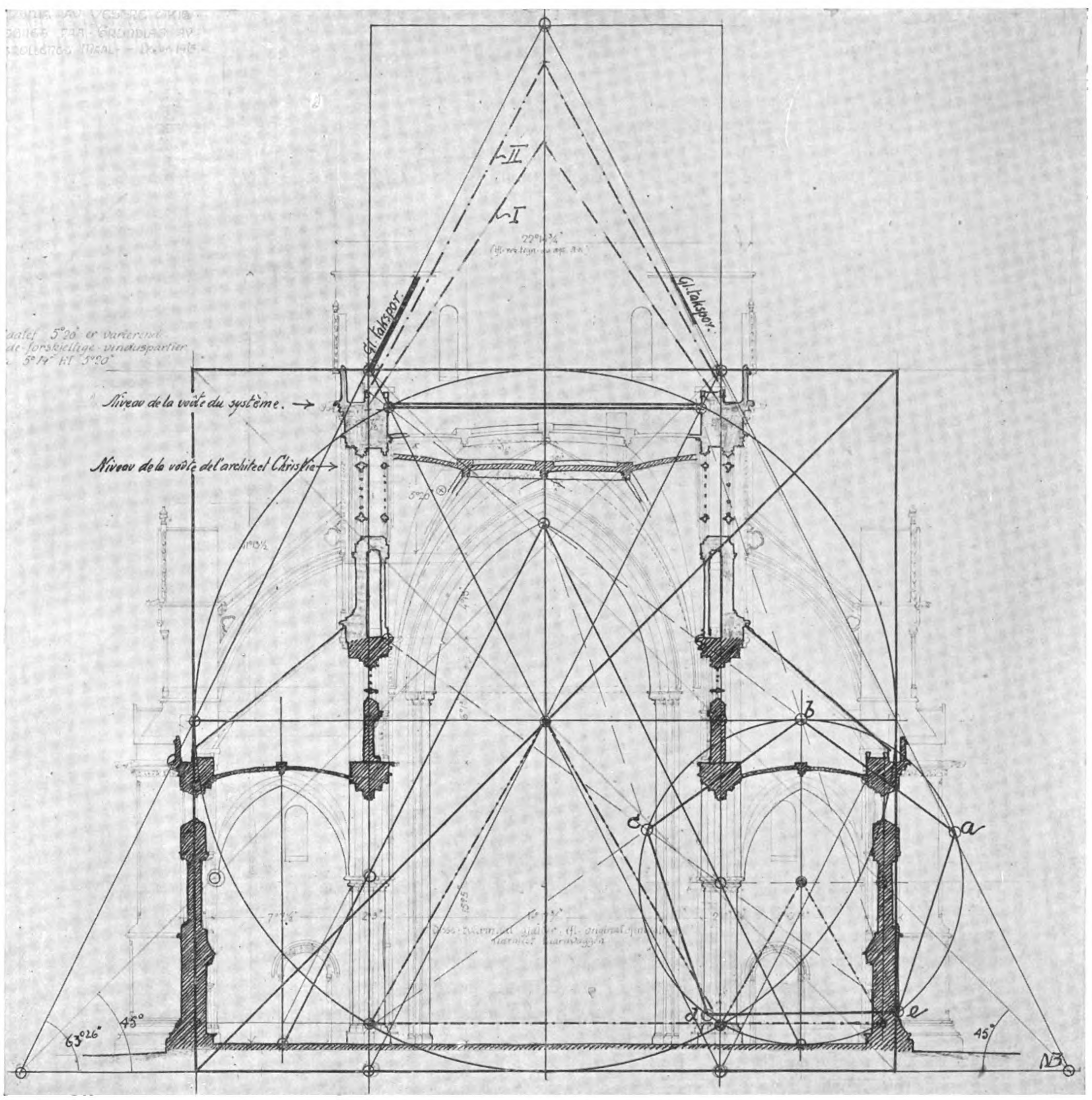


Fig. 290.—Cathedral of Nidaros. Transverse section of the nave after Nordhagen.

about 1 ell under the summit of the old trace, and the difference could be made still smaller if the edge of the coping was somewhat raised [!]. *To disregard clear archæological demands without necessity can scarcely be recommended* [!]." See the publication issued by the Ministry of Education (Documents concerning the last scheme of O. Nordhagen for the rebuilding of the west front of the cathedral (alternatives I, II, and III, 1911).)

To make a small loose stone more important as an archæological document than a fixed trace on a wall, several ells long, is quite in accordance with the obtuse way of thinking which makes two *professors of architecture* at the Technical High School of Trondhjem, after measuring the trace, state that the angle gives 63° 20′ with the horizontal plan, *without taking the hint, that this accidental*

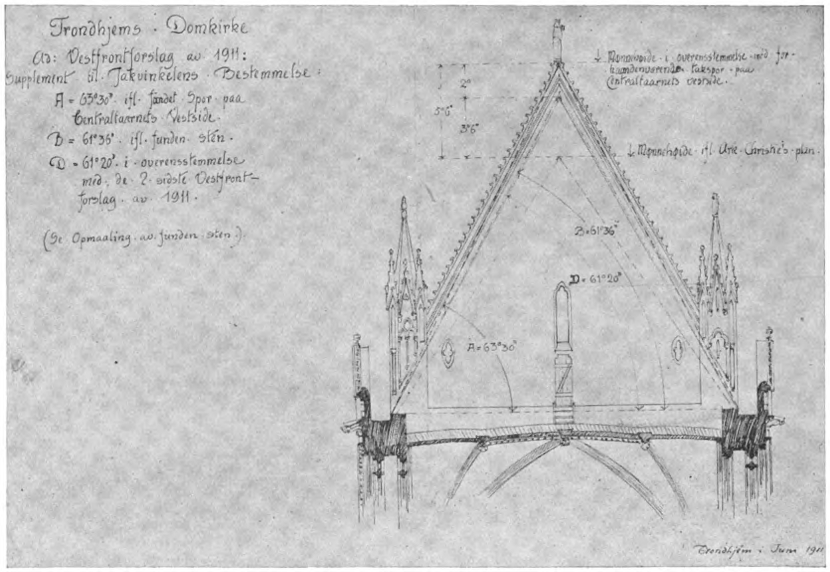


Fig. 291.—Cathedral of Nidaros. Documentation of the angle of the roof by Nordhagen based on the "skew corbel."

angle, having no geometrical connection, is only due to their inaccurate measuring of the angle **63° 26′**, *produced by the diagonal of a rectangle with proportion of* 1 : 2, *and this inaccuracy is the result of their empirical, artisan-like way of measuring*. It is all the more compromising as it shows that they have not connected any idea with the official statement made earlier by Mr. Solberg: that the existing part of the medieval church shows, in a conspicuous manner, this proportion of 1 : 2.

Ever since October 1894 there has been in the archives of the cathedral a measured drawing of the trace of the roof (fig. 292). As shown by our analysis, Mr. Nordhagen's measuring of the angle of this trace is not even correct, as it does not appear to have at the base the angle of 63° 20′, but exactly 63° 26′. It shows that the professors did not even use the most elementary scientific method in their investigation.

We cannot be astonished, then, at these gentlemen not understanding that, in this trace of old roof, the geometrical conception of the cathedral stands revealed, as witnessed by the trace continuing down and intersecting exactly the half-width of the front on the base of the plan of the construction. Just as in Cologne, this roof-line goes right through the geo-

metrical auxiliary construction from which the cathedral is proportioned, as shown in fig. 293. In direct connection with it, we notice the equally interesting fact that a line at an angle of 45° from the lower limit of the front, for example on the right, after having passed through the unquestionable height of the capitals under the ribs of the vault of the aisles, on the outer wall, runs directly to the lower edge of the old trace of roof.

It must also be remarked that all the main points in the existing parts of the side-walls of the nave up to the height of the floor of Christie's clerestory, the correctness of which we shall test further in our analysis of the longitudinal section, fall within the lines of the auxiliary construction. The case is the same with the summit of the arch under the tower, which we see is determined by the intersecting point of lines after 63° 26′ starting from the intersecting point of the axes of the aisles with the line of the floor (fig. 290). In connection with the further proportion of this arch to the building, we refer to what is demonstrated and marked in the analysis of fig. 190. Finally, it will be seen in fig. 293 that the propor-

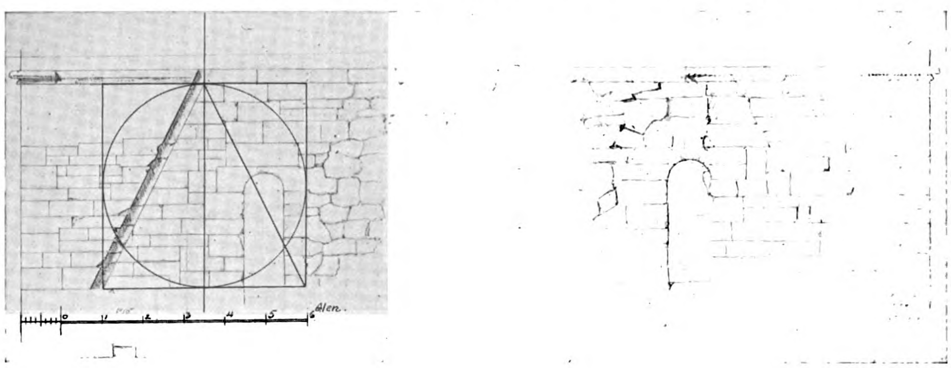


Fig. 292.—Cathedral of Nidaros. Analysis of the angle of the trace of medieval roof.

tion between width and height under the vault, in the existing aisle, is as 1 : 2—as shown by the diagonals of 63° 26′ drawn in. All these points, of which the two first alone would be enough to prove the correctness of the statement, as well as the previously indicated observations from the already analysed part of the cathedral, must form, for a reasoning mind, a connected row of irrefutable proofs in favour of its systematic construction, and prove also that it is the outcome of one united conception *ad quadratum*.

We have been able, in all security, to base our design for the reconstruction of the clerestory of the vault and of the roof on these observations (fig. 294). Just as the proportion between the octagon and the existing aisle is as 1 : 2, so is the proportion between width and height under the vault of the nave; and we found it realised also in the proportions of the bays of the chancel, the plan of which shows the length of four bays on the total inner width. The case is the same with the plan of the nave (fig. 275). When we come to examine the longitudinal section we shall see that the principle of the three-aisled church, the division of the square into four with proportion of 1 : 2, is now introduced in both plans of elevation and in the longitudinal and transverse sections, from our design done according to the indications of the plan, of the trace of the roof and of all the other points.

In connection with this, we must mention one of Mr. Nordhagen's arguments, continually brought forward in later years, against the correctness of our vault and thereby against the whole theory and the system.

This argument was immediately proved unfounded in 1915, but as, in spite of it, Mr. Nordhagen has continued to bring it forward during the last four years, we shall demonstrate here its unreliability.

In 1910 Mr. Nordhagen found, under the outer surface of the west wall of the central tower, only 2 ells above the capital, carrying the vault on the south side of the nave, a rift, or uneven slit, which was supposed to be a trace of the original vault of the nave.

During the course of these four years, in his contest against our work, Mr. Nordhagen has maintained that this "trace of vault" is *sharply marked on a length of 2 ells 8 inches*, and that "*through this most interesting find the transverse shape of the original vault can be approximately fixed*." As the vault thus reconstructed gets quite a different transverse section—according to Mr. Nordhagen's statement—than the one produced by the principle *ad quadratum*, it follows that the last-named vault, and with it the whole principle, must naturally be wrong.

We shall give the following exact information concerning this "conclusive" argument of Mr. Nordhagen.

(1) "The trace of vault" was found in April 1910, it was photographed, measured, accurately described, and then at once again walled in, and it has remained so ever since.

(2) The account given of the *sharply marked shape, etc.*, in support of the statement concerning the value of the trace as an archæological document, stands, since 1915—that is, four years after the trace had been walled in—in open contradiction to the official report of 1910 in April given by Mr. Nordhagen himself when he still had the trace before his eyes—a report which is *not mentioned* by himself, nor by the committee, in the heat of the discussion.

We quote the following part of this report:

After having named this interesting find, "*which can be thought to have some influence upon the problem of the vault*," Mr. Nordhagen continues directly: "*in the meantime I must state that this trace does not give a more satisfactory basis than the present* [loose fragment found by Christie] *fragment of rib for the transverse shape of the vault. As the trace follows the wall arch for a length of 2¼ ells, with a fairly uneven edge down on its surface* [!] *there could be described a series of arches* [!] *with radii varying between* 12 *and* **15** *ells, which would all coincide sufficiently accurately with the mentioned trace* [! !]."

(3) The vault strictly produced by the principle has accurately a radius of **15** ells—see transverse section (fig. 190), with the centres of the vault drawn in two summits of the pentagram.

This needs no comment.

A further demonstration of the raising of the nave in the transverse section, according to the system, is superfluous. We need only remark that the lower end of the trace of roof coincides exactly with the upper side of the square of the auxiliary construction, which shows that the construction of Nidaros cathedral is not raised, like the later one of Cologne, to the plan of the plinth.

There only remains now to explain the system of abutment which is so closely connected with a vault construction.

We refer here to fig. 294, the transverse section of the nave, where we have introduced the pentagon and the square placed diagonally, giving together the place where the flying buttresses had to be built to bear the thrust of the vault, as shown before. As it is noticed by comparing figs. 289 and 294, we have made an important change in the abutment system of Christie, reconstructed according to the indications of the pentagon and the square.

In fig. 295, Christie's reconstruction of the system of abutment and ours are put side by side for comparison. In the previous fig. 279 the stones can be seen, on which Christie has based his reconstruction of the flying buttress. In fig. 295, I, these stones are given in profile, the real voussoir marked *b*, and the "coping-stone" marked *a*. In our drawing of

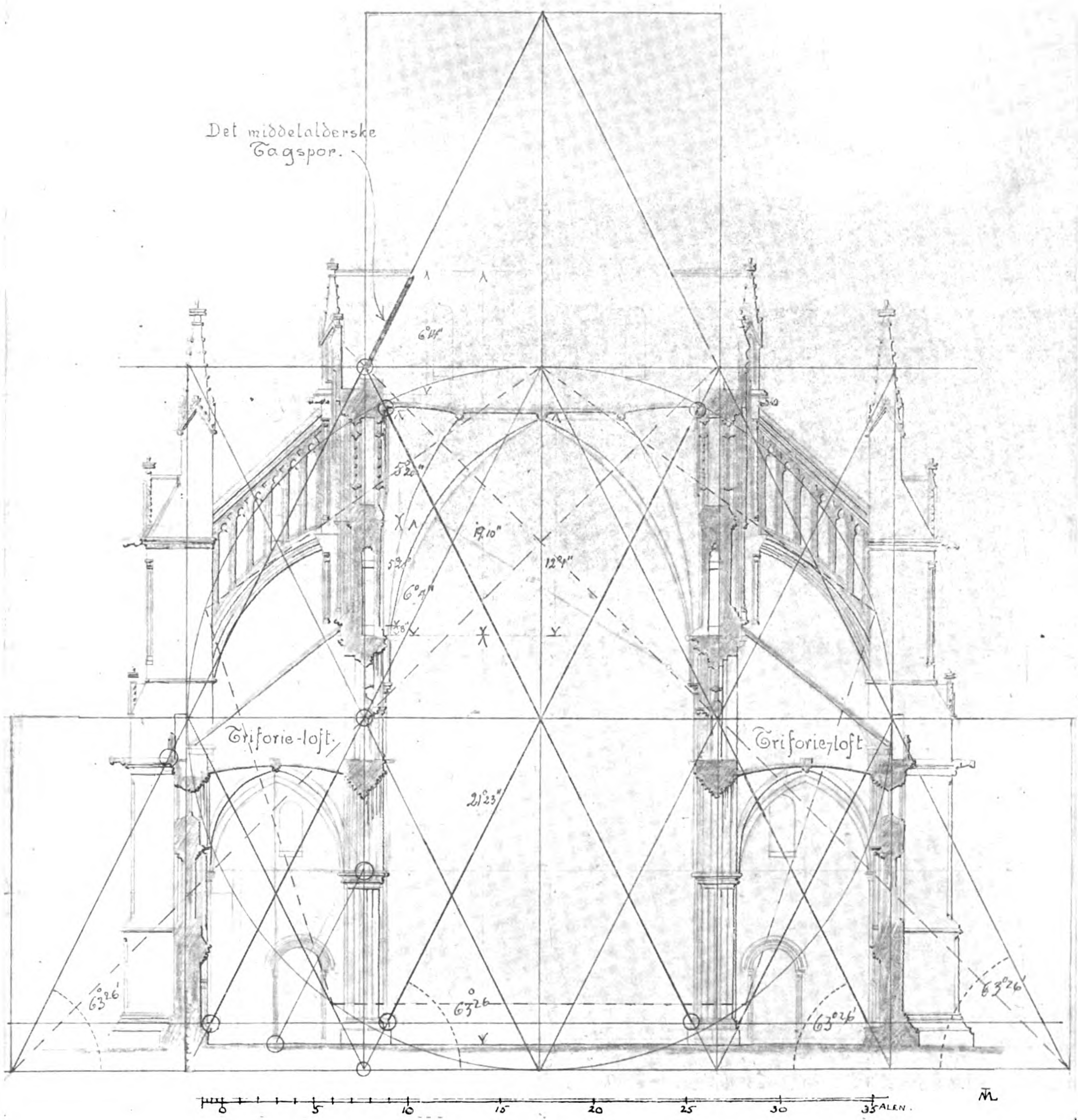


Fig. 293.—Cathedral of Nidaros. Transverse section of the nave by Macody Lund towards east, with clerestory, vault, buttresses and roof heightened according to the proportions of the cathedral.

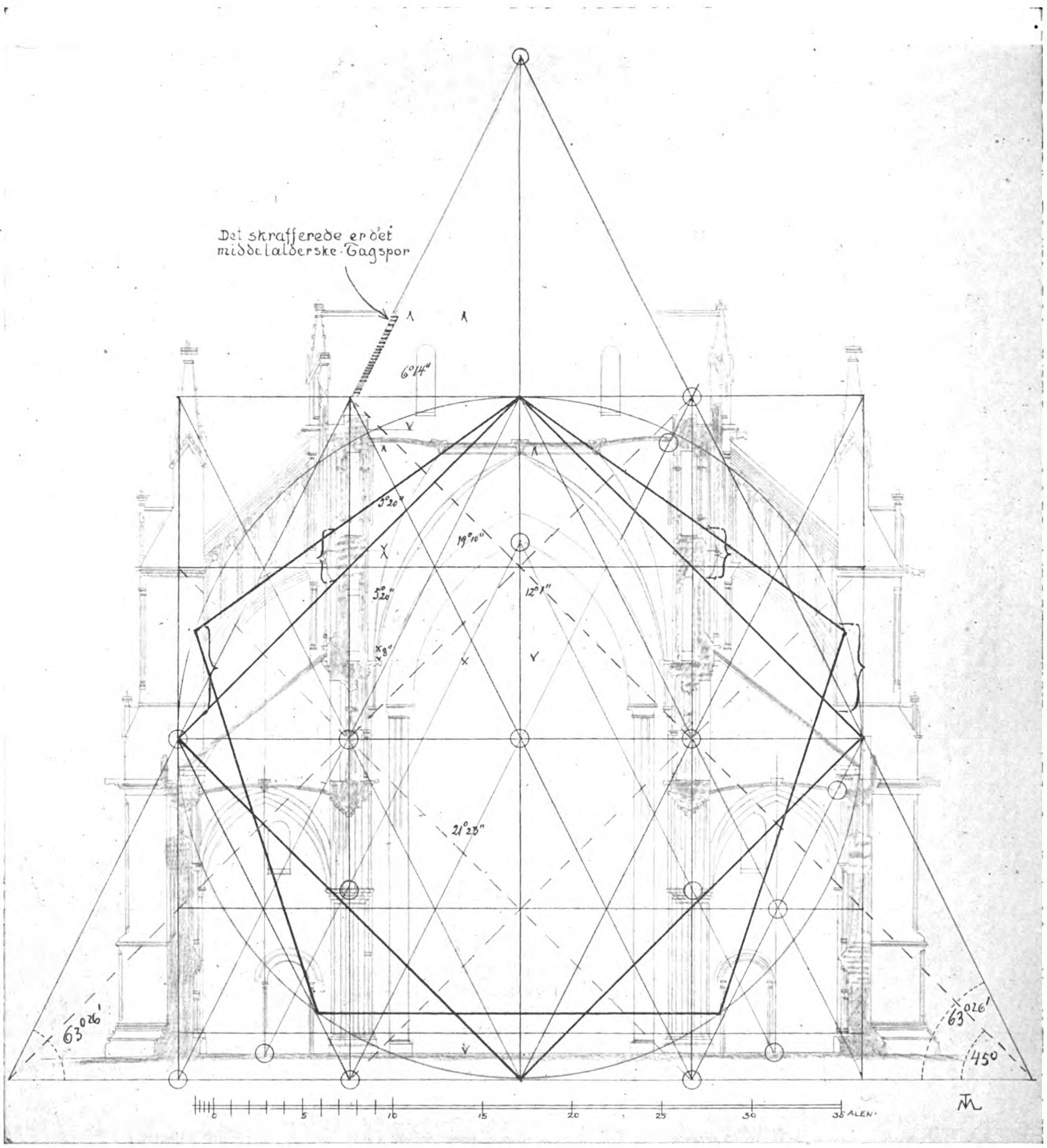


Fig. 294.—Cathedral of Nidaros. Macody Lund's transverse section of the nave, with its system of abutment.

the reconstruction, fig. II, the same profiles of stones are given and marked with the same letters.

It will be noticed that the materials were adequate to show that the system of abutment of the nave was developed in quite a different way than for the chancel, where the flying buttress, as shown by the photograph of the south wall of the chancel reproduced in fig. 296, has been covered with a real coping-stone having a ridge—that is to say, a profile resembling the one given, for example, in fig. 295, III, and whose function was only to serve as a weathering on the flying buttress. Therefore, if Christie's reconstruction of the system of

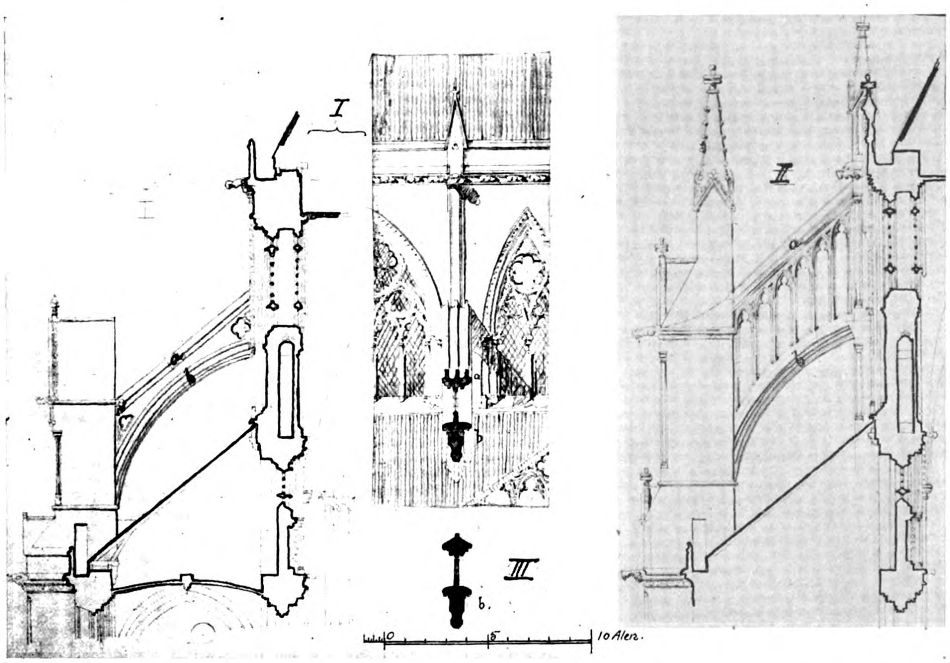


Fig. 295.—Cathedral of Nidaros. Reconstruction of the system of abutment by Christie and Macody Lund I and II.

abutment of the nave had been correct and like the original one, a coping-stone with a ridge would have been found and the flying buttresses would have had a profile as in fig. 295, III.

But, on the contrary, the found "coping-stone" has quite another profile, namely, with a twin drainage on the top, such as shown in the section (fig. 295, I*a*); consequently it has been a gutter, to receive the water from the spout of the parapet on the wall over the roof of the aisles and carry the water out through the buttress. So that when Christie uses this as a coping-stone or weathering directly on the flying buttress low on the wall, while placing just above it one spout, as in the chancel, the whole construction becomes meaningless—because here we have a double gutter placed more than 6 ells below a single spout, which can never find any part of the twin gutter, or a coping-stone having a profile like a useless gutter.

It is obvious that this twin gutter ought to have stood in such a position, as regards the gutter of the parapet, that its clearly defined profile could have been used as intended.

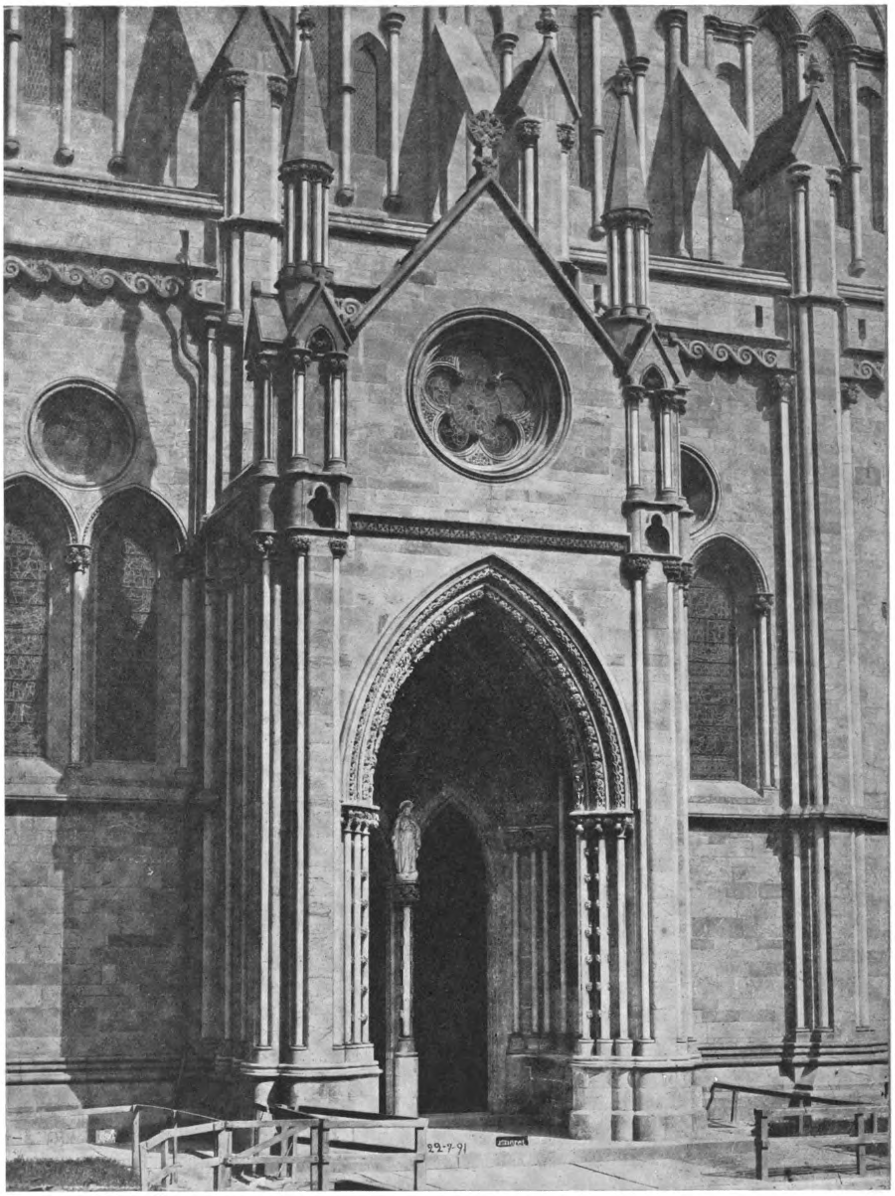


Fig. 296.—Cathedral of Nidaros. Photograph of south wall of the chancel.

We have placed it, as shown in fig. 295, II, with two fantastic monsters, one on each side of the pilaster strengthening the wall—that is to say, with one spout for each gutter.

We see, in this coping-stone with the twin gutter, indications of a construction well-known in medieval cathedrals, of combined flying buttress and gutter connected by means of tracery. As shown by fig. 295, II, we have produced the treble function of the system of abutment indicated by the fragments, viz.:

(1) A moulded voussoir, marked *b* as flying buttress, to receive the thrust of the vault.

(2) A moulding of gutter, marked *a*, to conduct the water from the roof.

(3) To counteract the wind pressure from the roof.

It is clear that the necessary increased height of the buttress against the wall of the aisles, in order to receive the double flying buttress, increases at the same time its weight and thereby its efficiency.

Our proposed construction of the system of abutment which is based upon the very fragments of the cathedral and its own geometrical system gives in reality a quite other firmness and stability to the building than the one designed by Christie and copied slavishly by Mr. Nordhagen—just as its esthetic appearance is increased by this stabilising function. The side elevation obtains from it an enchanting appearance and it raises the entire building from being a provincial church to a cathedral.

We have had the stability of our system of abutment, as also Mr. Nordhagen's, calculated by an engineer, Mr. Slettum, of the firm of Bonde and Norman, of Christiania, employed by the State as consulting engineers in bridge constructions. This is their conclusion after going through the case in detail: "In comparison with Mr. Nordhagen's, Mr. Macody Lund's design has a stability $37\frac{1}{2}$ per cent greater,—or, in other words, under the same conditions it will stand a $37\frac{1}{2}$ per cent. greater wind thrust," and they continue: "as for security against a crack, it is calculated, further, that in the design of Mr. Macody Lund it is 30 per cent. greater than in Mr. Nordhagen's, because a section at point *a* (that is to say, on the plinth of the farthest buttress to leeward) is the determining factor." It is mentioned, moreover, that the vault of our design is considerably more favourable from the point of view of stability than Mr. Nordhagen's, because among other reasons it does not lead to the rigidity of the side-walls.*

The Longitudinal Section

We are now going back to the *Octagon*, in our analysis of the longitudinal section of the cathedral (fig. 297, Pl. XIX). We see here also that the line according to an angle of 63° 26′ has determined the proportioning of the octagon—that is to say, according to the proportion of 1 : 2.

It can be seen that Christie has carried back the webs of the vault of the octagon to the original shape with the rising towards the keystone like an umbrella vault. It will also be noticed that the same angle determines the proportioning of the height of the capitals in the windows of the clerestory.

* The above referred statement, from a reliable source, puts the whole campaign of Mr. Nordhagen against our work and our commission in its true light. It is known that Mr. Nordhagen has tried to undermine our position beforehand in various ways, by claiming publicly, in a writing published at the expense of the State and with the approbation of the Ministry of Education, that, if our system were used, it would endanger the stability of the church, which argument he has tried to render plausible by calculations obtained from a teacher at the technical school of Trondhjem (see *Throndhjems Domkirke*, Mr. Nordhagen, October 1915). As the above mentioned calculations rest upon some tendencious drawings of Mr. Nordhagen himself, which are supposed to render our system, but in reality have nothing in common with it, his statements are useless. We claimed then at once that this production of Mr. Nordhagen's was a falsification, and we have good reasons to repeat it here.

We remember, from our analysis of the plan and the transverse section, that the rebuilding was tied to the fixed points in such a way that it had to be done on the foundation of the Norman chancel, and that the medieval architect, in spite of these difficulties, had succeeded in carrying out the building according to the principle *ad quadratum*. He has also succeeded in doing this in the raising longitudinally, because, in spite of the fixed points, he has understood to carry out the principle with four bays in the square, as shown by lines NB^1 and NB^2, and not as in Lincoln, where there are three, and as in Wells, where there are four and a half bays, whereby the vertical construction is out of proportion, as we have demonstrated.

We remarked before that the reconstructed vault of Christie can be looked upon as the original one. The same can be said, after what we have indicated before, of the side-walls as to their principal points. The proportioning of the windows of the clerestory is not correct, however, because the capitals should have been placed higher; see the drawing of the second bay from east. As the diagonals drawn in will show, the standing part of the chancel has the proportion of 1 : 2, for example, the twin lancet windows of the walls of the aisles and the arcading underneath. On the drawing, some lines give the various proportions of the sectio aurea, carried out also in arithmetical equations.

If we continue now to the *central tower*, we see that its principal points are also placed *ad quadratum*, just as it will be seen that the proportion, from the level of the floor of the triforium of the tower to the capital of the pillars and from here to the plinth, is according to the sectio aurea. This proportion is also used in the south gable wall of the transept in the alteration of the stage of the triforium, carried out in early Gothic times.

As regards the analysis of the nave, we are obliged again to consult the *plan* (fig. 275). It will be seen from the diagonals in strokes according to 45° that, both longitudinally and transversally, the plan is divided according to the principle of a three-aisled church: the square divided into four, there being the length of four bays inside each of the two squares on the interior width of the nave, as shown by the above-mentioned diagonals. We shall explain later the reason why the bay farthest west is shorter.

As regards the *side-walls* we have remarked before that the two lower stories up to the clerestory are reconstructed accurately by Christie, so to speak, within an inch.

We have proved previously that, in all its plans, the chancel is built strictly *ad quadratum*. The case is the same with the plan of the nave. It will be seen equally, as regards the latter, that the remaining part of the walls of the aisles and the part of the walls of the nave up to the clerestory, as reconstructed by Christie upon remaining parts, coincide exactly in the geometrical auxiliary construction, as proved by the diagonals with rings, introduced in fig. 297, Pl. XIX.

In accordance with these observations, we have determined the height of the side-walls up to the vault *ad quadratum*, on the length of four bays. Each double bay gets thus here, in the nave, a clear proportion of 1 : 2. And the side-wall, in its totality, just like the plan, will be designed within two squares on the interior width of the nave. On the west of the central tower we shall carry out the proportion which exists in the east part of it, which is medieval. A connected correspondence will be obtained thereby in the system of vaulting from east to west, right through the church; this is shown by the steep diagonals according to 63° 26′, starting from the line of the plinth of the east wall of the feretory and deflected in turn from the vault and from the line of the plinth—these points of deflection marked by two rings.

While the deflection points in the nave, where parts of older buildings have not stood in the way, coincide exactly with the axes of the pillars, there are divergences in the chancel, due to obliqueness in the plan. Anyone can convince himself of it by taking the average width along its longitudinal axis. But the determining factor is, that the deflected diagonal coincides with the ridge rib of the medieval vault, and in doing so it shows a distribution

of four bays within each square; this distribution is thus proved to go through the whole design, from east to west, as the circles traced in the plan (fig. 275) will show.

An interesting evidence of the correctness of our reconstruction of the side-walls of the nave is given by the fragments found and put together by Christie in his incorrectly reconstructed clerestory window.

We take this opportunity of saying that Christie had no direct guidance for the rebuilding of this part, as only loose stones could be found belonging to it; and, as the laws of proportion applying to the religious architecture of the past had not yet been discovered, he sought the guidance for this reconstruction in Lincoln cathedral, according to the acknowledged chronology.* We have noticed several times before that this church had so much in common with Nidaros cathedral in the character of its ornamentation that it has been thought that one cathedral had imitated the other. As the facts have been wrongly stated, on account of the before mentioned inexact chronology of the cathedral of Nidaros, it has been thought that the latter was an imitation of Lincoln, even designed by the same architect.

Starting from these false suppositions, Christie thought he could get there the truc model for the reconstruction of the clerestory. He had nothing else to rely on than the fragments of the rose tracery of the windows and fragments of columns under these, as shown by fig. 279, in addition to the springers of the vault whereby the character of the latter could be settled as a "star vault" (really a rib vault with ridge ribs longitudinally and transversally and "tiercerons," viz. a rib on each side of each diagonal rib, whereby a treble diagonal system is developed). The present illustration (fig. 298) shows how slavishly Christie has copied his model. It must be remarked, by the way, that for the reconstruction of the two lower stories, and their decorative ornamentation, Christie had full material in the remains found (see fig. 279).

We have indicated previously, and we mention again now, that the wrong proportioning of the elevation of Lincoln is due to the fact that there are only three bays in the square of the plan, whereby each double bay gets a proportion of $1 : 1\frac{1}{2}$, instead of $1 : 2$. It is clear that such an extension of the length of the bay must bring with it a heightening of the lower arcade at the expense of the two stories above, within the given height of the square. In its proportions, therefore, Lincoln cathedral is a misunderstanding, and, to judge from the given chronological data and illustrations, it must be considered as an imitation of Nidaros. We have already shown, in our analysis of the chancel of the latter, how genially its *Magister operum* has succeeded in carrying out the principle *ad quadratum*, and we have seen equally in the plan of the nave that here this principle is completely carried through—that is to say, with four bays within each square, or eight bays in the two squares for the whole nave. Therefore, when Christie fetched his model from Lincoln, it must follow that his reconstruction of the clerestory had to bring about the same disharmony as in his model, causing a complete break with the principle of proportioning, existing in the remaining parts. In the longitudinal section of Christie (fig. 297, Pl. XIX), we can see the compressed, inharmonious result of this imitation without critique and against historical evidence.

In fig. 299 we have reconstructed the window. The fragments found and put together by Christie are tinted. As it was to be expected, the top part, remaining from the Middle Ages, is constructed perfectly *ad quadratum*. In accordance with it, we have reconstructed also the lower part, from the upper edge of the capital to the upper edge of the base, according to the same principle. We obtain thus a window according to the proportion of the church itself of $1 : 2$, with a collective height of 12 ells from the floor of the clerestory to the top of the window.

* The chancel of *Lincoln cathedral*, with the small east transept and a part of the large one, was begun in 1192 and finished in 1200; the nave and the west front were finished about 1254. The angel-choir, which Christie took as model for his reconstruction of the clerestory of the nave, was first finished in 1282.

For comparison we have put side by side (in fig. 300) the reconstruction of Christie's window from his model in Lincoln, the window of Mr. Nordhagen and the committee, heightened haphazard and now being carried out, and our window reconstructed according to the system of the church itself.

The correctness of the latter must be apparent when the following measures are considered —the height from the floor of the church to the floor of the clerestory is correctly taken by Christie as 21 ells 23 inches, and the height from the floor to the beginning of the vault is, according to the system, 34 ells 12 inches, including the wall arch. Between the floor of the clerestory to the lower edge of the wall arch, the height will be therefore, when the

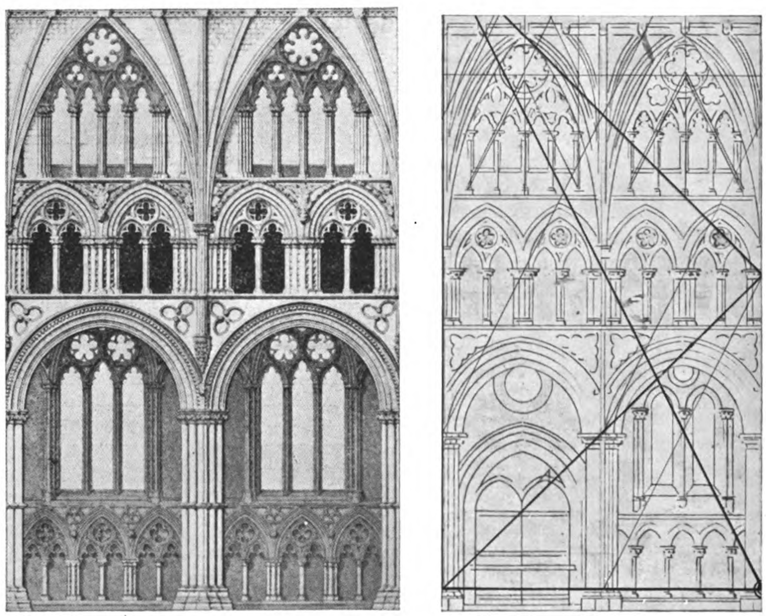


Fig. 298.—Cathedral of Nidaros. The angel-choir in Lincoln cathedral compared with Christie's reconstruction of the clerestory. The angel-choir is reduced to the scale of the Nidaros cathedral.

wall arch is reckoned as 12 inches, 34 ells ÷ 21 ells 23 inches = 12 ells 1 inch, *which is precisely the height of the clerestory window according to the geometrical system of the cathedral.* In other words, the window settles in its place automatically and it coincides exactly within the given frame of the system of the side-wall, as proved clearly and overwhelmingly in the reconstructed window of the third bay from the central tower, in the longitudinal section (fig. 297, Pl. XIX). We refer also to the measures of height written in the transverse section and showing the connection; we must also remark that the diagonals put in the longitudinal section show that the capitals of the window correspond with the principal points of the wall.

What we have said must be acknowledged as an irrefutable evidence of the correctness of both the reconstruction of the two lower stories by Christie and of our own reconstruction of the clerestory.

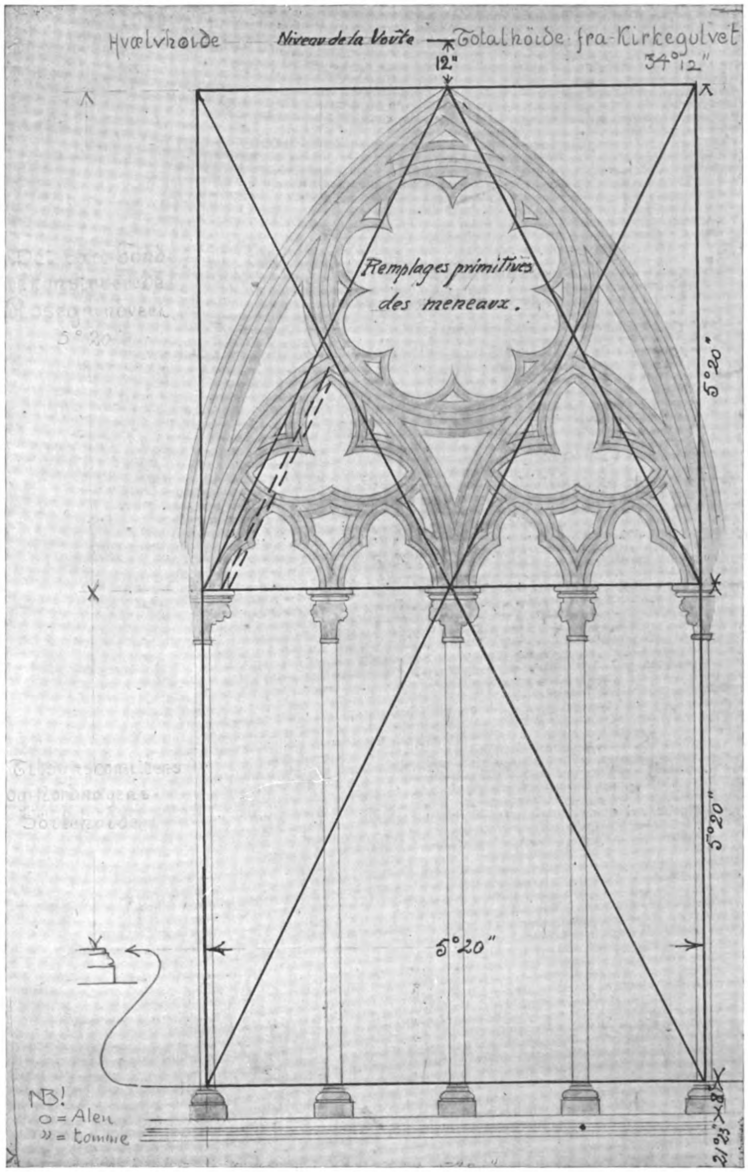


Fig. 299.—Cathedral of Nidaros. Window of clerestory of the nave reconstructed *ad quadratum*.

As another interesting evidence of the correctness of the latter we shall point out that, in order to get room at all for the smallest possible central window in his west front, Christie was obliged to lift the ridge rib of the vault of the nave in the last bay towards west, to a point coinciding with the vault produced by the very geometrical system of the church; see the longitudinal section (fig. 297, Pl. XIX). This height of vault is therefore necessary to give the front its free and natural architectonic growth and to bring about the necessary connection between the architecture of the front and of the side-walls both for the interior and the exterior.

The raising of the clerestory, as of the whole wall, is done from the indication contained in the plan, showing the length of four bays within the square, the constant fundamental

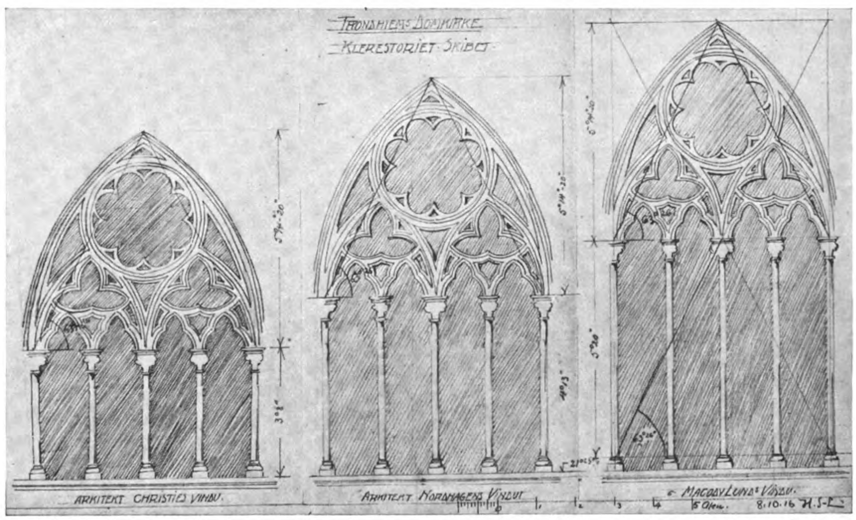


Fig. 300.—Cathedral of Nidaros. Window of clerestory, after Christie, Nordhagen, and Macody Lund.

proportion of 1 : 2. It is evident that this proportion must get its full corresponding expression in the exterior, so that not only the remaining part, but the clerestory also and the whole wall up to the cornice should be included, according to the same fundamental proportion, within a square on the length of four bays, with its base on the upper edge of the plinth, as shown clearly in figs. 301 *a* and *b*. There exists now a correspondence in the exterior as in the interior, between the principal points of the elevation, and there will be the same harmonic proportion between the three stories inside and outside.

Above the windows of the clerestory some canopies are placed externally in connection with the pierced balustrade of the parapet. This arrangement of canopies is founded on fragments which are reproduced here in figs. 302 and 303. This find was considered earlier, by Christie and other architects, as belonging to the large central window of the west front. We shared this view ourselves, but, when measuring them, however, and when experimenting with models executed from the measured drawing, we soon came to the conviction that these fragments were too small and not sufficiently divided to have belonged to a central west-front-window about 30 feet broad, where the canopy would reach a height of 94 feet above the ground, and also,

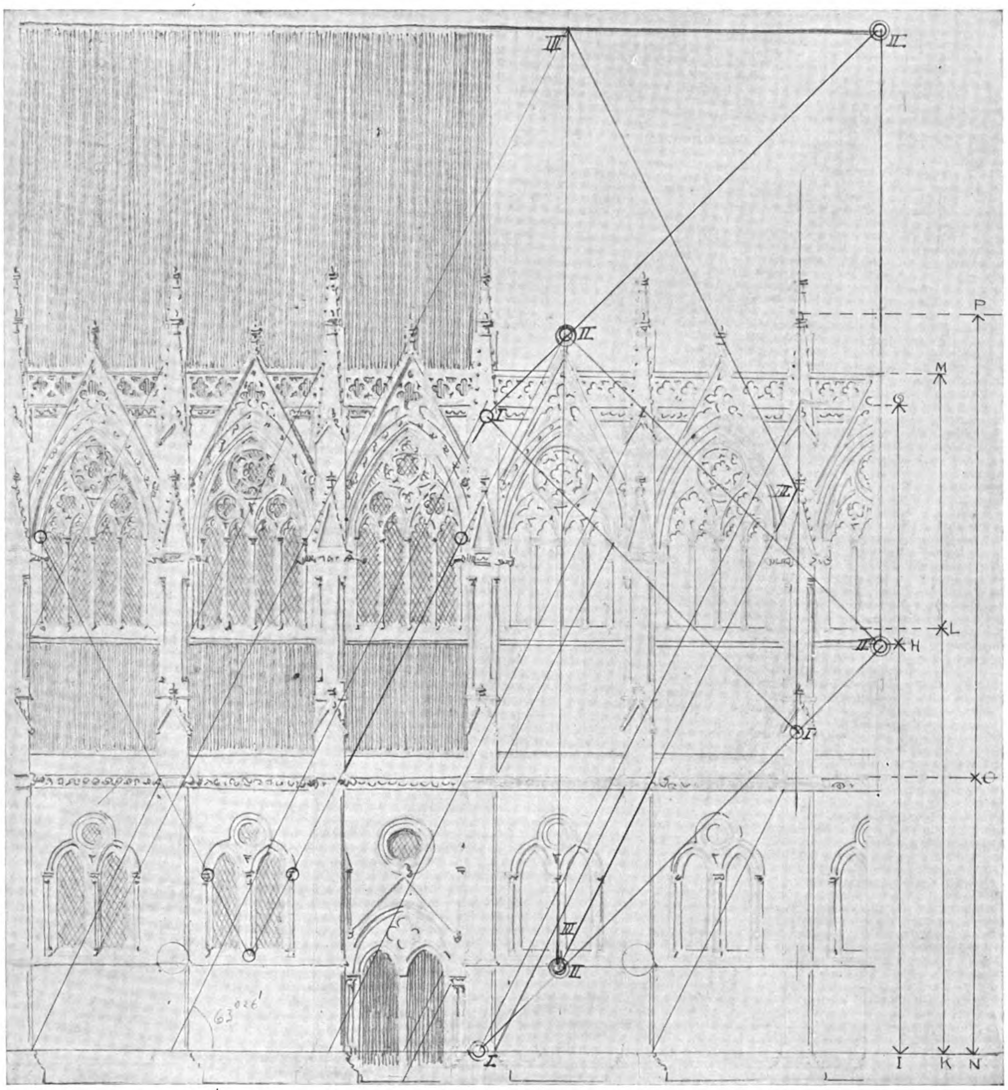


Fig. 301*a*.—Cathedral of Nidaros. Part of side-wall of the nave, exterior reconstructed *ad quadratum*, according to fragments found.

it must be remembered, where it would receive and co-operate with the balustrade on the powerful corbelled cornice, necessary to such a screen front.

Those remains show, however, that this canopy did not have a projection greater than nine inches altogether. Of these, as it appears from fig. 303, six inches are taken by the

balustrade with which it combines. There only remains for the profile of the gable a projection of three inches. The moulding, as will be seen, is indicated. These facts are sufficient to relegate this canopy with balustrade to a light situated lower down and of smaller dimensions. From their characteristics these fragments have indisputably belonged to the cathedral; in the meantime there is not to be found in the whole building any other light where they can be placed in connection with the parapet other than the clerestory windows of the nave. It will be seen from fig. 301*a*, that the canopy corresponds accurately with the lines of the system, while it acquires a great artistic value.

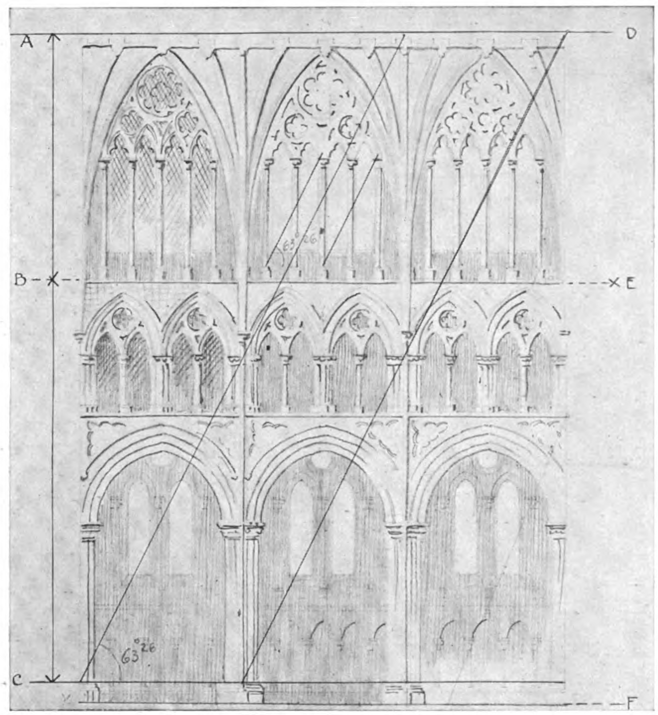


Fig. 301*b*.—Cathedral of Nidaros. Part of side-wall of the nave, interior reconstructed *ad quadratum*.

It has been argued that this decoration being French, it has nothing to do with the cathedral, which, according to the old theory, is entirely English in character. In answer to these commonplaces, it is sufficient to say, after the previous developments, that the cathedral is neither French nor English. It is situated in Nidaros, in Norway, and it is older than the buildings which are supposed to be its models. As we have shown, it is a construction quite independent in character, created from a single conception of design, built by a people who, in the Middle Ages, had travelled more than any other, and who were in no way behind others in artistic development.*

* Note.—In connection with this we advise our readers to consult the *Quarterly Review*, 1877, p. 51 and following.

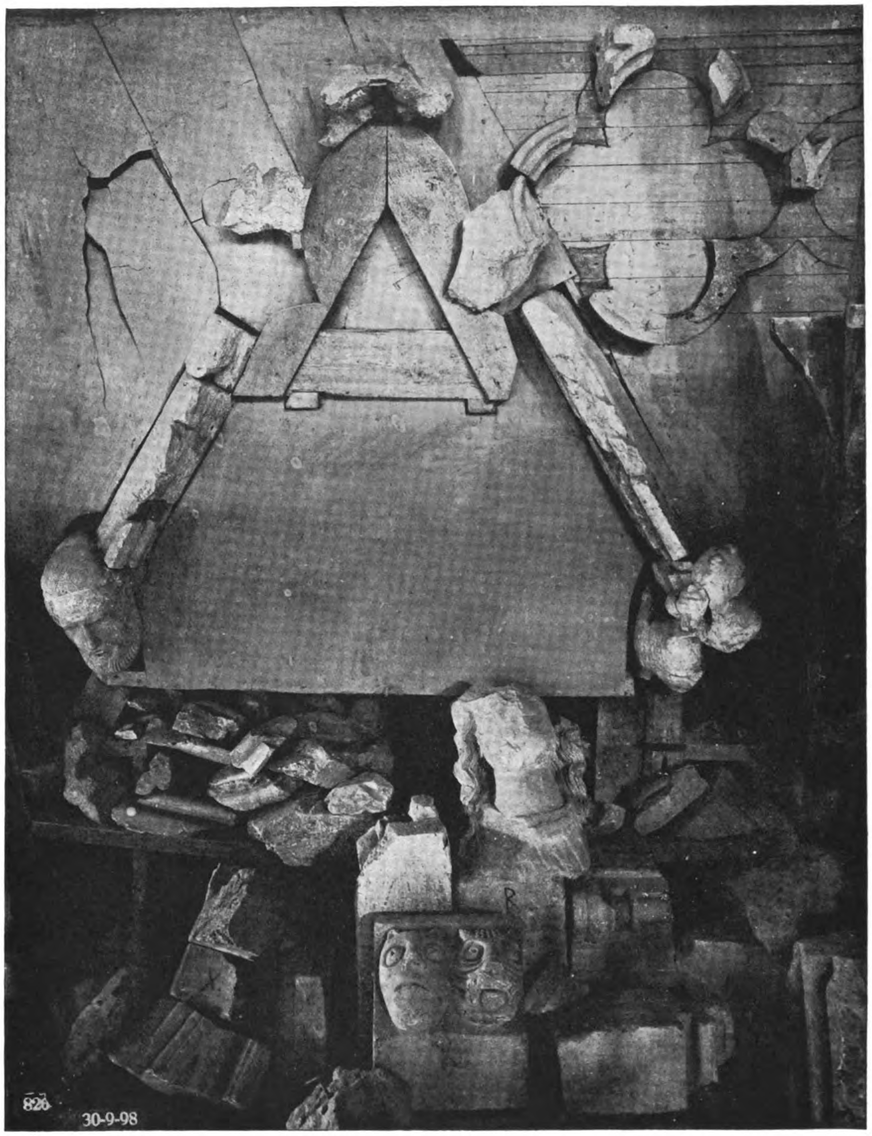


Fig. 302.—Cathedral of Nidaros. Loose fragments of canopy, photograph.

In order to appreciate our alteration of the side-wall we have put together (in fig. 304) four alternatives for the raising of this wall.

First alternative reproduces three bays of Christie's north wall.

Second alternative reproduces part of the drawing with raised clerestory, which the committee, after we called attention to the wrong height of Christie's wall and roof, gave as an accompanying document in the architects' competition after Christie's death.

Third alternative reproduces Mr. Nordhagen's north wall approved up to the main cornice

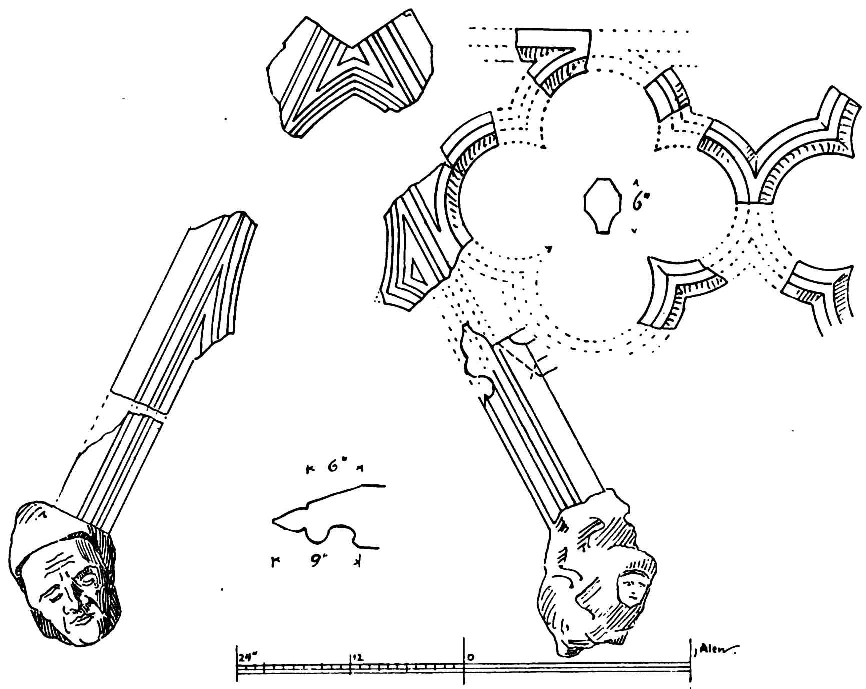


Fig. 303.—Cathedral of Nidaros. Measured drawing of the fragments of a canopy combined with a balustrade, photographed in fig. 302.

in 1912. As it will be seen, this is exactly like the preceding one, but with an open balustrade on the parapet and some small crockets on the finials.

Fourth alternative reproduces finally three bays of our archæological reconstruction of the north wall. The comparison is obvious.

For further appreciation of our elevation of the side-wall, we have introduced, in the previous figs. 301 *a* and *b*, various lines to which we refer. After our earlier examination of the system it needs no further explanation to understand that the exterior, as the interior, longitudinally and transversally, is strictly determined *ad quadratum* in our design, not only as concerns the wall, but, as it will be seen, right up to the ridge. We get further conclusive proof therein of the authenticity of the trace of the old roof, and its connection with the system of the church. There is no room here for the haphazard attempts of our two architects,

First alternative. Second alternative. Third alternative. Fourth alternative.

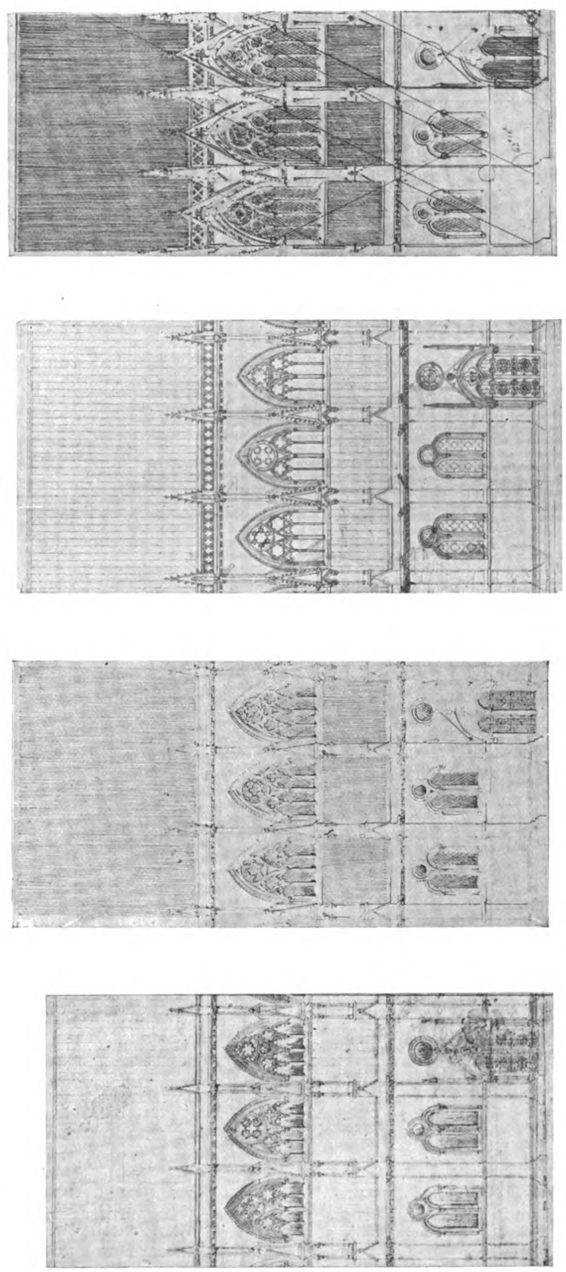


Fig. 304.—Cathedral of Nidaros. Comparison of the four different reconstructions of the side-wall of the nave exterior, proposed by Christie, the Supervising Committee, Nordhagen, and Macody Lund

which are a gross breach of those proportions so completely according to the laws of geometry governing this harmonious building.

* * *

It might seem natural or convenient to continue this partial analysis of the exterior of the side-wall and examine it in its entirety. But, as the side elevation must stand in intimate relationship with the elevation of the front, we prefer to examine this first and to make note of its correctness. Before we begin our task, however, we shall first end our remarks upon our reconstruction of the side elevation with a demonstration showing that the proportioning of the principal parts coincides strictly according to the sectio aurea—that is, in complete accordance with the old remaining parts—with the octagon, the pierced chancel wall, the central aisle of the chancel, the central tower and the transept, both its Norman part and the parts reconstructed in the transition style.

It will be remembered, from the analysis of our reconstructed transverse section of the nave (in fig. 190), that we found its distribution proportioned according to a geometrical harmonic progression, the development of which is based on the diameter of the columns forming the arcade of the aisle, used as unit—the modulus. We found, in this way, that the square of the design had a side length consisting of 22 such moduli.

The diagonal of the pentagon in circle I in that drawing has 21 moduli, the diagonal of circle II has 13, in circle III 8, in circle IV 5, and in circle V 3; in other words, the progression of the "sectio aurea" 3 : 5 : 8 : 13 : 21 : 34, etc. The side of the decagon in circle V gave this modulus. It is obvious that a transfer of these geometrical values in round arithmetical numbers can only be approximate.

We have applied this modulus = the diameter of the column to two of the buttresses in fig. 301*a*. When we see how our construction of the side-walls, of their main lines, has led to the proportioning according to the sectio aurea, it will be important to control also the distribution vertically. It will be no less important to establish the correctness of the position of the canopies.

It will be remembered that, in our analysis of fig. 190, we found that the height from the floor of the church to the lower edge of the trace of medieval roofing was exactly equal to 21 moduli = the diagonal of the pentagon in circle I.

As it may be interesting, however, to use this rough arithmetical "Vitruvian" measure, the modulus, on the side-walls also, we must first give a progression of moduli proportions starting from the plan of the plinth. The reader can ascertain for himself the correctness of these by means of compasses.

In order to compare the proportions between the remaining medieval wall and our reconstructed clerestory, we begin with the original wall of the aisle. In the various heights, all reckoned from the plinth of the foundation. There are:

Three moduli in the height up to the plinth under the windows of the aisle.

Five moduli to the capitals in these windows.

Eight moduli to the cornice of the aisle.

The reconstruction of the wall of the clerestory coincides harmoniously with the following heights:

Thirteen moduli to the lower part of the columns belonging to the clerestory windows.

Twenty-one moduli to the top of the canopy.

Here it may be interesting to add in advance that the height to the top of the gable of the west front contains 34 moduli; see the scale of moduli introduced in fig. 314, Pl. XXIII.

A more accurate result than through this synthetic manner is obtained naturally by an exact

geometrical analytical measuring. As an example of this we give the proportion of the heights to the geometrical figures. We indicate, consequently, that the height from the plinth to the cornice of the aisle = the side of a pentagon in circle II, and the height from here to the top of the canopy = the side of the pentagon in circle I. Similarly the height from the base of the columns of the clerestory window to the top of the canopy = the side of a pentagon in circle II. Furthermore, the height from the lower edge of the cornice of the aisle to the upper edge of the plinth of the clerestory window = the side of the pentagon in circle III. The height from the string-course under the window of the aisle to the lower edge of the cornice of the aisle = the side of pentagon in circle III.

As regards the interior it will be seen that, just as we raised the clerestory according to the system of the church, so the proportion between this story and the increased height of the wall is according to the sectio aurea.

The height from the plinth to the lower edge of the string-course of the clerestory = the side of the triangle in circle II and the height from here to beneath the web of the vault near the wall = the side of the triangle in circle III, that is to say, the proportion of the sectio aurea. Finally, we remark that the height from the plan of the plinth to the upper edge of the base of the columns belonging to the window of the aisle and from here to the top of the vault of the aisle = the side of the pentagon in circle IV and the side of the pentagon in circle III respectively and are in proportion to one another according to the sectio aurea, which is also the case with the height from the floor to the upper edge of the capital of the column belonging to the main arcade and from here to the lower edge of the string-course of the triforium.

The height from the upper edge of the plinth to the string-course of the triforium = the side of the pentagon in circle II = 8 moduli. Moreover, it can be said that the axial distance of each double bay = the side of a square in circle II, and that consequently two sides of this square = the height of the wall from the plinth to the lower edge of the cornice of the nave.

We have thus been able, with the reconstruction of the transverse section carried out, to convince ourselves that the rediscovered ancient auxiliary method *ad quadratum* and *ad pentagonum* has brought the rebuilt clerestory, inside and outside, with its canopies over the windows, in full accordance with the remaining medieval part, and we have found that the proportioning of the principal features agrees strictly with the antique rules of religious architecture, which we saw were the secret of the wonderful harmony of the Doric temples—the same rules which we have previously proved to have been used in proportioning the cathedrals of Paris, Laon, Cologne, York, and Wells. Rheims cathedral will be analysed in an appendix. Later on it will have to be acknowledged that this progression of harmony, in which the various features of the side elevation are tuned, includes also the distribution of the front and all the different distributions of the three towers—right up to the top of the spire of the central tower, whereby the cathedral, the old and the new parts, are tuned together in a harmonious unity.

Our principal task, the reconstruction of the west front, is now the next decisive point.

CHAPTER XVIII

THE RECONSTRUCTION OF THE WEST FRONT

WE have proved in the preceding explanatory analysis that the principle *ad quadratum* has been originally used in the design of the remaining parts of the cathedral of Archbishop Eystein. It will also be acknowledged that our reconstructed clerestory with the roof according to the same method, grew naturally from the medieval remains. We shall now make use of the theoretical principle in the much greater task of reconstructing the missing upper part of the west front. Of this there remained only in our time what can be seen in the present reproduction of a photograph (fig. 305) taken before the restoring. These standing remains of the front were restored by Christie, which accounts for the three porches being again open.

As regards the appearance of the front, we have only general descriptions, foreign as well as Norwegian, right from the beginning of the sixteenth century, by men who had either seen it or described it from other people's accounts. They are unanimous in their praise of the church as being unique in splendour and beauty. The oldest evidence known to us dates from the famous German writer, Sebastian Münster,* in the sixteenth century, who says, in his *Cosmographia Universalis*, Book IV, chap. xxii., that "the cathedral of Nidaros is a great evidence of the splendour and the magnificence of the ancients," and that "in the whole of Christendom its equal will hardly be found as regards size and as to the skill of its sculpture." Another German, Martin Zeiller,† says in his *Neue Beschreibung der Königreiche Dänemark und Norwegen*, Ulm, 1658, p. 251 (see *Minutoli*, p. 19, note 1):

"To this damage was moreover added that, in the year 1530, the cathedral was ruined by fire, so much so that only the choir remains at present, and here they still sing and preach. This church had formerly been the most beautiful building in the whole of Europe, even of Christendom, being made entirely of artistically cut stones and pillars: the old walls can still be seen." ("Zu welchem Schaden Anno 1530 noch weiter dieser gekommen, dass auch die Domkirche durch Feuer ins Verderben gerathen, dass zu dieser Zeit nichts mehr als der Chor, darin noch gesungen und gepredigt wird, vorhanden ist; da vorhin diese Kirche das allerschönste Gebäu in ganz Europa, ja in der ganzen Christenheit gewesen; als die von eitel ausgehauenen Steinen und Pfeilern, sehr künstlich gebaut und aufgemauert, ganz prächtig da stand: davon noch altes Gemäuer zu sehen.")

* Sebastian Münster, 1489–1552, Franciscan monk, afterwards Protestant Professor of Theology in Heidelberg, Hebrew scholar, mathematician, and geographer. Among his many works, which were much esteemed, his *Cosmographia* was translated into German, French, and Italian.

† Martin Zeiller, 1588–1661, author of many writings; among which topographies of France, Sweden, Denmark, and *Norway*, also of Hungary.

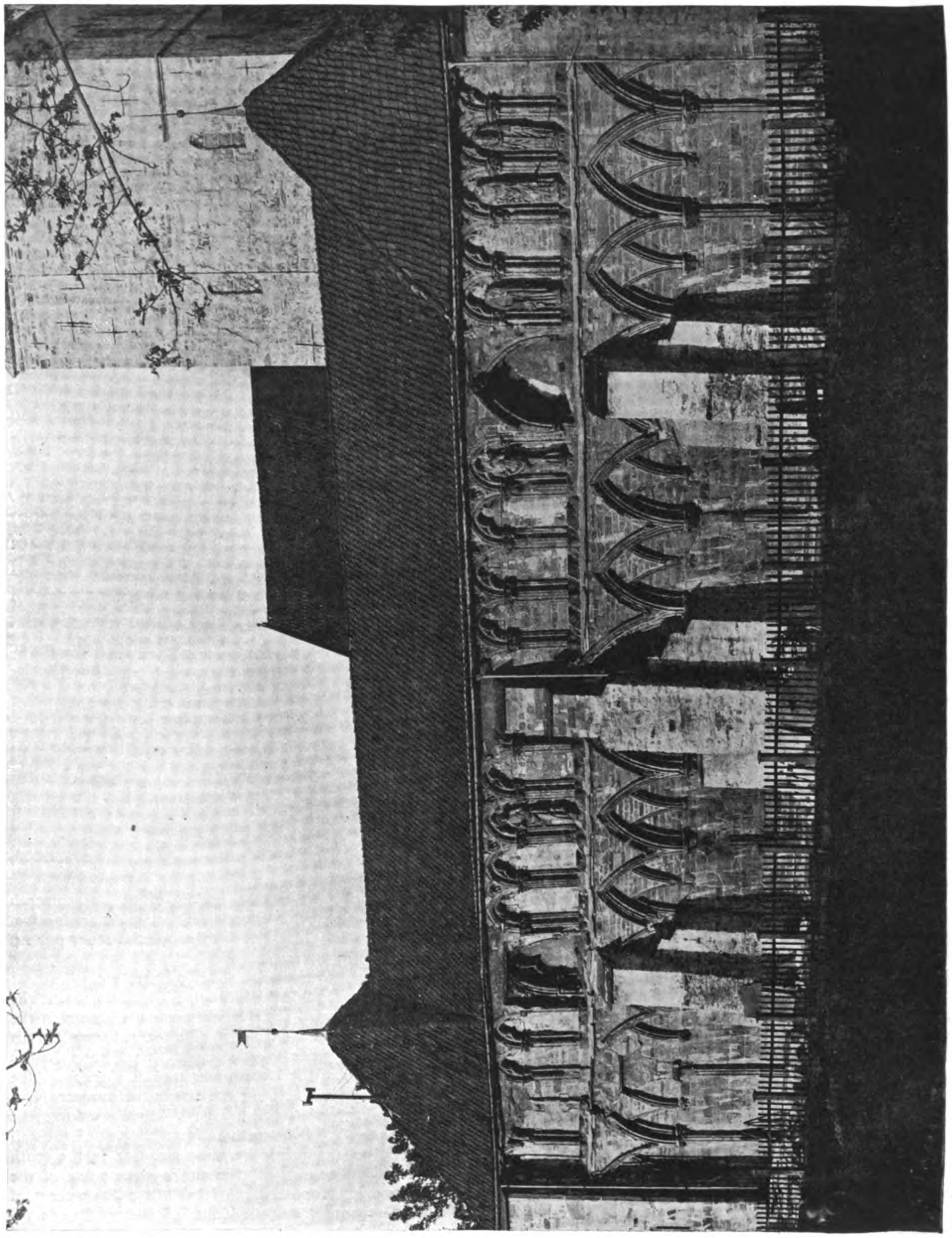


Fig. 305.—Cathedral of Nidaros. Ruin of the west front, from a photograph.

The Dutchman, Rudgerus Hermanides,* says, in his description of Norway: "The cathedral of Throndhjem has been a most magnificent building, surpassing all other churches in Europe; it had no equal even in the whole of Christendom."

As regards Norwegian evidence from the sixteenth century, we shall only quote some statements of the famous royal chaplain of Bergenhus, Master of Arts, *Absalon Pedersen Beier*, and the well-known historian, *Peder Claussön Friis*. In his *Norges Rige* (1567) the former states that the equal of the cathedral of Nidaros is not found "in the whole of Europe; cardinals and pope's legates cannot praise it and admire it enough; it can easily be understood how desirous the kings were to advance religion and what wisdom there was in building such a house for God's service." On another occasion, page 65, he makes a statement helpful in the rebuilding of the west front, that on the gable of the cathedral there was "a splendid rose, which had been gilt with the best gold." †

Peder Claussön says: "But of how this church is built, or how big and largely planned it is, I cannot write much, except from what I have heard; that it is built as a cruciform church, of cut stones only, which are also carved in different kinds of figures round the whole church, both inside and outside, so that one is filled with astonishment; and in the western gable of the church, which is gilt, the twelve apostles are carved in very large figures and gilt; there are many pillars both inside and outside the church of various kinds of polished marble, both white and black, and various columns, so polished that some think that they are cast, and in the south porch (viz. the destroyed porch before the door of the south aisle) are sixty pillars made so beautifully that one cannot well reckon or think what this door must have cost, not to speak of the whole building." After having described the wealth of the cathedral and the number of its treasures, he finishes with this remark: "And what I have described of the building of this cathedral, I have meant and understood alone of the walls themselves and the beautiful stone whereof they are built, as I know well that many other beautiful buildings in Christendom are enriched and adorned with much more gold and silver and other rich materials; but, as regards the walls, this church finds hardly its equal.‡

According to his own statement, Peder Claussön obtained his knowledge of the church from the Judge of Agder, Jon Simonsson, who was born in 1512 in Thröndelagen; he had been a pupil of the cathedral school of Nidaros, and moreover was a member of the chapter ("Choersdegn") of the cathedral—which means that he was an eye-witness, having seen the church before its nave and west front fell in ruins.§

Although these written evidences can only be considered as quite ordinary reminiscences,

* Rudgerus Hermanides, Dr. jur. Prof. politices et historiarum in Harderwick, in the middle of the seventeenth century, author of *Britannia Magna, Peninsularum regnum Sueciae, Daniæ, Norwegiae, Slesvici et Holsatiae descriptio nova.*

† Hist.-topograph. *Writings on Norway and Norwegian Districts*, ed. by Dr. Gust. Storm, Christiania, 1895, pp. 14, 35, etc.

‡ *Peder Claussön Friis*, collected writings, edited by Dr. Gustav Storm, Christiania, 1881, p. 348, etc.

§ Bishop Bang (and the lecturer Kolsrud) have tried to prove that the "gable" of Absalon Pedersön meant the triangle of the gable, and that the gilt rose meant a rose carved in stone or a small circular gilt window right up in the triangle of the gable and not the large rose-window in the middle of the front. This idea of a carved rose reveals only ignorance of architecture, just as the explanation of the meaning of "gable" only shows that Bishop Bang's science of the Norwegian tongue has deteriorated through the long supremacy of the Danish language in Norway. The word "gavl" (gable) means, in the Norwegian language, both in old Norwegian and still in the dialects of modern Norwegian, the shorter side of a built object, a house, a bed, or even a box. The expression "Sengjargavl" is still used to mean the short end of a bed, just as one calls "Gavlsneiden" the triangle of the gable to distinguish it from the part of the wall lower down called "Brystet" (chest). It is undeniable that the word "gavl" had also the same meaning in the tenth century, as appears from the irrefutable expression of Peter Claussön quoted above, "and in the western gable of the church . . . the twelve apostles were carved." Those that are preserved stand precisely in the "Bryst," or chest of the gable, in the lower row of figures in the west front, and serve to demonstrate the value of Bishop Bang's remarkable interpretation.

yet their unanimity shows that esthetic opinion is right when it pronounces that the splendour shown by the remains of the lower story of the front must have logically continued in height —developed itself richer and richer round a rose in the centre of the gable, and this magnificence was considered by foreigners and Norwegians, without exception, as unique.

Of pictorial evidence from former times there exists only a copper engraving (fig. 306) from 1661, when the top part of the west front had disappeared. This engraving was done by the map draughtsman at the silver works of Kongsberg, who became priest later, Jacob Mortenssön Maschius. Yet, both from an architectonic and iconographic point of view, it has great value for the reconstruction of the front and for the pictorial decoration, as well as for the understanding of its dramatic meaning, because it reproduces an important part, which disappeared later—that is to say, two stumps of the triforium stage of the front, as well as some missing statues. These fragments are clearly stumps of piers between two lights, one very wide in the middle and two narrower on both sides. We get in this an indication of how the vertical distribution of the front has been originally.

It can be seen that the north stump still retains some of its architectural setting—that is to say, on the north splayed jamb there are columns with capitals and springers for an arch over a narrow light; on its front it has also three niches, where there still remain two statues. The south side of the pier-stump shows, like the north one, the splayed surface of a window-jamb. Corresponding splayed jambs are also to be found on the south pier-stump. These splays indicate lights, a wide central window, and, if one starts from the axes of the side-porches, one smaller window over each of these porches.

Some years ago (in *Teknisk Ugeblad*, 1909, No. 45) we demonstrated, in a geometrical analysis, that the rendering of the front in the engraving did not only stand in the right proportion to the plan of the church, but that it gives correctly the architectural distribution of it in its most important lines; its proportion vertically and horizontally and of these parts mutually—and that, in spite of the outer sides of the front which are compressed together, whereby its total width has become too small in proportion to the height.

After the proofs produced in this work of the geometrical proportioning of the cathedral we would complete our previous demonstration by comparing the drawing of Maschius with the measured design which Christie did of the remaining part of the same front (fig. 307, Pl. XXI).

This comparison is obtained by analysing both drawings together. It will be noticed lower that, although the engraving of Maschius has the appearance of a perspective, it is really orthographic and can be used in the geometrical test undertaken here.

On both drawings we have traced vertical lines from two points on the line of the plinth —marked by one ring on Maschius' drawing and with three on Christie's. These represent the axes of the north and south side-walls of the nave. It will be seen that on both drawings they correspond with the same architectural part of the front. After having introduced in Christie's design, according to the indication of the engraving, the pier-stumps in relation to the remaining architectonic distribution, we see that the mentioned axes in both drawings form the central axes of the front of the pier-stumps. With the axial distance as radius we have also introduced in these drawings the circle of the auxiliary construction in the same manner as in the reconstruction of the transverse section. We see that the vertical tangents of this circle in the engraving coincide very nearly with the corresponding architectural part of the lower arcade in Christie's measured drawing. In other words, the engraving is correct for the main part in its direction towards north and south.

On Christie's drawing we have tested with the construction of the sectio aurea that the arcading or row of niches is in proportion, as we could expect it, according to this section. It will be equally noticed that we have determined the original height of the piers according

to the system of the church. The construction shows also the same sectio aurea proportion, between the pier-stumps of the triforium and the height of the row of niches. If we now try to divide the engraving according to the same constructions of the sectio aurea, it will be seen that in height this engraving has the same proportions as the measured drawing of Christie. The historical value of Maschius' engraving is therefore perfectly proved, and we could proceed to an examination of how correct here also are the vertical distributions of the height of the triforium as reproduced by Maschius; or, in other words, if the large width of the central light and the width which we introduced for the smaller lights on each side are correct. To that end we have introduced vertical lines in the engraving, showing that the outer lines of the central light in the plan of the front coincide each exactly with the axis of the column belonging to the arcading just in the middle between the porches. These vertical lines are transferred as axes of columns in the measured drawing; and in the light under the drawing we have written for guidance: "Centralvinduets Bredde paa Maschius' Stik" * (Width of the centre light on the engraving of Maschius.)

Before we begin this analysis we must indicate an important point in explanation of the character of the front. A glance at fig. 307 (Pl. XXI) shows already that the front lacks entirely the buttresses which should receive the constant thrust which the main arcades and the arches under the two flanking towers exercise against the gable wall. Through this lack of buttresses the front seems to be a so-called screen front for the church lying behind. It has been, as Maschius' engraving shows it, a wall richly decorated with sculptures, and having a fixed iconography. This is obvious from the preserved fragments of sculpture, and also from the double row of figures on the engraving.

We find the evidence of the intended character of a sculptured wall, both in the plan and in the longitudinal section.

It will be remembered that, in the drawing of the plan, fig. 275, we have marked in dotted lines according to 45° the rectangle composed of two squares, such as the interior of the nave represents. But while this rectangle is limited south, east, and north by the walls of the aisles and of the transept, we can see that the wall of the front towards west lies *inside* the west side of the rectangle. As a result of this, the most western bay of the side-walls is contracted accordingly.

The constructive aim of this arrangement appears clearly in the longitudinal section, of which we reproduce here the four most western bays (fig. 308). The diagonals of 45° and of 63° 26′ respectively, introduced at the summit of the vault and on the height of the plinth, show where the axis of the last pillar against the wall ought to have stood, if the front had been an ordinary front with exterior buttresses. Through the contraction of the last bay the master-builder has in the meantime obtained an inner system of abutment of much more mechanical efficiency than in the first case, because he would then have been obliged to receive the constant great thrust of the arcades of the side-walls with greatly projecting outer buttresses; with the construction he has chosen, he has not only succeeded in obtaining a powerful pile of abutment in the wall itself and the pier belonging to it, but the contraction of the last bay causes it to be included in and made a part of this interior system of abutment. The construction is a good example, therefore, of the consciously scientific way of building in the Middle Ages.

In this we see undeniably the conscious and clear object of the master-builder—to finish the church in the west in such a way as to be able to cover the whole building behind a sculptured wall wholly continuous, unbroken, and undisturbed by any static element.

* We have already demonstrated this in 1903, 1907, 1908, and 1909 in *Teknisk Ugeblad*, without its truth being admitted by the wise superiority of the committee of direction.

In the analysis of Christie's measured drawing carried out in fig. 307 (Pl. XXI), after having first convinced ourselves of the strict raising of the front *ad quadratum*, we shall acknowledge that the width of the lights at the stage of the triforium, in the drawing of Maschius, is the original one and thus in full and harmonious accordance with the elevation of the existing ground-floor.

It will be noticed that, in addition to the vertical tangent mentioned before, we have

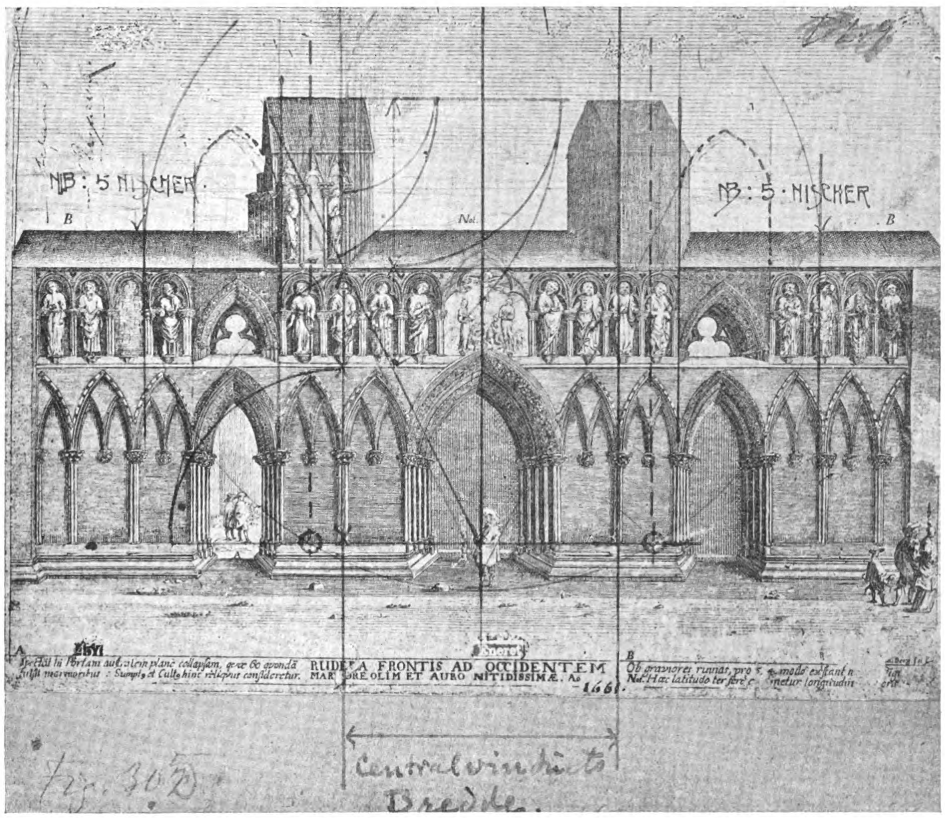


Fig. 306.—Cathedral of Nidaros. Copper engraving of Maschius, the west front.

introduced a lower horizontal one, which is traced through the nadir of the circle to serve as base to the auxiliary construction. We have equally put in a horizontal diameter and lengthened this right across the front. From the zenith of the circle (the top part of which is cut off on the drawing) we have drawn the diagonals of the half-square through the intersecting point of the horizontal diameter with the circle, that is, at an angle of 63° 26′, and lengthened these to the base. From the intersecting points of these diagonals with the base, we have drawn up vertical lines limiting the plan of the construction.

It will be seen, as already mentioned before, from the plan of the west front introduced

under the drawing, as well as from the adjoining walls on the east, that the north and south surfaces respectively of the two flanking corner towers coincide exactly with these vertical lines. It will be seen, furthermore, that the axis of the auxiliary construction and the vertical tangents introduced, divide the front exactly into six equal parts, four inside the circle and one on each side (see also the plan, fig. 275). It is clear from this that the front is already, in this way, conceived inside a square divided into six parts, in order to obtain through this width, as already remarked, the powerful façade impression of a five-aisled cathedral in the front of a *three*-aisled one. Moreover, we have introduced the vertical main axis, under the plinth marked *a*, *b*, *c*, *d*, *e* and *f*, for the architectural division of the lower arcade. To show the regularity of the standing parts, we take our starting-point from the central axis through the main porch at the line of the ground and from here we place the diagonal according to 63° 26′. We shall call it diagonal 1. It will then be seen that, for example, the diagonal on the left, that is on the north side, intersects the axis of the third column from the middle of the row of niches; from here it is deflected and ends on the line of the ground at the intersecting point of this with the axis of the smaller porch. From here it is deflected further to the lower part of the capital of the third column on the left of this axis, and is deflected again from here to the ground-line, where it is intersected by the boundary line of the screen front against the small projecting stair turret of the north tower. It can be seen that the case is the same on the south half of the front.

If we now take our starting-point from the centre of the main door on the line over the plinth, it will be seen that the diagonal from here, which we shall call diagonal 2, passes through the upper edge of the innermost capital of the main door, also through the upper edge of the base under the second column from the middle in the row of niches, and intersects the axis of the third column at the level in which diagonal 1 intersects the outside opening of the central light according to Maschius' engraving. From here diagonal 2 is deflected; it passes through the axis of the fourth column from the middle at the upper edge of its base, and further through the south front column of the northern porch—that is to say axis *b*, at the height of the capital in order to meet on the line of the plinth with the axis of the northern porch. From here it is deflected again through the capital of the opposite porch, it crosses the string course under the row of niches at the intersecting point of this with the axis of the second column from the window over the smaller porch, it passes through the capital of the next column in its axis, and it is deflected from here through the axis of the last column of the arcade towards north at the upper edge of its capital, in order to end over the line of the plinth, exactly at the boundary of the front against the mentioned stair turret. This is repeated exactly in the same manner on the south side. It will be seen from the diagonals introduced in the arcade and in the row of niches, that the proportion is everywhere determined by the angle of 63° 26′ or the proportion of 1 : 2. The slanting diagonals according to 63° 26′ from the line over the plinth in the centre of the main porch show that the whole wall also on each side of the central axis, to the small flanking stair turrets, has the same proportion.

* * *

If we now proceed in the same manner with the analysis of the west front as we proceeded with the examination of the octagon, we shall find that the architectural raising of the preserved remains of the front is determined exactly as in the octagon (see fig. 283), as in Vezelay (fig. 20), in Paris (figs. 53 and 54), in Cologne (fig. 72, Pl. VII), and in York (fig. 94). This is apparent from the diagonals drawn in the smaller porches and in the windows over these. As to this it will be sufficient to point out the two diagonals, which start from

the intersecting point of the axis of the side-porch with the line of the plinth going each in a different direction and determine the proportion of the porch. It can be seen that, after being deflected at the height of the capital of the porch, they cross each other on the base of the middle column of the window, and finally they intersect the lengthened axis of the columns of the porch at the very level (the horizontal broken line) where diagonal 1, from the intersecting point of the central axis of the main porch with the ground-line, intersects the north boundary-line of the central window in the plan of the front. At the same level the diagonals from the line of the plinth in the middle of the centre porch and the side porches can be seen to intersect one another. From this we have determined the thickness of the disappeared string-course over the first row of niches.

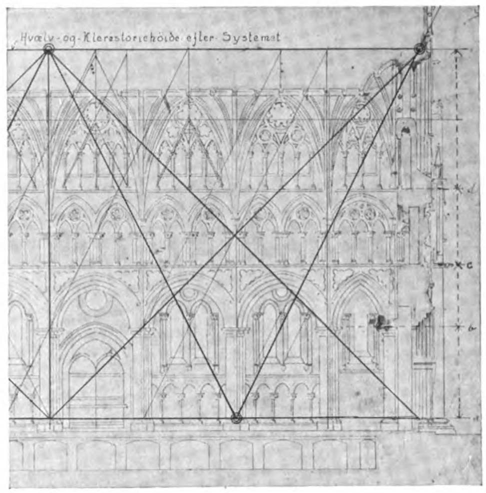


Fig. 308.—Cathedral of Nidaros. The four west bays of the longitudinal section.

After studying the diagonals drawn in, more or less slanting, according to 63° 26′ and 45°, the reader will convince himself that there is full correspondence between all the principal points of the ornamental architecture of the remaining lower story. It will be seen equally that the six points which determine the width of the lights of the triforium, four of which are exactly the same as in the engraving of Maschius, take part in this correspondence. Furthermore, the division we have made of the axis according to the sectio aurea, marked *c*, shows that the horizontal distribution, or, to be precise, the division of the vertical lines, is done according to this proportion. The scales of proportions introduced on the left of the drawing, show the same thing as regards other distributions.

The principle for proportioning the standing part of the remaining front is therefore established, and we have obtained in this way a firm and secure basis for the raising of the front *ad quadratum*.

Our next problem is now to find the correct height of the first missing part of the front, the pier-stumps reproduced in Maschius' engraving, or the triforium of the front. It can be done in two ways. We have ascertained that the front is proportioned exactly according to the sectio aurea, and we can proceed quite synthetically in the manner of Vitruvius by adding part to part according to that proportion. In other words, we could raise at once, not only the separate parts, but the whole front in this manner. We refer, for instance, to our drawing where there are introduced, on the right, the height of the arcade as five moduli and the height of the first row of niches as three moduli. Or we can reason thus: as the first row of niches is five ells high, it follows that the triforium must have a height of eight ells. An architect of the Renaissance would have added the corresponding number of moduli for the missing stories. But, in the meantime, we have shown, when dealing with the sectio aurea, that this way of proceeding would lead to inaccurate results, while it is not satisfactory in the present case.

Therefore we prefer to use the more laborious but more accurate and interesting geometrical

method. In a way we have used it already, when we determined through it the height of the missing triforium. It will be seen that the diagonal from the line of the ground in the middle of the main door, after having crossed axis *c*, or the north boundary-line of the central window at the top edge of the found hood-mould, intersects the axis, marked *b*, of the south front column of the north side-porch. The correctness of this is ascertained by the diagonal starting from the height of the capital in the axis of the north column of the same porch, marked *a*, as this diagonal intersects the northern boundary-line of the central window in the plan of the front at the same height. From this the height of the pier-stump is determined by the very fundamental proportion of the church, of 1 : 2; the measurement taken shows it to be eight ells. If we now carry out the construction of the sectio aurea, as shown in the central window of the drawing, we see that the triforium is in proportion to the first row of niches according to this section. We saw that this was also the case in the engraving of Maschius; therefore we have demonstrated its correctness in every direction.

The same result is also obtained naturally as regards the south pier-stump. Later on we shall prove in another way the correctness of the engraving as regards this floor.

It is obvious that the central axes of the smaller porches which determine the remaining lights over the porches conducting to the aisles have also been the central axes of the lights above in the triforium.

If the boundary of these windows against the pier-stump is now established, the symmetrical limit of the windows of the triforium on the other side of the axis is also established. This point is equally determined by the diagonals from the front capitals of the smaller porches; for instance, from the capital on axis *b*.

If we measure the front width of the pier-stump between the central window and the window of the triforium it appears that this is exactly four ells, while the height from the upper edge of the string-course on the first row of niches to the lower edge of the string of the triforium is eight ells.*

To ascertain this height in proportion to the width of the front, we have drawn, from point G on the axis of the central window, on the upper edge of the mentioned string, diagonal lines after 45°, of which, for instance, the left one intersects axis *c* a little under the two rings, which mark the *upper edge* of the string over the triforium. The broken horizontal line through this intersecting point, marked H, gives the *lower edge* of the string. Between this line and the upper edge of the string over the first row of niches the diagonal mentioned is deflected at points H, I, and K, to end accurately at point L, where the farthest boundary line of the front intersects the already found line of the upper edge of the string over the first row of niches. There are, then, at the stage of the triforium eight squares altogether right across the total width of the front. But the height found for the triforium fits also *ad quadratum* in the very frame of the screen front, because the diagonal from point L on the inside of the small stair turret which corresponds with point M on the outside of it and is deflected at N, marks out two squares up to the north front boundary of the pier-stump, the axis *b*. There is, further, half a square for the surface of the pier-stump and a square to the central axis of the church, the latter marked by the diagonal HG—that is to say, altogether three and a half squares at the stage of the triforium, on each side of the central axis of the church. We have pointed out previously the correspondence between the six points which mark the width of the windows in the plan of the front. The correctness of the main proportions of the triforium is irrefutably witnessed by diagonals LKT down to the intersecting point of the central axis with the ground in the middle of the main porch and diagonal MN, which, after having passed through the corner of the window on the side, marked by two rings, ends at the line

* The triforium built by Mr. Nordhagen, which we criticised, is 16 inches too low, by which the harmony is disturbed in the proportioning of the front.

of the plinth at the central door, the point o with arrow. The standing remains of the front, judging from the various diagonals drawn in, appear to be strictly governed by the rules—in their main proportions as well as in their divisions—and they continue thus governed in the triforium which we have reconstructed.

We have, then, established the correctness of the exterior width of the central window as well as of the windows on each side, in the engraving of Maschius.

We can proceed now to determine these lights and their height and we shall use for this, fig. 309 (Pl. XXII), where the geometrical auxiliary construction with its circle is entirely carried out in thin lines, while the auxiliary diagonals from the fixed main points in the drawing of the front are drawn in thick lines.

It is, of course, a generally acknowledged natural rule that the angle of incidence of all the lights in a wall must be identical; it is only in this way that evenness of light and shade can be obtained. A glance at the plan of the wall of the front introduced under the drawing shows that the preserved door and window openings in the standing part of the front have the same angle of incidence, 45°. This gives us also the angle of incidence of the reconstructed lights.

It will be seen that we have projected on the plan the exterior width of these lights as obtained in our analysis; it is indicated by axes *a* and *b*, *c* and *d*, *e* and *f*; we have equally introduced the angle of incidence of their splay jambs according to 45°.

Concerning the inner width of these lights, this depends how deep the windows are set in the wall itself. As regards the secondary windows at the side, the question is settled at once by the porch underneath and the windows in the first row of niches. Our interest is therefore concentrated round the dominating central window.

We have already demonstrated how the front is in reality composed of two parts, viz. the west wall of the church and the screen front raised before it. It is, then, obvious that this central window must be pierced through this screen wall, and so deep that it reaches the wall of the church itself—that is to say, the wall under the triangle of the gable. The window is placed according to this, as shown by line *g h* in the plan under the drawing of the front. This position will also be seen in the transverse section of the wall of the front in our reconstruction of the longitudinal section of the cathedral (see fig. 310, Pl. XX). The inner window opening, thus determined, has been consequently projected on the front from the plan.

The top limit of the inner and outer lines of the central window, or, in other words, the level of the springers of the arch, can only be found, however, after having determined the level of the inferior horizontal limit of the light—that is to say, the level of the upper edge of the window-sill; it is from here obviously that the light and the geometrical tracery contained in it must be proportioned. We have found that the limit line of the window in the plan of the front coincides with the axes *c* and *d* of the two columns in the first arcade, just in the middle of the wall between the doors. We found also that the level of the lower edge of the window-sill is determined *ad quadratum* on this axial distance, reckoned from the ground. It will be seen, furthermore, that the position, or rather the outer side of the base of the farthest column in the splay-jamb of the window, is indicated by diagonals from the line of the plinth in the middle of the main porch. It is, then, obvious that to determine the upper edge or height of the sill, we must start from the intersecting points of the line of the plinth with the two axes *c* and *d*. It can be noticed, then, that the diagonals from these two points, after having crossed each other at the central axis of the front, intersect the inner lines of the central light. The three principal points are marked by rings. Furthermore, it will be seen that diagonals from the line of the plinth on both sides of the inner lines of the main porch intersect the inner lines of the central window exactly at the level of the intersection of the diagonals coming from axes *c* and *d*. Through this point a horizontal line is drawn to

both the points just found on the limit line of the window. From this we have determined the upper edge of the sill and of the springers of the arch over the window, also the centre of the large rose, the former *ad quadratum* on the axial distance *c d* from the plinth = the front width of the window, the two latter, the springer and the centre of the rose, *ad quadratum* on the inner width of the window.

A tangent is drawn under the circle of the rose. The diagonals introduced according to 45° show that the level of this tangent is also *ad quadratum*, on the half width of the window, just as the diagonals after 63° 26′ show that there is complete accordance between the main porch and the central window.

It is interesting to notice that the circle of the rose which gave the shape of the arch over the window gives it also the same proportion as the rest of the church, because the height from the upper edge of the sill to the centre of the rose, and from here to the top of the arch of the light, stand in proportion to one another according to the sectio aurea.

The parts of the building still standing contain a point of practical importance to test the correctness of this reconstruction, the base of which is to determine the level of the upper edge of the sill. At the back of the first row of niches, as shown on the drawing, there is a passage in the wall which forms the communication between the north and south triforium and ten steps lead down under the central window. The top part of it has disappeared with the destroyed triforium and window-sill. It will be seen that if the passage is given the usual height of a light, three ells, it will succeed, stair included, to get clear of the sill, which was determined by the geometrical system of the church. We shall content ourselves, among the many theoretical points of interest, to point out here the central one, the absolutely conclusive proof for the correctness, not only of the reconstruction of the window, but of the whole front, even of the geometrical theory of the cathedral, namely, the fact that the level found for the centre of the rose does not only lie *ad quadratum* on the width of the central light, but—we repeat it here decisively—it lies *ad quadratum on half the screen front*, because the centre lies at the intersecting point of the diagonals after 45° from the corners on the base of the auxiliary construction. In direct connection with this geometrical phenomena, the diagonals of the auxiliary construction, from the centre of the rose, marked by a very large C, on their way to points D and E on the base of the square of the front, pass through the corners of the base of the central light, and, after being deflected at the named points D and E, continue respectively to points G and F on the sides of the square of the front, which, as already remarked before, coincide with the surfaces of the north and south outer wall of the two flanking corner towers, as shown in the plan under the drawing of the front. It will be seen that the horizontal line between points G and F of these auxiliary constructions runs straight through the centre of the rose. This horizontal line is intersected at the points marked by a ring and NB, by the line of the inner side of the small stair turrets, against the screen wall. It will be seen, then, that diagonals after 45° from the line of the plinth in the middle of the main porch, deflected at these points marked NB on the line of the inner side of the stair turrets, cross each other at a point on the west wall of the central tower, marked with four rings. The diagonals of the half-square from here to both sides downwards coincide (as emphasised on the drawing) with the *original trace of medieval roof*, to end at the line of the plinth in the corners, near the lines of the inner side of the stair turrets. From the proportion of this central window as regards the standing part of the remaining front, we have obtained still another proof of the historical reliability of Maschius' engraving, in whose width of window we found the guidance for our reconstruction of its principal divisions.

We shall see, now, that we can also rely upon Maschius' rendering of the architectural decoration of the triforium. We have established the width of the front upon the pier fragment of his engraving, which—when introduced in the front—shows a width of 4 ells or

96 inches. The stump seems fitted with three niches. Upon each of these there falls, then, a width of 32 inches. In this we have an indication regarding the decorative distribution of the stage of the triforium. If this measure is reckoned in the distance from the flanking stair turret to the window of the triforium there is room for eight niches exactly. If this same measure is used in the central light, we obtain here also exactly eight panels. This conformity carried throughout the decorative distribution of the stage of the triforium gives us again a confirmation of the correctness of our reconstruction of the central window and of those of the triforium.

* * *

We go back now to the auxiliary construction for the further raising of the front.

It will be noticed that, in order to determine the sculptured front, in its architectural as well as in its decorative distribution, we have taken our starting-point from the line over the plinth (the thick diagonals), while the auxiliary construction drawn in thin lines is carried out in accordance with the same construction used for the church lying at the back. This manner of proceeding is indicated by the edifice itself, as the width of the sculptured front is directly determined by the points of intersection of the line of the plinth with the outward going diagonals of 45° from the farthest points of the base of the construction.

This produces two starting-points: one to determine the proportion of the front in its totality, another to determine the screen front and how it stands in proportion to the church at the back. It is remarkable how these two systems play into one another in the raising of the front, just as they coincide in the most intimate way with the raising of the interior according to our earlier development, as we have already seen and as will be further corroborated.

On the drawing of the front it will be seen that we have traced the circle entirely and carried out all the constructions developed in this way in thinner lines. It will be obvious, for instance, that the height of the triforium determined by the very construction of the screen front coincides exactly with one of the horizontal lines in the auxiliary construction. The horizontal tangent through the zenith of the circle establishes the total height of the wall of the screen front, or, to be more accurate, the level of the projection of the main cornice. It will be seen, furthermore, that in the front we have put in the exterior outline of the central window, axes *c* and *d*, from the plinth and right up to the tangent through the zenith of the circle. From the intersecting points of the large circle with the plinth line, marked by three rings, we have traced diagonals after 63° 26′ and continued them up, until they intersect the mentioned outer line of the central window. Through these two intersecting points on the outer lines of the central window we draw a horizontal line which is to be the top of the arcade development of the screen front itself.

To test the correctness of this height we have introduced new diagonals, namely from the intersecting points of the lines (axes *c* and *d*) of the central window with the line of the plinth. These diagonals intersect the opposite axes of the side-walls A and B of the nave, exactly at the level of the height found for the arcade development of the screen front. If we allow these diagonals to be deflected from here downwards, they are seen to coincide exactly with the intersecting points of the boundary lines of the front and the plinth line. To test further we can start with diagonals of 63° 26′ from the intersecting points of the line at the base of the clerestory with the lines of the inner sides of the stair turrets. These diagonals will be seen to deflect from the upper boundary of the screen front, down to the top of the triforium window, to be deflected from here upwards to the intersecting point of the outer lines of the central window with the top boundary line of the screen front, in order to end, through deflection, at the intersecting points of the plinth line with axes A and B, marked with three rings.

These manners of proceeding are, however, of a theoretical kind. It could be useful to undertake a counter-test with starting and finishing points in the architecture of the front still standing, or, in what we can consider as still standing, namely, the vertical continuation of the small, slightly projecting stair turrets, which frame the screen front, as, by means of the square standing diagonally between these turrets, we arrive at the top of the ridge on the nave and to the old trace of roof. Therefore, to establish finally the correctness of the theoretical result concerning the height of the clerestory of the front, under the main cornice, we must point out that a diagonal starting from the height of the capital on axis *b*, for instance on the south front capital of the smaller north porch, runs up to the intersecting point of the line of the inner side of the stair turret with the theoretical boundary line of the height of the clerestory under the main cornice.

As another evidence of the correctness of this solution we can remark that the height of the clerestory of the screen front, thus determined, stands in the proportion of the sectio aurea to the reconstructed triforium just as this story stands in that proportion to the first row of niches in the standing remains of the front, also that the whole standing remains of the front over the plinth stand in the proportion of the sectio aurea to the height of the whole reconstructed part above, right up to the projection of the main cornice.

* * *

Our next problem now is to find the axial distance for the distribution of the panels of the clerestory of the screen front.

This distribution can also be found in several ways. We choose first to find it by means of the diagonally placed square between the inner sides of the stair turrets. It will be seen that the left side of this square, from the middle of the main porch, intersects first the horizontal boundary line between the triforium and the clerestory at a point marked with a ring, and where a vertical line called 2 is raised; after being deflected from the side of the stair turret, it runs up to point 4 on the line which gives the level of the projecting part of the main cornice. From here it is deflected down to the boundary line of the central window in the plan of the front, axis *c*, at the height of the springer, from where it is deflected down on the boundary line of the stage of the triforium at a point marked by a ring.

If we now take our starting-point in the still remaining ornamental architecture, for nstance the capitals of the columns in the middle of the blind arches on each side of the northern side-porch, we can see that the diagonals of the half-square cross each other at the top of the arch on the small window over the side-porch; one of them runs up to the height of the springer of the arch over the central window, that is, to a point on axis *c*, or the axis of the arcade of the clerestory, No. 8, after having crossed a diagonal from point 4 on the line of the cornice at a point on the base of the clerestory marked by a ring; the other runs to a line under the main cornice, to a point marked by a ring. A perpendicular coming down from this point intersects, at the base of the clerestory, a diagonal from point 4. It would be tedious to carry our operations further; from the drawing the reader will see that we have distributed the clerestory between the central window and the inner side of the stair turret, in eight exactly equal axial distances for the division of the panels, measuring each 2½ ells in width.

Now concerning the horizontal distribution of these high, narrow panels, various diagonals show that the horizontal line through the centre of the rose window intersects each panel according to the very proportion of the church, as 1 : 2. Here we have the indication for a horizontal division of this high panel.

The correctness of the distribution of the panels of the clerestory thus carried out in

horizontal and vertical directions is corroborated by the fact that the new lower part of the panel stands in the proportion of the sectio aurea not only to the triforium underneath, but to the upper part of the clerestory. It can be seen, furthermore, by the diagonal from the corner between the wall of the stair turret and the base of the clerestory that the second axis is intersected at the height of the thin horizontal line in the auxiliary construction, as it divides the upper part of the clerestory according to the proportion of the sectio aurea, and we get from this the height of the capitals of the columns in that panel.

It is obvious that the canopy above the large central window must also have been raised

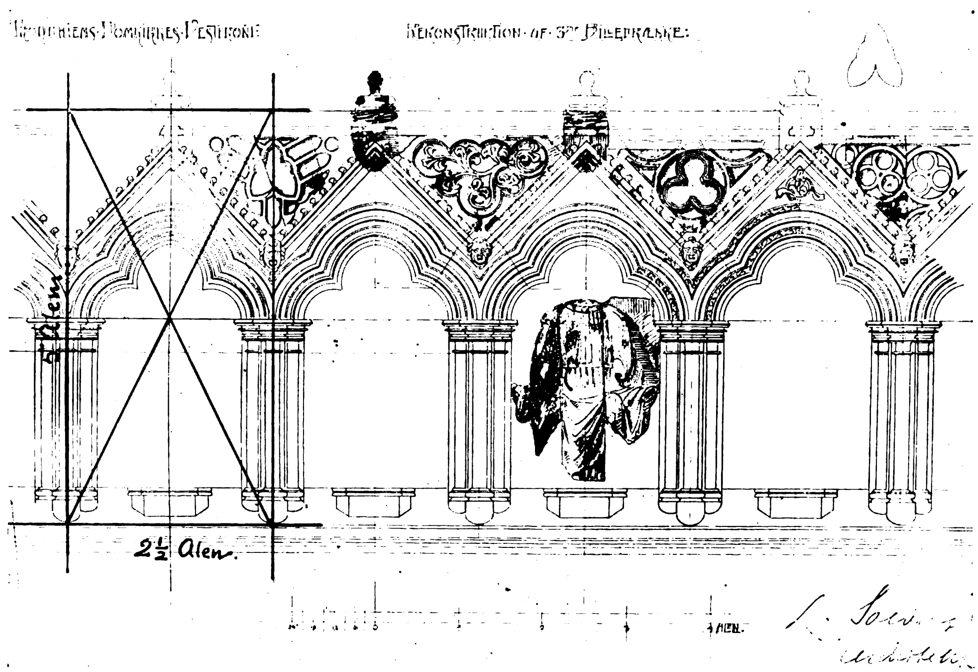


Fig. 311.—Cathedral of Nidaros.—Reconstruction of three rows of figures by the architect Solberg.

in logical and intimate connection with the architecture of the front—as indicated on the theoretical drawing. With this canopy raised up high, the dominating central part of the screen wall obtains its finish. The distance between the plinth to the centre of the rose and the distance from here to the top of the canopy stand in proportion to each other according to the sectio aurea—as it will be seen indicated among the scales of proportions attached to the drawing.

From this the horizontal and vertical distributions of what we called the screen wall or sculptured wall are determined.

We have seen that the front grew automatically, from the preserved standing parts, thanks to the geometrical method according to which these have been drawn, and we have also seen that the distributions produced by this method fell into the law of proportion demonstrated

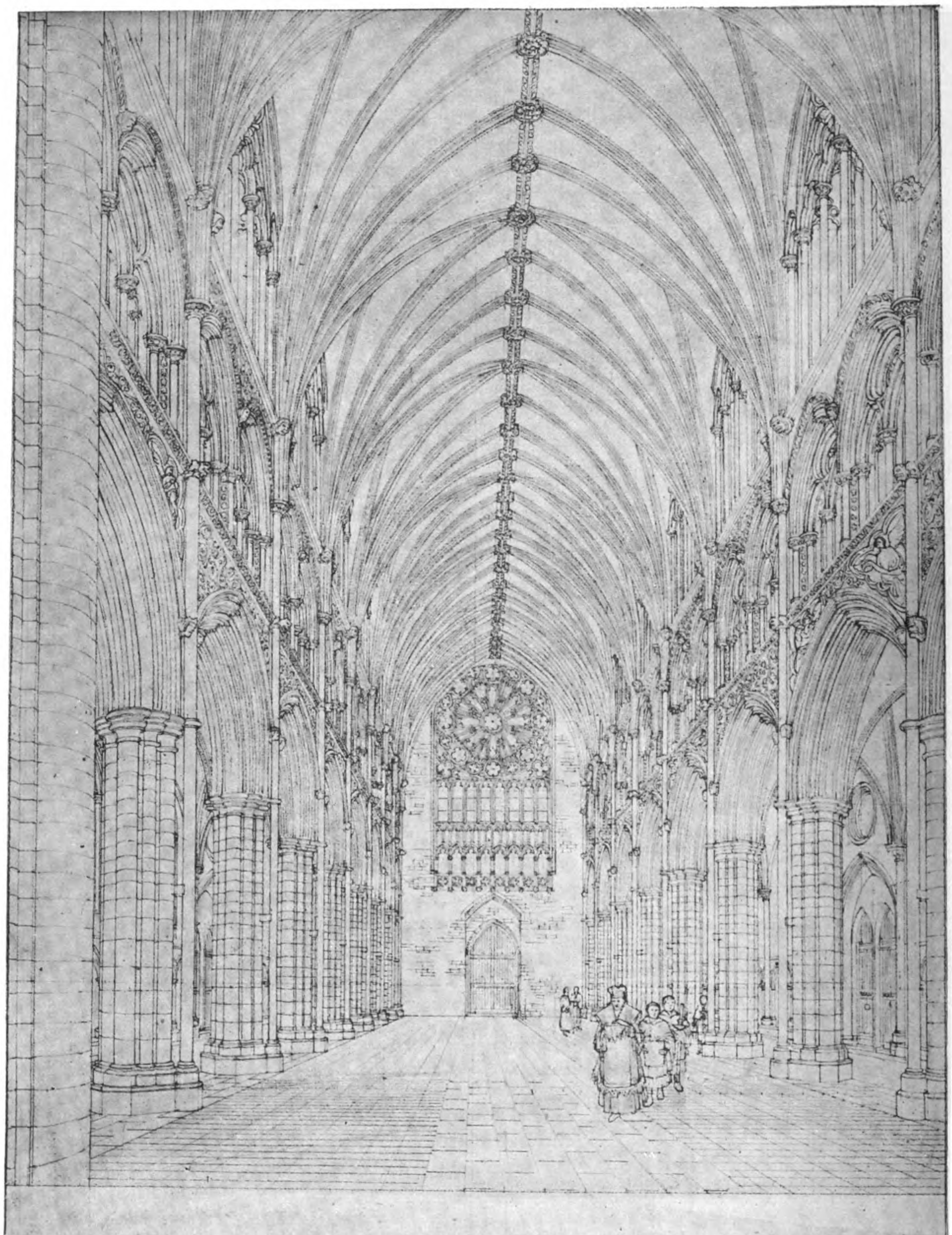


Fig. 312.—Cathedral of Nidaros. Interior of the nave towards west after the perspective drawing of Harald Sund.

everywhere else in the church. We have seen, moreover, that the vertical distribution of the horizontal lines produced by the method, as well as the distribution of the lights and of the decorative niches of the remaining wall surfaces of the triforium coincide completely with the evidence of the oldest pictorial proof.

We give now two important evidences from the cathedral itself as regards the correctness of the niche division of the clerestory produced by the geometrical system. In the collection of stones belonging to the remains, some fragments of architecture have been found which, on being put together, give indications of panels having an axial distance of exactly $2\frac{1}{2}$ ells. Moreover, there are fragments in the collection, of a strongly projecting relief, almost in "ronde bosse", representing a seated figure. On the left corner of the background preserved intact there is the incision of a capital. If we imagine this figure reconstructed in relation to the first loose stone mentioned, as it was done by the previous architect of the cathedral, Mr. L. Solberg (see fig. 311), we get the height of the panel determined up to the capital and also—with due regard to the room occupied by the plinth and the socle of the seated figure as well as by the bases of the flanking columns—a panel exactly after a proportion of 1 : 2, or, expressed in measures, $2\frac{1}{2}$ ells wide and 5 ells high—that is to say, the measure, within an inch, which we obtained for the lower part of the clerestory. In our subsequent development of the iconography, as this is indicated by the remaining sculptures on the front, and also by the engraving of Maschius, we shall prove that this figure must have been placed necessarily in this very row of niches.

We have demonstrated above and we leave it to the reader to convince himself, by studying the diagonals drawn in, how the architecture of the standing remains, and the architecture of the reconstructed part of the front stand clearly and remarkably in intimate and firm correspondence with the original trace of the roof of the church at the back, found on the central tower. In connection with this we must mention, as a witness of the accordance between our reconstruction of the front and our reconstruction of the clerestory of the church itself, that the centre of the rose window—the main light of the nave, the position of which is determined by the indications of the screen front, that is to say, quite independently from and without thinking of the reconstruction of the side-walls—coincides exactly with the level of the capitals of the windows of the clerestory reconstructed by us, and which were determined by the construction of the side-walls, according to the system. See the interior fig. 312, and fig. 215 (Pl. XXIV). Therefore there is and should be a perfect harmony between the central window of the west front and the interior of the church. Finally it will be seen that over the screen front we have indicated an open arcade which in fig. 309 (Pl. XXII) is only inscribed as: "Hoiden af den aabne Kolonnade" ("height of the open colonnade.") It is drawn in full in fig. 313. This colonnade has a double function: architectonic and iconographic. Architectonic, it serves as a gradual finish to the ornamental architecture of the front and to help solve the screen problem. Over the large window, with its rose forming the central feature of the front, the great canopy or ornamental gable raises itself up in the colonnade, whereby the arcades of the screen wall are resolved in an interplay of detached white marble columns. Through these glimmers the church at the back, behind the sculptured front. In this manner the building appears to be receding still more and the rich ornamental wall gives the impression of being relieved of any static function.

The iconographic problem of the arcade is contained, as mentioned, in the iconographic programme as presented in Maschius' engraving and the remaining statues of the front. We shall examine this programme entirely later on.

* * *

In fig. 313 we start our reconstruction of the raised part of the gable lying at the back of the screen front, and also that of the two flanking towers and of the central one. We have

indicated recently, in our demonstration of the connection between the old trace of roof on the west wall of the central tower and the architecture of the screen front, that the continuation downwards of the line of the trace on both sides leads to the extreme points at the base of the screen front. In connection with this we found that the middle of the rose in the central window coincides with the middle of the square of the theoretical auxiliary construction, within which the front has been designed in its totality. As we have repeated several times, this appears from the fact that the outer wall surfaces of the two flanking towers, north and south, coincide with the vertical sides of the auxiliary square. It is, then, evident that the height of the gable must be determined by the diagonals of that square. In connection with this, the top of the gable wall, under the coping, is determined by the diagonals from the base of the auxiliary square, while the summit of the gable, its decorative crockets and finials, are determined by the diagonals from the intersecting point of the line over the plinth with the sides of the auxiliary square.

It is evident that the gable is similar to those of Paris, Cologne, Stavanger (fig. 145), of the church of Akr (fig. 133*c*), of Gran (fig. 135), and of the church of St. Mary in Bjorgvin (fig. 158), raised *ad quadratum*. (See also figs. 94, 99, 102, 104, 107, 109, 110, 111, 112, 113, 114, 115, 117, 119, 122, 124, 125, 128, 152, 153 and 154*a*.) The analysis of the cathedral of Rheims in the Appendix shows the same proportion.

The height of the flanking west towers, up to the cornice, is also obviously determined by the same principle, as it can be seen by the diagonals of 45°. We have obtained the interesting result that the centre of the large rose window stands in the proportion of the sectio aurea to the centre of the whole picture of the front, and its principal divisions as well as to the open colonnade above, up to the towers, as shown by the sectio aurea scales of proportions affixed on the left of the drawing. The upper edge of the parapet of the west towers, as well as the upper edge of the parapet in front of the central tower, is drawn a little too low owing to carelessness. They should be raised in such a way that their upper edges coincide with the dotted level line marked C on the left of the drawing, whereby the upper edge of the parapet also is in proportion to the rest of the elevation according to the sectio aurea. A glance at fig. 93 shows that the front of the cathedral of Wells related to it is designed exactly in the manner used here.

The height of all the pinnacles on the west towers is naturally also determined *ad quadratum*. The top of the spires of these towers, as will be seen, is equally determined according to the same proportion. The same applies to the central tower, viz. the tower itself, its pinnacles and its spire (see fig. 92). The top of the latter is determined by the intersecting points of the vertical boundary lines of the plan of construction with a horizontal line, drawn at the summit of the spires of the west towers. Perhaps it will be considered strange that we did not start here from the top of the spires of the west towers. But it must be remembered that these points are the result of the intersection of the main diagonal of the auxiliary construction with the axis of the west towers. It is, therefore, a practical result. We must allow a horizontal line to take us back to the theoretical auxiliary construction, making our start from this, so as to determine the height of the spire of the central tower. That this is right is shown by the introduced scales of proportions and equations. In the following drawings (figs. 314 and 315, Plates XXIII and XXIV) to which we now turn, we shall find the proof of the correctness of this way of reasoning.

We have carried out the final scheme of the front from the auxiliary construction in fig. 313; the architectonic formation is again traced on this scheme so as to use it in our data.

This auxiliary construction is introduced and traced in thinner lines in fig. 314 (Plate XXIII).

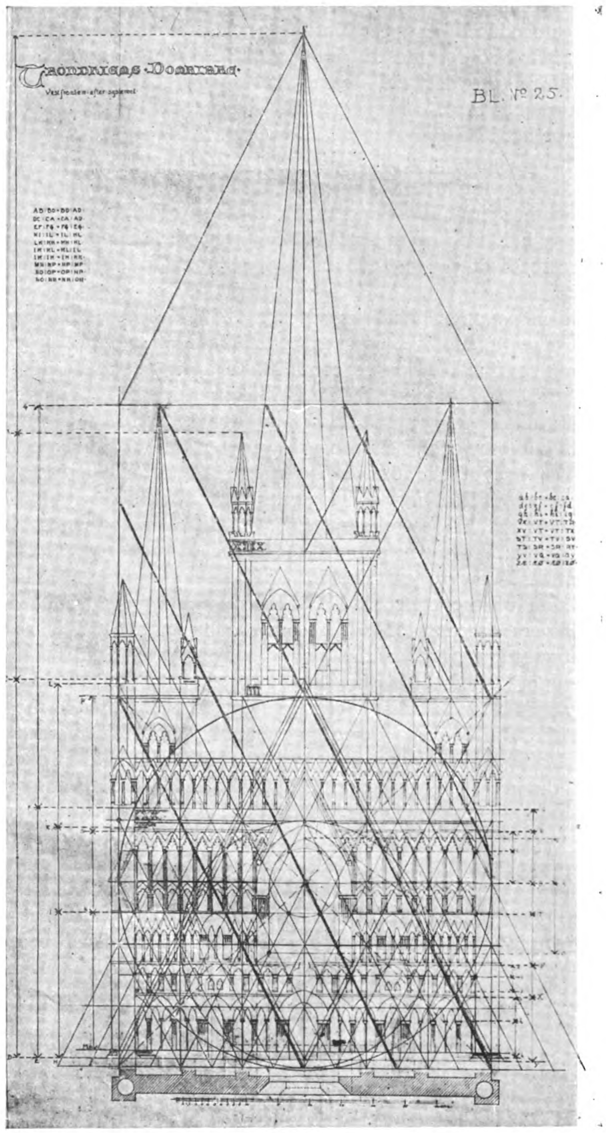


Fig. 313.—Cathedral of Nidaros. West front with flanking towers and central tower, reconstructed *ad quadratum*.

So as to make a general examination of the correctness of the scheme in its entirety we have introduced it in fig. 307 (Pl. XXI), which is the analysis of the standing remains of the front; we have introduced also the outward going auxiliary lines for the proportioning of the reconstructed parts and for the distribution of their decorative architecture.

It will be seen from the diagonal of the half-square starting from axis A at a point on the plinth line marked by three rings, that the height of the central tower contains four squares on the distance between the axes of its walls. We have the same height for the spire. It appears also from the square placed diagonally, with an angle in the middle of the porch and its summit at the top of the ridge following the line of the medieval trace, that the height of the tower with spire is = three times the side of the square on the width of the screen front, one and a half side of the square for the tower, and one and a half for the spire. This proves the correctness of the method used in fig. 313 for determining the height of the spire of the central tower.

As regards the demonstration made previously of the connection between the decoration of the remaining parts with the reconstructed part of the trace of the roof, the reader can convince himself now that this accordance continues right up to the top of the spires.

On the right of fig. 314 (Plate XXIII) is affixed the before mentioned moduli scale—a little too high, unfortunately; for this reason we have introduced another one in the scales of proportions carried out previously, showing the proportion of the sectio aurea. Similar ones are carried out on the left of the drawing. These show that right up to the top of the spires, we have succeeded in carrying out logically the proportioning of the church according to the sectio aurea, with the help of the system, just as we had found it to be the principle going through all the preserved older part of the church.

It may be of interest now to examine in what proportion the reconstructed front stands to the transverse section of the church. To this end we have introduced in fig. 315 (Plate XXIV) half the transverse section of our reconstruction of the nave, as we did previously for Cologne. The connection between this and the front is so evident that any comment is unnecessary. We shall only point out that the lines of surface of the pyramids of the spires correspond not only with the auxiliary construction, but also with the transverse section. It will be seen that these lines come down to the point which is marked with two rings on the diameter of the large circle of the front, on the left of the centre of the rose. In the transverse section of the nave the line of the spire comes down to the corner on the base of the square of the transverse section, the vertical side of which is the axis of the wall of the aisle—that is to say, a phenomenon similar to the one we observed in Paris and in Cologne. Similar to these, we have found the same connection to exist between all the principal points of the edifice, outside and inside.

Later on we shall see that this connection does not only exist between the front, the central tower, and the transverse section, but that it includes the fixed points in the east gable wall of the chancel—in other words, that it exists in the whole edifice from east to west.

CHAPTER XIX

THE SIDE ELEVATION

AFTER having previously given extensive arguments for our reconstruction of the transverse section of the nave, of its side-walls and of the west front, we go back finally to the side elevation to treat it in its totality.

It will be remembered, from the analysis of the chancel, that the conditions did not allow the medieval architect to give the chancel the same height as the nave west of the transept, where its width could be the base of its vertical construction without any hindrance.

It had been part of the problem of the architect to work together these two features having such widely different data, and to unite them according to the same main lines in a harmonious unity in order to get thereby grandeur in the whole and avoiding putting together piecemeal the chancel and the nave, which would be little dignified in an episcopal church, the fourth international pilgrim church of Europe. It is manifest that such a fusion has taken place.

Christie held also the correct historical and esthetic view that the dignity of the cathedral demanded a united architectural appearance. This is shown clearly in his scheme for the reconstruction of the nave, according to which he has given the wall a height which would be least noticed when compared with the different one of the chancel (fig. 322). It will also be seen that he has united the whole church from east to west under one ridge line so as to give it unity and grandeur. But he has obtained this result at the expense of historical truth. As regards the nave, we have already proved that his scheme was an infringement of the system of proportion of the church, and as regards the roof it will be also proved, when we explain that the level of Christie's ridge is a whole 5 ells 6 inches (if not 6 ells) lower than the level which we obtained according to the system of the church and the remaining trace of medieval roof, the correctness of which we have shown in so many ways previously.

The former architect, Christie, in contrast to the present leading one, was scientifically educated and well schooled in Gothic art; moreover, he had obtained a thorough knowledge of Norwegian medieval architecture, through long and fruitful researches when working for the Norwegian Archæological Society, in the place of H. E. Schirmer, as leading architect for the restoration of the cathedral. When we mentioned above his reconstruction of the windows of the clerestory we said that he sought his model in Lincoln; but we omitted to explain that, nevertheless, he did not work without method and according to "feelings." Christie was well acquainted with the theory of proportion of Viollet-le-Duc, and it is from this that he used the equilateral triangle when proportioning the principal divisions of the cathedral, its transverse section, its front, its height of ridge and its spire, as shown clearly in fig. 318. But, as it happens that the cathedral is not built *ad triangulum*, but *ad quadratum*—

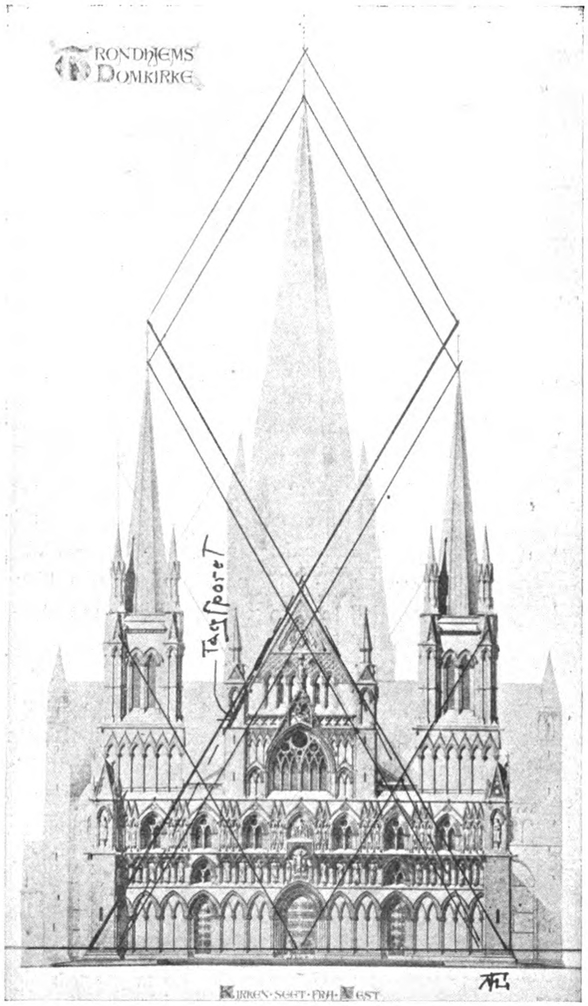


Fig. 318. Cathedral of Nidaros. Christie's reconstruction of 1903 *ad triangulum.*

as we have proved undeniably—it will be understood at once that Christie's reconstruction of the roof is not historical.

For the reconstruction of the hood over the octagon he had fully satisfactory materials. Before its restoration the octagon had a baroque cupola dating from about 1730 (fig. 282) resting upon the clerestory stage terminating horizontally. An examination of the courses of the wall showed that this horizontal finish was not the original one, but it was caused in the sixteenth century by a walling in of the space between the small gables into which seven of the sides of the octagon had terminated; refer to fig. 319, 3, section *a—b*. It will be seen as well from the section as from the plan, No. 1 in the same figure, that at the top of each of these gables there is a mortise (marked *e*) for the tie-beams, which had stretched horizontally from top to top, by which a new octagon is produced in the plan of the hood, having its angles just in the middle of the sides of the first octagon. As concerns the height of the pyramid intended here, it will be seen from section 3 on the line after the plan *a—b*, that a channel with a square section is found running from point *h* at the east gable of the chancel (see No. 2, section *c—d*). One of the beams of the hood fitted into this. It follows that the height of the pyramid is given by the point where the angle of inclination of the channel continuing in height intersects the vertical axis of the octagon. The reconstruction of the hood is thus undeniably correct historically in every point.

The height of the roof of the chancel was less obvious. When the old roof was taken down in 1888, no less than four different traces were found on the east side of the central tower (see fig. 320).

No. 3 is the trace of the roof taken down, while No. 4 is the one chosen by Christie. According to the measured sketch, drawing No. 522, in the archives of the architecture of the

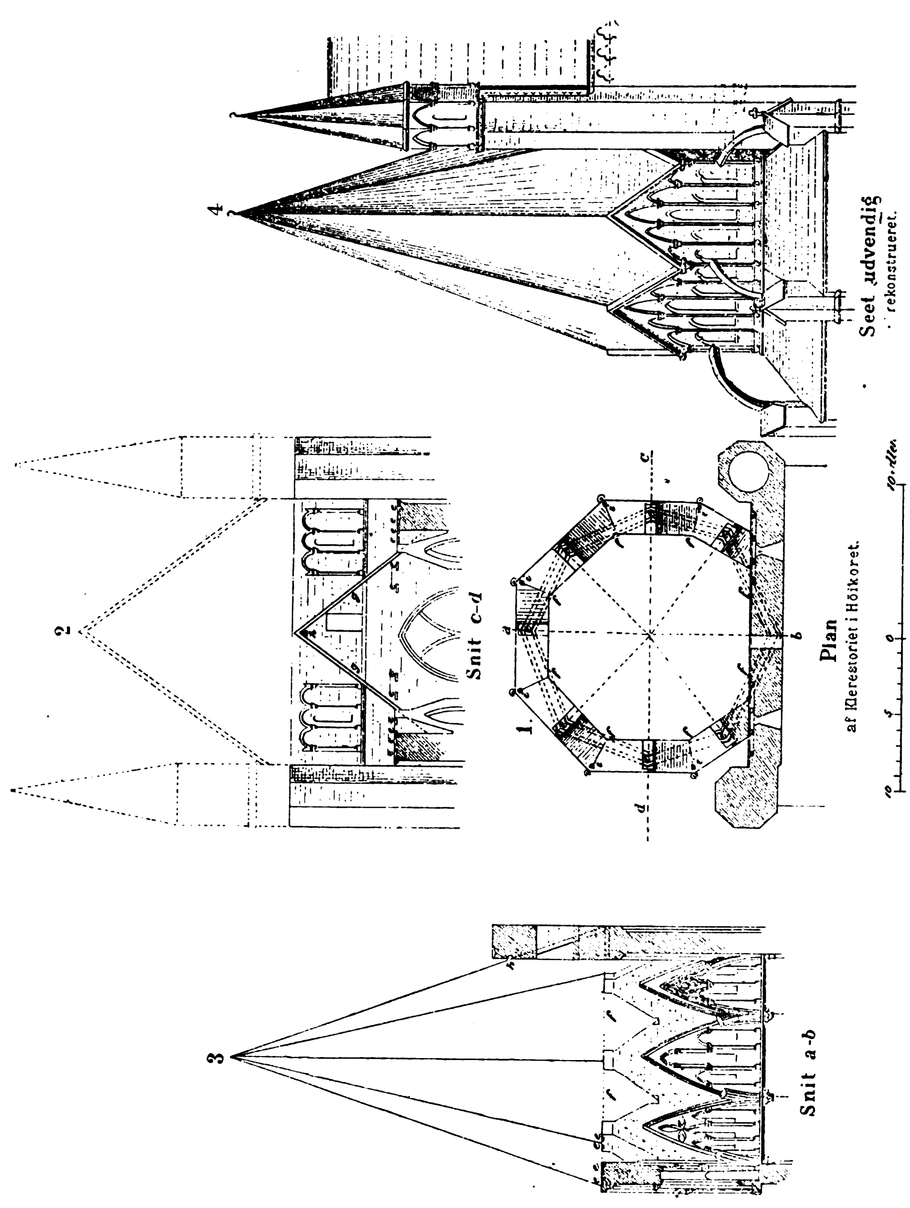


Fig. 319.—Cathedral of Nidaros. Christie's reconstruction of the hood of the octagon.

cathedral, the trace has an angle of inclination of about 58°, which gives for the ridge a level of 17 ells 3 inches measured above the main cornice. As regards the east gable there does not exist in the archives any measured drawing of the interior, but there is a measured sketch of it externally (Archive No. 516). In the middle of the east side of the south octagonal stair turret—that is, the side which is across the longitudinal axis of the church—about 2 ells above the main cornice and 2 ells from the south side of the tower, there is a part put in hatchings in the sketch, about 14 inches long and 8 inches wide. Just near the hatchings there is written: "Height 8 inches, deep trace of old roof on the inside of the south tower." The trace indicates an angle of about 50° and a height of ridge of about 15 ells 13 inches above the main cornice. As the difference of angle, 58° on the central tower and 50° on the east gable, can be a natural outcome of the chancel, being, as we explained, about 3 ells wider towards east, it was decided, without further reasoning, that the difference of height of 1 ell 15 inches, as the short trace of the measured drawing shows according to the ruler, could on the whole be done away with, and that this trace of roof ("tagspor" on the figure) on the east gable and trace No. 3 on the central tower might be from one and the same roof. As the drawing is made in very faint pencil lines and cannot be photographed we have transferred it, from notes, to the photograph of the authentic drawing of the reconstruction of the east gable (fig. 321). As it will be seen, the trace ("tagspor") lies over the before mentioned remains of wall, where we have emphasised it by a thick line. The continuation of the trace is indicated by a line in crossed strokes, and an arrow marked "NB. Tagsporets forlængede Linie" ("Lengthened line of the trace of roof"). To make a general survey we have introduced the square of the whole design of the west front, as well as the examination of the whole design of the nave, with the axial distance between its pillars G 2 H 2 as data, refer fig. 286 (Plate XVIII). The square on the width of the screen front is indicated by the diagonals of the half-square marked with four rings. The traces upon the west wall of the central tower are indicated in thick strokes along these diagonals. The summit of Christie's roof-construction is indicated by two rings. This lies 17 ells over the main cornice, while the one marked "tagspor" at the height indicated on the east gable lies 1 ell 15 inches lower. With Viollet-le-Duc's theory of the equilateral triangle in his mind, Christie has tried the height of the ridge according to trace No. 4 on the central tower in proportion to the front, and he found this to be *ad triangulum* on the width of the screen front, as shown in fig. 321—therefore, in full accordance with Viollet-le-Duc's opinion on the method of proportioning in medieval architecture. It follows that trace No. 4 was thought to be the original one. After this observation Christie built the roof above the chancel, just as he afterwards proportioned the transverse section of the nave, the front and the spires of the towers *ad triangulum*, tied as he was to the best theory of the time. Incapable of making by himself new independent observations which could lead him further, it never occurred to him to attach any importance to traces Nos. 1 and 2, the angle of which he was not even interested enough to measure. Consequently, no measured drawings of these traces exist in the archives of the cathedral.*

* We owe this information, the correctness of which is quite supported by fig. 318, among many other proofs, to the late town architect, G. Bull, who, from the appointment of Christie in 1872 until his death in 1906, was chairman of the committee of direction for the restoration of the cathedral. When G. Bull saw our drawings in 1916 he struck his forehead, and this native of Bergen, then in his eighty-fifth year, broke out in true Bergen dialect: "We have been fools, all of us!" regretting at the same time that his age did not allow him to put up a fight for our scheme. He wrote, however, concerning it and said: "Unfortunately my health is so shattered that I am not capable of doing anything for Macody Lund and his work, which has interested me greatly, as I see in it the basis of a satisfactory solution" (Sinding-Larsen: *Macody Lund*, Christiania, 1919). With our knowledge of Christie's scientific earnestness and of his modesty, we have not the slightest doubt that he also would have admitted his errors unreservedly, and with pleasure given his approbation to our scheme—as his friend and colleague did.

This information does not throw a flattering light on the present chairman of the supervising committee, Mr.

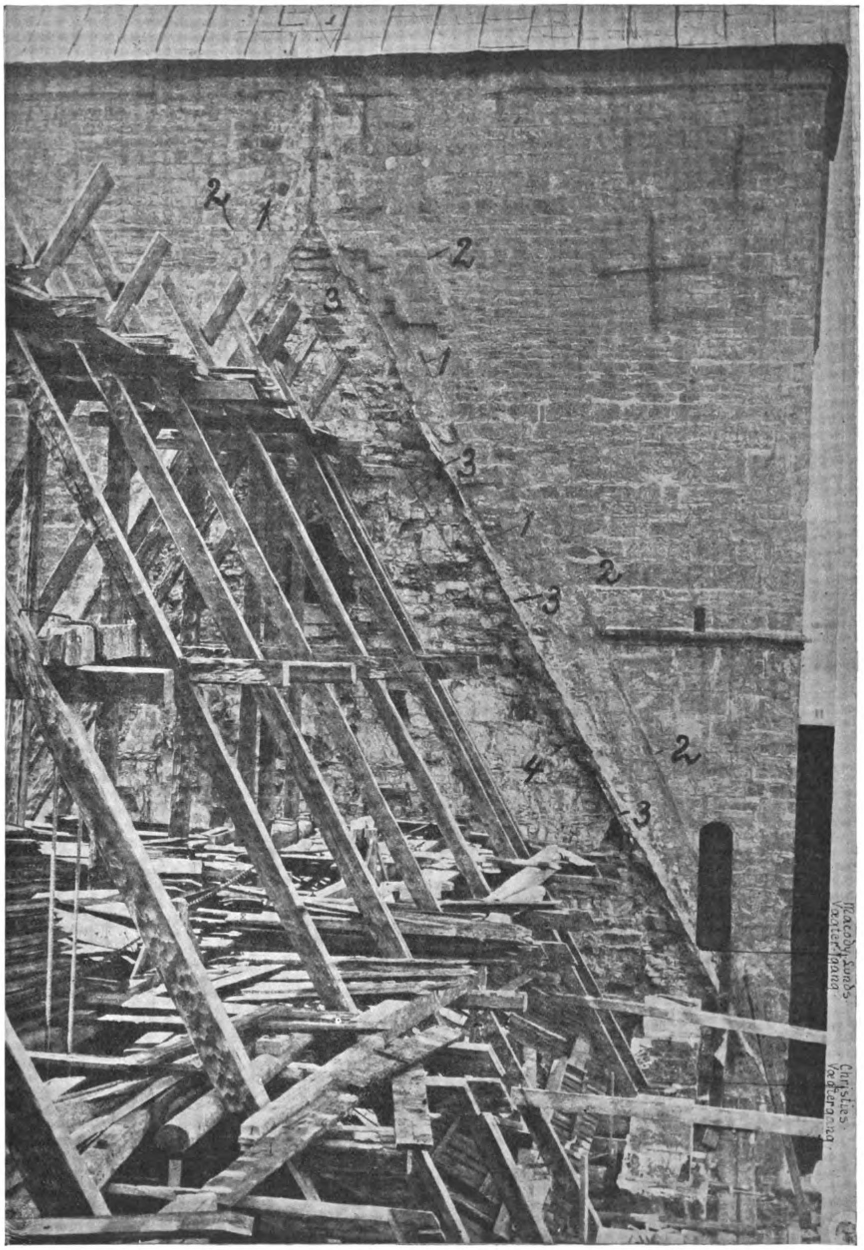


Fig. 320.—Cathedral of Nidaros. Photograph of east wall of the central tower with the old trace of roof.

The restoration of the cathedral had not been decided yet as a whole, but only in bits, first the chapter-house, then the octagon, then the chancel. Instead of seeing in this a danger they thought, on the contrary, through sheer ignorance, that this was quite in accordance with the medieval manner of proceeding.

Nor did it occur to Christie to measure the angle of the trace on the west side of the central tower before he designed the roof of the chancel. It was only after the transept was restored and when the question arose of restoring the nave that they undertook to measure it—in 1894, fully six years after the roof of the chancel was designed and four years after it was put up. Even if Christie had subjected the angle then found to a trigonometric calculation, he would hardly have attached any importance to it, because the square as means of proportioning was not used then in the recognised medieval method. Christie was convinced until his death that this had never been a trace of old roof, and that his own roof over the chancel was historically correct, so that its continuation over the nave would be correct also.

When the roof of the chancel was built in 1890, Christie's statement published in 1876 concerning the correctness of the reconstructed hood of the octagon was all forgotten, and the critique found it ugly—as compared with the "amusing" and "picturesque" cupola that had been pulled down (refer fig. 282). It did not occur to anybody that the reason of this unpleasant appearance was that the architectural appendix rises in an absolutely illogical way above the roof of the main building—as much as 6½ ells (refer fig. 319 and fig. 297, longitudinal section, Plate XIX).

After the previous statements showing that the church is built *ad quadratum*, and not *ad triangulum*, it does not need much explanation before it can be understood that it is not the hood, but the roof which is not historical. This appears already from the fact that the compressed roof does not harmonise with the historically reconstructed hood. An examination of fig. 321 will show, moreover, that the roof-line indicated by "tagspor," on its way downwards, passes through the upper inner corner of the preserved fragment of wall before mentioned, marked on the drawing by an arrow, and it intersects it as well as the door leading out on the parapet, to end just over the projection of the main cornice. If there has been a roof according to this trace, then there has been no parapet. The same is shown by trace No. 4 on the photograph, fig. 320. Thus this trace must be either older or later than the Gothic building. In the first case, it must date from the roof of Olaf Kyrre's chancel, and this is impossible, for the reason that the two semicircular arched windows on the west side of the central tower correspond with the windows on the south and north sides, where their position is determined by the roof rising between them; refer to figs. 289, 322, and 323. There is no doubt, then, that the nave of Olaf Kyrre had the same height as the transept, and the ridge of the roof of his chancel has passed through the point marked by a ring and a cross in fig. 321. In connection with it, it must also be explained that the arches inside, and the blind arcade composed of semicircular arches outside on the gable of the chancel just at the back of the hood of the octagon, are built from arches and parts of arcades from the pulled down chancel of Olaf Kyrre. The trace No. 4, used by Christie, can thus date only from the time of the decline of the church. An attentive examination of the

Johan Meyer. He was a member of it in Christie's time, and he must have known that Christie worked according to a method. Mr. Johan Meyer, this worshipper of everything that Christie did, was the emissary of the supervising committee, going round to architects' societies and to newspapers in order to cover up our criticism with his learned saw-dust. When we remember that he is among those who have ridiculed the geometrical system of proportion, and, with the compiler of Nidaros, has treated it as "a symptom of the 'naïveté' and of the ignorance of a certain group of people," when these opinions can find approbation, then it shows that Mr. Meyer and the other gentlemen of the present committee cannot have understood anything of Christie's work and of the method which directed it, or it suits them to be silent about what they must necessarily know.

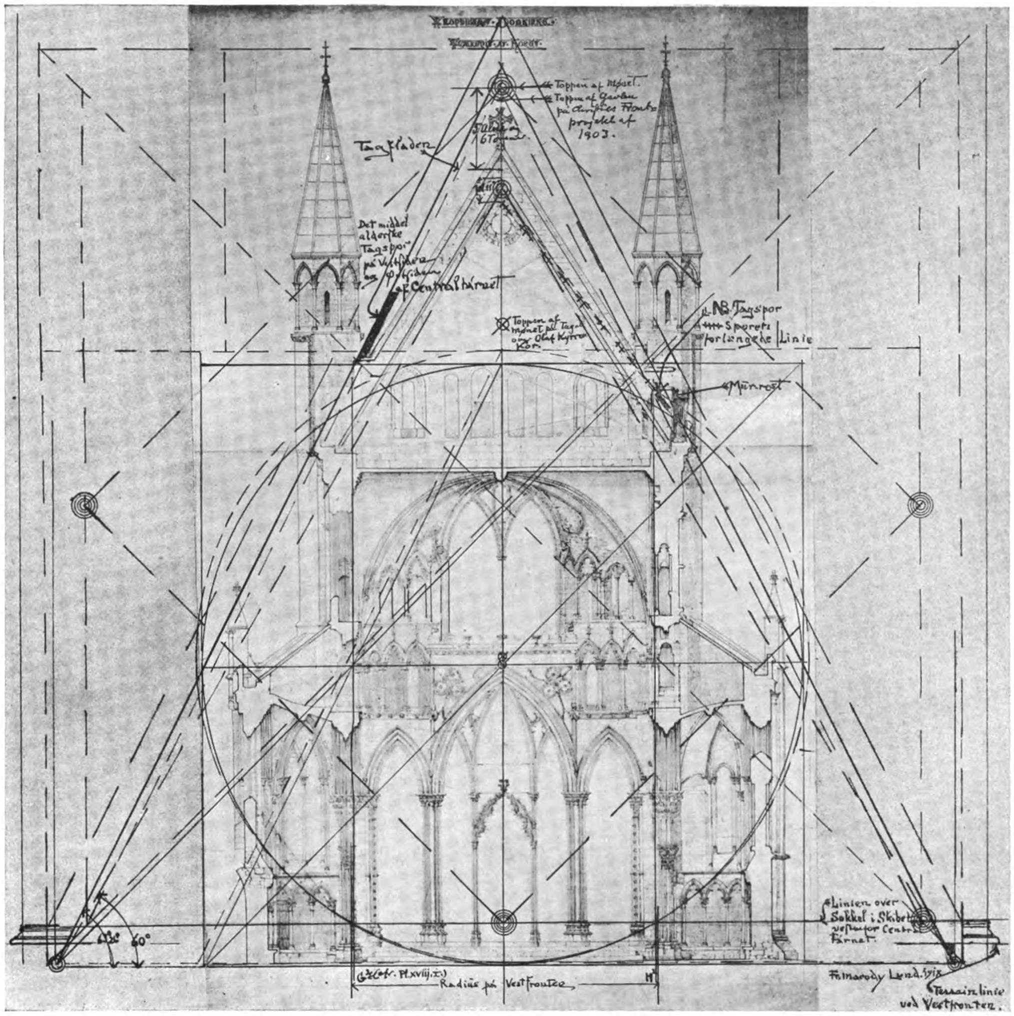


Fig. 321.—Cathedral of Nidaros. "Tagspor" on the east gable wall of chancel.

photograph, fig. 320, gives a confirmation of this, because it shows clearly that the surface of the wall under trace No. 2 is patched-up masonry.

To this is added the conclusive fact that trace No. 2 is according to the same angle as the trace on the west side of the central tower and gives thus exactly the same height of ridge. If so many different traces of roofs are found on the east side of the tower, and only one on its west side, it is explained by the different reconstructions after seven conflagrations, most of which seem to have been due to lightning. Peder Claussön, who had his evidence, as we mentioned it, from an earlier member of the chapter, Jon Simonsson, says that the

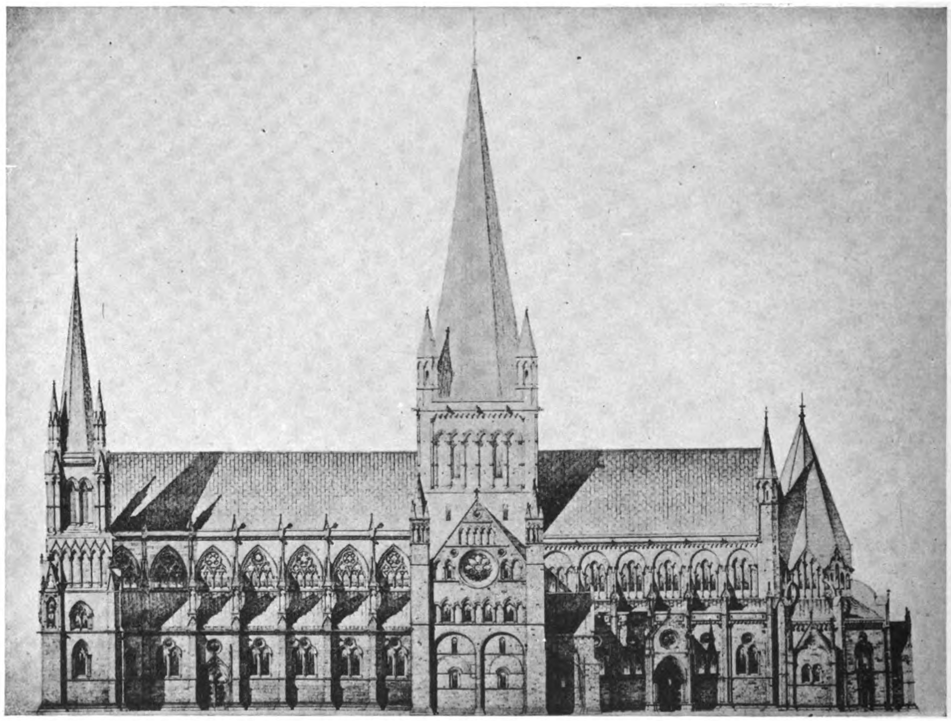


Fig. 322.—Cathedral of Nidaros. Reconstruction of side elevation by Christie.

three fires in the Middle Ages, in 1328, 1432, and 1531, were caused in that way. In Nidaros, as on all the west coast, thunder-storms come mostly with the west or north-west winds, so that the burning spire fell every time on the chancel and set fire to it, whereas the nave escaped. As shown by the traces, the nave was burnt only once—in 1531, when the fire from the burning tower reached the church and laid it in ashes. It has not been rebuilt since.

After having finally succeeded in getting the old trace on the west wall of the central tower acknowledged as such, the reinforced committee met our demand for the raising of the roof only half-way, because, while admitting its correctness as regards the nave, they relied blindly on Christie's authority, and allowed the roof of the chancel to remain unchanged. The clauses of the architects' competition were changed according to this. Later on, however, the committee has allowed Mr. Nordhagen to lower the ridge more or less according to his changing "feelings" for the sake of the "outline" and for "the more harmonious distribution of the masses," and also to get a "more natural proportion between the nave and the chancel" (this means the unchanged roof of Christie) to repeat the arguments quoted previously. In fig. 323, which is the design of Mr. Nordhagen of 1914, and which gave rise to our present work, a proof can be seen of this "more natural proportion"—as the ridge of the roof of the nave is four feet lower than the height of the old trace.

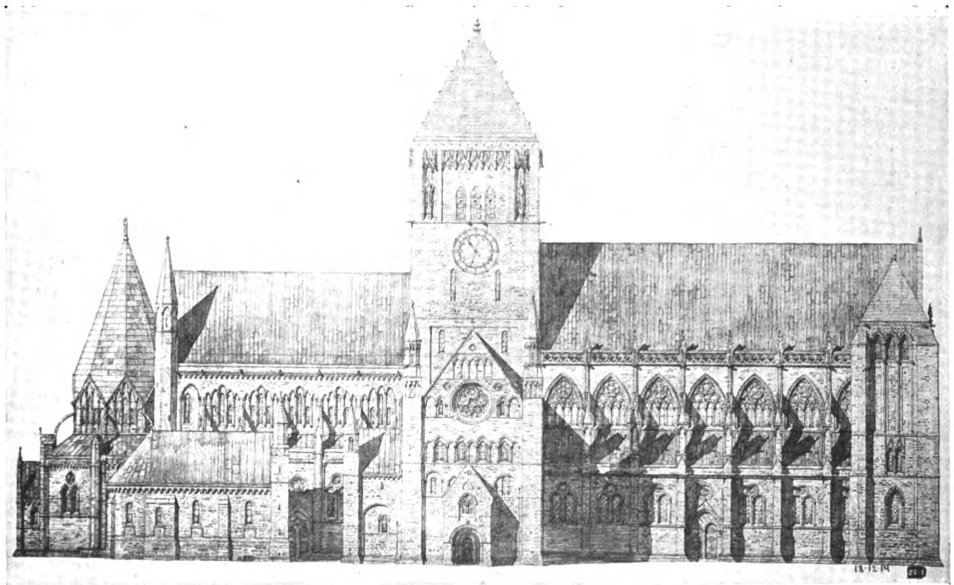


Fig. 323.—Cathedral of Nidaros. Design of Nordhagen, December 1914.

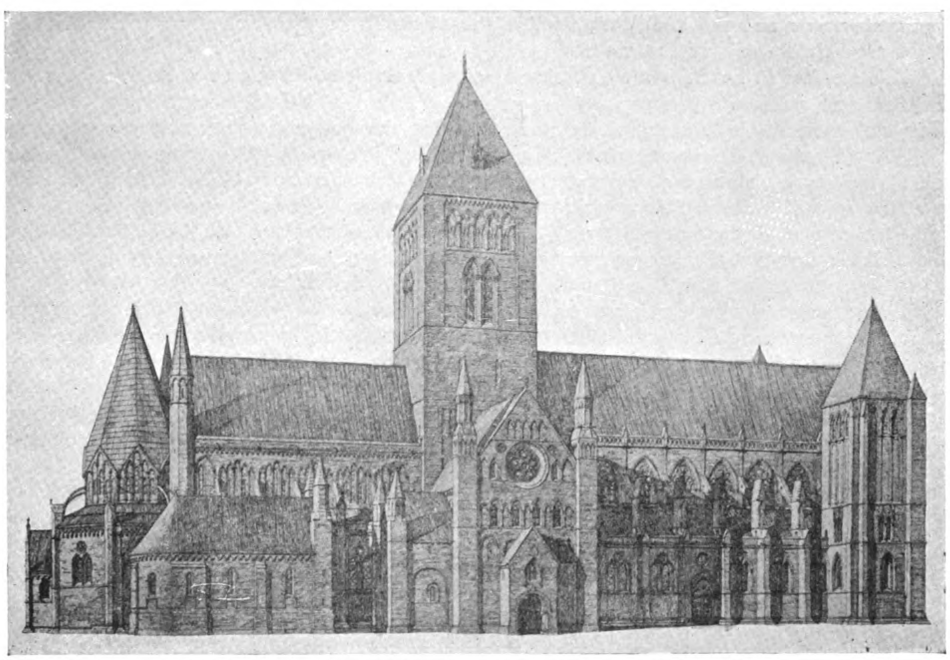


Fig. 324.—Cathedral of Nidaros. Design of Nordhagen, December 1916.

In his design of December 1916 (fig. 324), which is the eleventh outcome of his feelings, the "improved" proportion between the nave and the chancel is exactly the same; but after a new feeling, the distribution of the masses is improved by giving the central tower the same height, exactly to an inch, as in our design, after our discovery of the geometrical system, so ridiculed by him and the supervising committee.

Christie's design was done after a methodical reasoning, although from incorrect data. A comparison between this and the result of Mr. Nordhagen's feelings shows at once the hopelessness of the committee's adherence and lack of critique as regards Christie's roof over the chancel. Instead of the unity of Christie's scheme, they have obtained an accidental, cut-up outline, making the church smaller, while the hood of the octagon above the compressed chancel is more and more dominating—a sad evidence, truly, of the danger of restoring bit by bit without a general outlook and without method.

As a contrast we show our side elevation in its entirety (fig. 325, Plate XXVII), as it is produced *ad quadratum*, according to the law of proportion of the church and the fixed and sure traces of old roofs indicated before, on both sides of the central tower; these traces, which we proved to belong entirely to the auxiliary construction used for the conception of the cathedral and its architecture, and which show irrefutably, by their connection throughout, that the Gothic cathedral has been designed with and has had a roof as high on the east as on the west of the central tower. Thanks to this, the church is united under one line, it has the high and dominating elevation of a cathedral, and the hood of the octagon, which before was "ugly," is now beautiful, because, without losing its distinct architectonic character in the slightest degree, it takes a secondary place as an independent chapel, when compact and high it terminates the whole building in a harmonious manner.

Not only the roof, but the walls also, west and east of the central tower, are under the command of one and the same line, because the side-walls of the chancel are given the same height as the side-walls of the nave.

This increase in height is a technical and necessary result of the increased height of the roof of the chancel, and is proved historically correct thereby. But, as it leads to such important architectural alterations, we feel obliged to point out that the building itself still possesses unmistakable marks of this increased height. This is shown already in the photograph of the east side of the central tower (fig. 320), where it will be seen that trace No. 2 ends a good way up at the side of the small door which led out to the parapet from the winding stairs of the north-east corner of the central tower. This photograph is supplemented by fig. 326, which reproduces a drawing of the reconstructed tower by Christie. Over Christie's roof, according to No. 4, a trace No. 2 will be found with an angle of 63° 26′.

When analysing the transverse section of the chancel, we explained why the proportion of its central aisle is as $1 : 1\frac{1}{2}$ instead of $1 : 2$, and also why the main cornice, which must necessarily mark the limit of the clerestory, happens to be a whole three ells lower on the wall than it would have been if the architect had had a free site to build on. As explained before, the main cornice remains close to the central tower and to the east gable; and on the same level as the upper edge of this cornice, the opening of the door is kept in the south stair turret of this gable (fig. 321). These circumstances, as well as the remark made before, that trace No. 4 gives a height of ridge according to Viollet-le-Duc's triangle theory—that is to say, as a series of superficially agreeing points—have led Christie to the easy conclusion that the wall of the chancel had originally stopped at the main cornice. A glance at fig. 293 will show that the level of the projection of the main cornice determined *ad quadratum* on the wall of the central aisle, lies 6 ells 14 inches lower than the string-course remaining from the Middle Ages, showing the division between the clerestory and the belfry stage in the central tower of Olaf Kyrre; these stages have become respectively the triforium and the clerestory in the

Gothic rebuilding. The door mentioned in the central tower, which lies much higher than the roof of Olaf Kyrre's chancel, is thus pierced in the new triforium of the tower to gain access to the parapet of the new Gothic wall of the chancel. Now it is remarkable that the threshold of this door lies exactly at the level of the parapet of the central aisle, as shown in fig. 327. But in Christie's reconstruction the threshold lies 1 ell 6 inches higher than the top of the parapet and 2 ells 17 inches higher than the floor of the latter. It follows that he has been obliged to put a steep stair, unprotected, and therefore dangerous under the climatic conditions of Nidaros, going up to the threshold of the door; then he has been obliged to hide this part of the stair, which rises over the top of the parapet, by making the latter higher, as fig. 326*b* shows. It is superfluous to say that the conditions were never such originally.

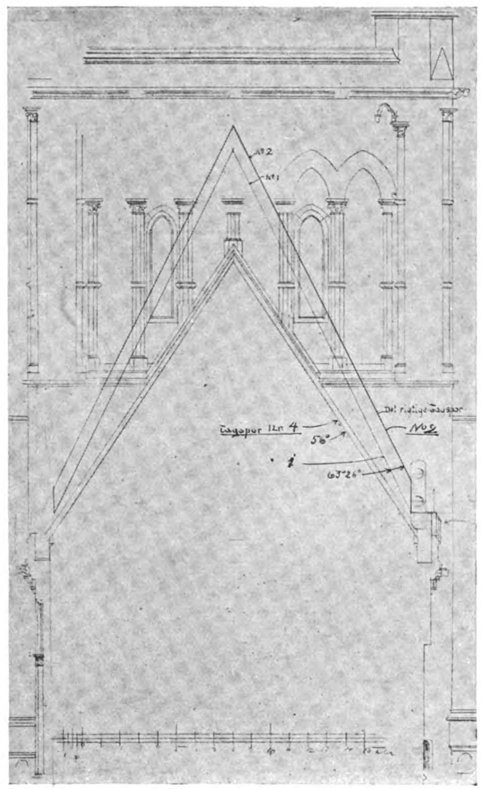


Fig. 326 *a*.

Finally, we shall call attention to the remaining fragments of wall previously mentioned on the south stair turret on the gable of the chancel—see the drawing of the ruins (fig. 274). This is a rudiment the importance of which Christie has not understood, partly on account of the restoring done piecemeal, and partly because of his use of the incorrect triangle method, but which he smoothed and cleaned up nicely; refer to figures 321 and 286 (Plate XVIII), where it is marked by NB and an arrow.

In this rudiment we have a material proof that the wall of the chancel has reached the height given to it by the system of proportion of the cathedral and the preserved traces on both sides of the central tower.

Fig. 326 *b*.

Figs. 326 *a* and *b*.—Cathedral of Nidaros. Christie's reconstruction of central tower against the roof of the chancel and photograph of Christie's stair masked by the parapet.

Whether the door in the stair turret near the east gable of the chancel is the original one or whether it has stood higher at somewhat the same height as the door in the central tower, is a question which depends upon the ascent of the winding stairs and the number of steps inside the circle, but having otherwise no connection whatever with the manner in which the increased height ought to be done. In figs. 327 and 328 our scheme will be seen with the increased height of the wall in its entire thickness. If the door is the original one, the access could be adjusted by a covered stair coming down from the heightened parapet, or, if it is not, by a new door at the height of the new masonry. But we believe the door to be the original one. The heightening of the wall of the central

aisle has not in reality been carried out in the whole thickness of the wall, but, like the cathedral of Reims, it had an attic wall 4½ ells high to receive the rafters of the roof, while the floor of the parapet has had the same level as at present. From here a stair hidden and protected by the parapet has gone up to the door in the central tower. In both cases the heightening is hidden by an arcade; in the first case, built together with the wall heightened in its whole thickness, as demonstrated in figs. 327 and 328, as in the cathedral of Prague (fig. 329), in the other, with an open colonnade, as in Reims (fig. 330).

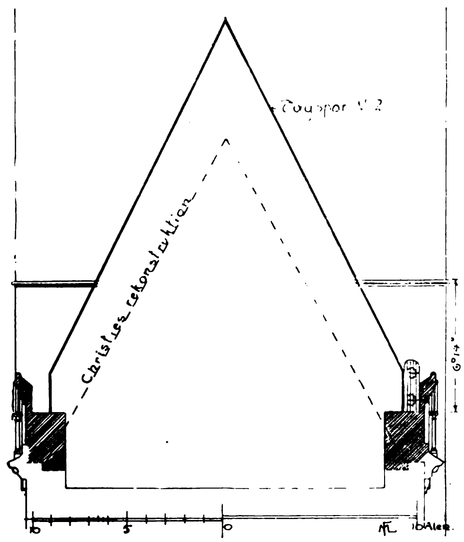


Fig. 327.—Cathedral of Nidaros. Increased height of chancel wall, towards the central tower.

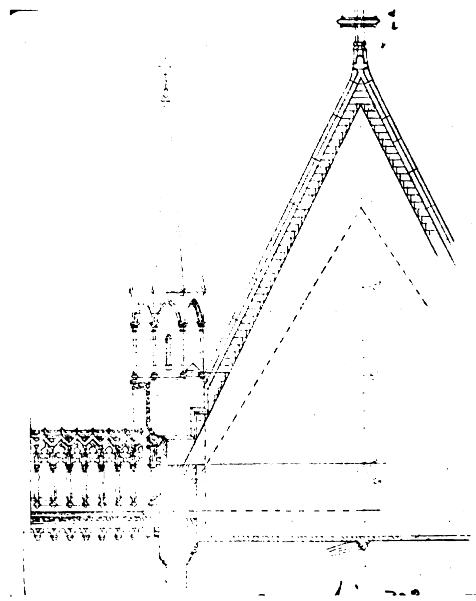


Fig. 328.—Cathedral of Nidaros. Increased height of chancel wall, towards east gable.

In *Prague*, the heightening is made necessary because the umbrella-shaped vault rises 3½ metres higher than the wall arches over the windows; in *Reims* it was to equalise the difference of height of two different parts, the same reason as in Nidaros.

In both illustrations (figs. 329 and 330) it can be seen how the main cornice marks the finish of the clerestory, while the gallery above hides the raised wall of the central aisle.

We see that Minutoli noticed that the wall must have been higher, because it occurred to him, in connection with the frieze going upwards on the stair turret, that it must have had the decorative function of receiving the gallery at its junction with the latter.*

* "Hoch über diesem Friese [viz. chancel cornice] auf welchem gegenwärtig das Dach ruht, werden noch am östlichen Treppenthurme die Reste einer Verzierung bemerkt, welche es wahrscheinlich machen, dass die oberen Theile der Mauer früher noch durch eine hohe und prächtig durchbrochne Gallerie verziert gewesen sind.

Die Verzierungen dieses Oberbaues scheinen, soweit sie erkennbar, gegen die unteren Theile des Ostkreuzes eine vorgeschrittene Entwicklung des Styls zu verrathen." (Minutoli, *Der Dom zu Drontheim*, p. 21, sp. 1. Berlin 1833).

(" High above the cornice on which the roof now rests, there can still be seen, on the east stair turret, an ornament which makes it probable that the upper part of the wall had been decorated with a beautiful high open parapet.

" The decoration of these higher parts of the church seems to show, from what is left, a development of style from the lower parts of the east chancel.")

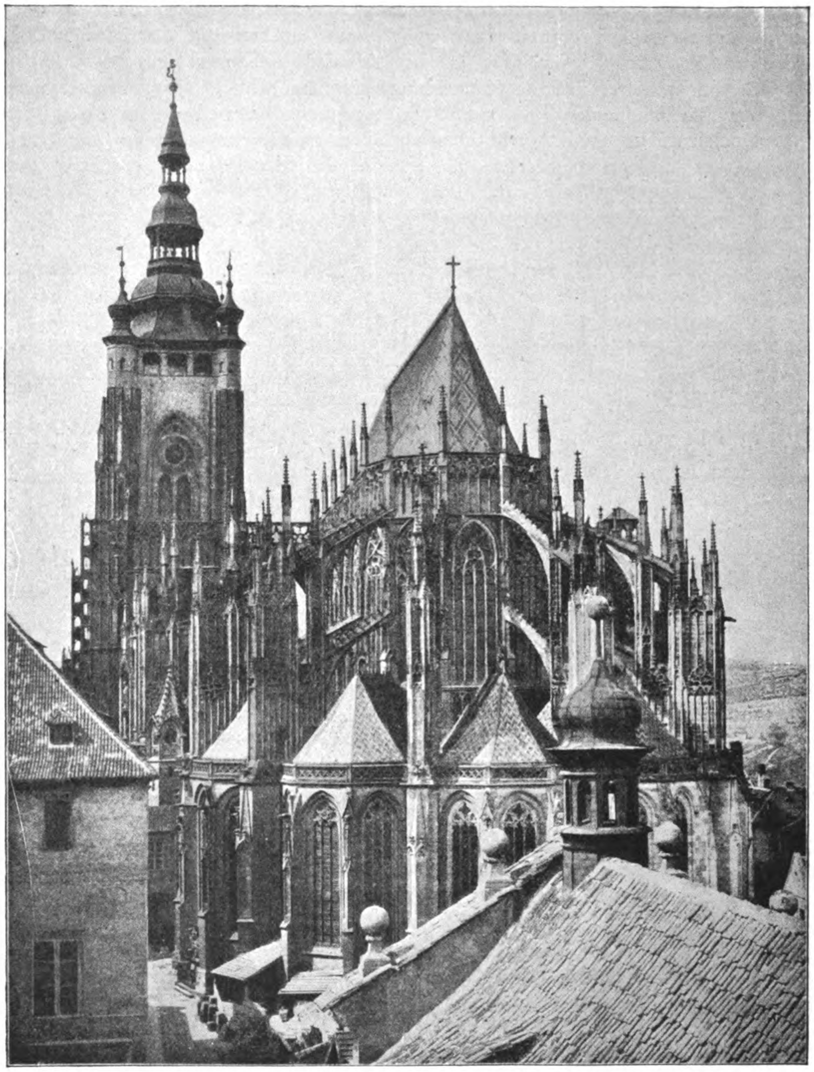


Fig. 329.—Cathedral of Prague. Masked heightening of chancel wall.

When objections have been raised against this heightening of the walls of the chancel they reveal a lack of knowledge, not only of Nidaros cathedral, but of medieval building art as a whole. We find a confirmation that we have followed the right course in obeying the mentioned indications of the church itself, as we did for the roof of the chancel, first in the esthetic fact that the pyramid over the octagon does not now compete with the church itself as if it were the spire of a tower rising over the roof of the chancel, but instead, subjects itself to the whole, like a beautiful finish; secondly, because the base, unused until now, of the triangle of the gable of the chancel, comes in this way into its function; furthermore, what is quite conclusive, because the bases of the triangle in the gables of the chancel and the nave are brought to the same level, while in all the earlier schemes they are at different heights. In other words, a complete accordance is brought between the east and the west gables (see fig. 321).

In fig. 331 (Plate XXVIII) we have made an analysis *ad quadratum* to ascertain the proportion between the towers, the height of which we determined by means of the theoretical auxiliary construction of the west front, as of the whole reconstructed building.

After what has been demonstrated previously, the test construction needs no explanation; the whole exterior of the church is united inside a large square with the whole length of the church as one of its sides—although we did not seek this result. It will be seen that Nidaros cathedral has exactly the same proportion as Cologne, where the height of the towers represents half the length of the church—the same with the spires—and just as the height of the church to the points of the spires coincides exactly with its length (see fig. 73, Plate VIII, and fig. 79, Plate XI).

We saw that the case is the same with Notre-Dame in Paris; the height of the west tower, which has no spire, is exactly equal to half the length of the church (fig. 59, Plate IV); this applies to the cathedral of Roskilde (fig. 125), and also to the church of St. Peter in Malmö (fig. 129). It is characteristic, as we mentioned it previously, that Viollet-le-Duc, this judge of medieval architecture, which he knew so thoroughly, in his scheme for raising the spires on Notre-Dame, gave half the length of the church as the height of these spires. Under his articles, "Flèche" and "Clocher," in his *Dictionnaire*, he makes this a rule for medieval cathedrals.

Finally, we will only remark that the pinnacles of the central tower, as well as of the west towers, are included in the test construction; that the altered east gable, with the roof of the chancel, comes into the system according to the diagonal of 63° 26′, from the east extremity of the church; that the axis of the octagon corresponds with the lines of the construction; furthermore, that the horizontal lines through the summit of the gable of the transept fix exactly the level of the bases of the triangle of the gables east and west; see the tangent through the zenith of the fully traced circle in fig. 286 (Plate XVIII) and fig. 321. Finally, it must be remarked that the gable of the single-aisled Norman transept is raised *ad quadratum*, just as we noticed also for the interior of the side-wall (fig. 268).

We remind, again, that this result is not sought, but that it is produced directly by the logical raising of the west front according to the principle *ad quadratum*.

As a general test of the whole analysis and the construction based upon it, we have done as with the existing cathedrals of Paris (fig. 38, Plate III) and of Cologne (fig. 79, Plate XI), and carried out the test construction in all the plans, as reproduced in fig. 332 on Plate XXIX. This drawing does not require explaining. The reader can see for himself how the whole church rises from the plan, how the front develops itself inside a large square with the height of the spire = the length of the church as side; how diagonals after 45° in that square go exactly through the intersecting points of the upper side of the auxiliary square of the front with the axes of the two flanking towers, while the level of the cornice on the latter coincides

with the side of the mentioned square. From this level the two diagonals on their continued course determine the height of the spires, *ad quadratum* on the axial distance of the side-towers. It will also be seen that the height of the central tower is *ad quadratum* on the length of the transept. This surprisingly direct connection, which appears in all plans as by magic, is seen in this drawing to be the result in reality of a very simple geometrical function. It is seen here, more clearly than in the theoretical demonstration of Cologne cathedral (Plate XI) that this connection is brought about only by the fact that the plan also is designed

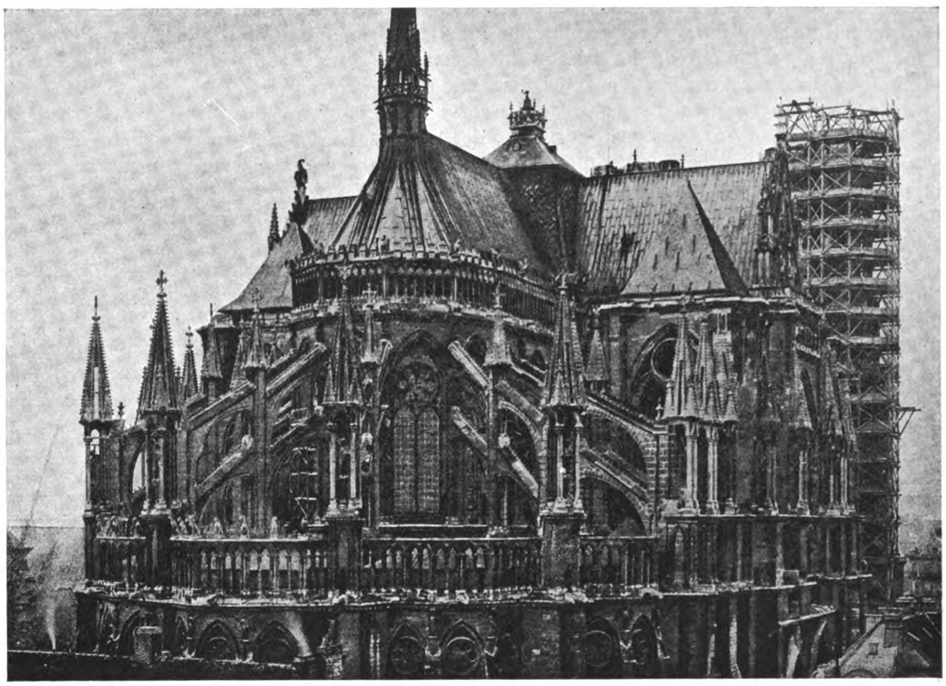


Fig. 330.—Cathedral of Reims. Masked heightening of chancel wall.

inside a large square upon the total length chosen for the whole foundation = twice the length of the transept. After a regular division of this square into four, the plan itself is then designed inside two squares upon the length of the transept; after another regular division of these, the square of the design of the front and of the transverse section are at last obtained. It is, then, obvious that through a complete geometrical raising of the various parts of the building inside each of the squares, a connection must be at last brought about between them all. When treating of the principle, we saw that in classic architecture the square formed also the base of the plan of the Greek temple, just as we found that the front of the temple was developed from the plan as part of its square. We saw also that in the harmonic geometrical progression, which is produced by and from the pentagon in the circle inscribed in the square, a unity was found which formed the introduction to the play of harmony between all the

parts of the temple, from the unit to the totality—because the various parts, while measuring one another, are measured by the whole, or by its major or minor. In our analysis and reconstruction of the cathedral of Nidaros, we have proved the corresponding harmonious accordance between the transverse section, the side-walls, and the front. We found the unity of this progression of harmony by the development of the pentagon in the circle inscribed in the square of the transverse section (fig. 190). When this, as well as the square of the front, is obtained by a regular division of the large square of the whole, it is evident that there must be an inner harmonious accordance between the harmonic progression developed in these circles and the one developed similarly in the large circle. We give, as an instance, that the minor of the diagonal of the primary pentagon in this circle = the diameter of the circle in the square of the front, or, if preferred, the side of this square. Moreover, the side of the pentagon in the circle of the transverse section = the diagonal of the pentagon of third order in the circle of the whole. As example of the proportion of the circle to the building itself, we can say that the diagonal of the pentagon of first order = the total interior length of the cathedral from the gable wall in the west to the gable wall of the small chapel farthest east. Moreover, the diagonal of the pentagon of third order in the same circle = the exterior width of the central tower. It can be interesting to explain, further, that the total length of the plinth of the west front = the minor of the side of the large square.

Finally, we must not forget to mention—as the last evidence of the reliability of Maschius' engraving, important for the architectural and iconographic reconstruction—that the width of the screen front is exactly equal to a third of the total length of the church, which Maschius explains in a note attached to the drawing. Now, as we proved that the summit of the ridge —according to the old trace on the west side of the central tower—lies *ad quadratum* on the width of the screen front, the silhouette of the cathedral under the line of the roof contains three squares in height up to the ridge—just as in Notre-Dame (Plate IV). It must be noticed that the roof line in the drawing of Plate XXIX has been put too high, by accident; it should have gone through the intersecting point of the diagonal of the half-square lying horizontally, coming from the east corner of the square on the base, with the diagonal of 45° from the west corner on the base.

As it will be understood from our explanation, the cathedral of Nidaros, instead of being an agglomeration of parts of buildings of various epochs, accidentally put together, is, on the contrary, a regular plan, a single conception, and proportioned in such a perfectly scientific manner that, in this respect, it is hardly exceeded by any other cathedral in Europe.

Therefore this is not an empty boast, nor the effort of a people exhausted nationally, indulging in provincial dreams and fancies, trying to find a lying consolation for a fate half self brought; on the contrary, it is the expression of a living tradition from Catholic times, when, according to the quotation of Peder Claussön before mentioned: "As to the walls," the cathedral of Nidaros "had hardly its equal." This could not have meant the extraordinary decorative splendour only, but the unique and harmonious proportioning, obviously and systematically carried through.

It is by studying this proportioning during many years that we found the principle *ad quadratum*, and with it the common basis for all the various expressions of religious architecture, as widely different as the Parthenon and the Norwegian Stavkirker, or, in other words, the fundamental thought underlying them all—the square and the circle.

In our reconstruction, we started from the square, we moved among squares, and we came back to the square. It is reasonable to think that this geometrical phenomenon, which contained surely a moral in the astonished mind of the ancients, has been the fundamental idea when raising a Christian temple. The *square* was for the Pythagoreans the symbol of matter and stability, the *circle* was the symbol of eternity and perfection, the pentagram stood for the

harmony-creating principle of existence. And, just as these figures were fundamental in pagan architecture, so they became fundamental also in Christian religious buildings. We have shown that the logical use of these principal figures of geometry creates perfect unity between all parts of the temple.

This accordance, found in the cathedral of Nidaros, among so many various expressions, gives us the certainty that our reconstruction carried out *ad quadratum* reproduces, in the main, the original appearance of this edifice.

CHAPTER XX

THE CHRONOLOGY OF ARCHBISHOP EYSTEIN'S CATHEDRAL

IN our explanation of the historical and artistic data concerning Olaf Kyrre's cathedral we gave clear evidences of an old and artistic development in Norway, dating from the eighth century already. As we shall now concern ourselves with dates of the various parts of Archbishop Eystein's cathedral, the history of which we owe entirely to Icelanders, and first of them all to the great Snorre Sturlason, we think it natural to indicate briefly the background on which moved the characters of the sagas. Snorre's emotional interest was not only due to his Norwegian origin, but perhaps quite as much to an inherited legal sense of Odel (similar to English law of primogeniture concerning landed property).*

Icelanders belonged to the best Odel (means also landed property aristocracy) of Norway —seen from a racial and material point of view. It was not only the justified aristocratic satisfaction of having high-born ancestors, but the knowledge of having a right of succession, although often quite fictitious, to vacant Odel in the mother-country, which made these proud Icelanders keep their genealogies in the manner of good Odel men. The fact of being able to reckon in their pedigree, the names of Norwegian kings and of the best members of great families, descended from the remaining kinsmen of chiefs who had occupied the new land, not only drew closer their Norwegian ties, but it kept them strong and warm in their country with its long, raw winter.

Snorre Sturlason grew up in one of these centres of the cult of family and traditions. He was forty years old when he first came to Norway, learned in history, in law, in the art of the bard and of the saga. His first visit lasted two years, from 1218–20, his second one lasted as long, 1237–9. Not reckoning a short summer visit in Gautland, he was therefore altogether four years in Norway, of which, fully three years in Nidaros. Moreover, his son Jon was there between 1221–4, partly as the King's and partly as Duke Skule's guest. Snorre could trace his pedigree back to King Olaf Kyrre and to Archbishop Eystein.

The cathedral of Eystein, with its transept dating from Olaf Kyrre, was the first great cathedral that Snorre had seen. It is evident that it must have roused his interest greatly, not only because of its greatness and wealth, but principally because he found, no doubt, a family tie in the whole building. As the honoured guest of the King and of his father-in-law, ranking as he did among the highest at the Court, he had full opportunity, in his intercourse with the Archbishop and his high officials, of obtaining knowledge of the history of the church, because, as proved by *Heimskringla*, he had access to the archives of the cathedral and of the King.

It is, then, evident that such a man, related to the founders of the cathedral, had ob-

* Translator's note in ( ).

tained full and sure knowledge enabling him to write with the greatest accuracy, as his short monumental style of narrative demand.

We gave previously a quotation of a part of the *Saga of St. Olaf*, pp. 258 ff., where Snorre speaks of the cathedral in connection with Archbishop Eystein. He mentions here explicitly the great "musteri" (minster) of the latter, which stands now in opposition to the former Christ Church of Olaf Kyrre.

We have convinced ourselves that the nave of Olaf Kyrre still existed in 1179, where the south-west tower is mentioned in the account of the battle of "Kalfskinnet." *

As the saga of Haakon Haakonsson contains a clear narrative of how the foundation of the now standing west gable wall, with its two flanking towers, was laid in 1248,† and as we can conclude with as much certainty that in 1179 Archbishop Eystein could not have possibly reached west of the central tower with his work of reconstruction, we have here the undeniable evidence, as we said, in Sverre's saga, that the nave of Olaf Kyrre was still untouched in that last-named year.

The question is, then, how far had the Archbishop reached in his reconstruction east of the central tower before his flight from the country in 1180? We have given proof previously that the octagon was finished before that year. Judging from the ancient Byzantine character of the ornamentation of its periapsidal gallery and chapels, as well as of its Romanesque construction, it may possibly have been begun before, or in any case at the same time, as the Gothic modernising of the transept.

As concerns the time of the completion of the chancel, the accounts of royal burials give complete indications. King Sverre, like all usurpers, was anxious to prove his legitimacy and he seized every occasion to do so. Thus Snorre relates that Sverre removed the body of his pretended brother, Haakon Herdebred, from Veöy in Romsdalen, where he was buried after his fall in 1162, to the stone wall of the cathedral, "fyrir sunnan í Kórinum" ("buried south in the chancel").

Cecilia, the daughter of the legitimate King, whom he also pretended to be his sister, and her husband, Baard Guthormsson, were also buried in the stone wall, but on the north side of the chancel; the former in 1185 or 86, the latter in 1194.

It would be in full agreement with the astute politics of King Sverre to give the son of the legitimate King, his pretended brother, a demonstrative and royal burial, just as the old legitimate Kings had utilised St. Olaf, in order to strengthen their own legitimacy and to found their throne and fix their crown in the mind of the people.

In the meantime we have here a valuable mark to judge how far the Archbishop had gone in his rebuilding of the chancel at that time, because it would agree little with a show of honour, nor with the aim of the burial, if King Sverre had caused these to take place in a church which was being pulled down or on the point of it.

* The word "Kalfskinnet" (calf's hide) is the actual name of a part of the town of Nidaros, west of the cathedral. We do not know where this name comes from. We identify it with "á Akrinum" in the *Saga of Sverre* where it says that Sverre encountered King Magnus and Jarl Erling "á Akrinum." After a critical examination regarding this place in Sverre's saga it occurred to us that this name of "Kalfskinnet," which is in reality the name of a kind of taxation, is based upon a misunderstanding of Protestant times. Peder Claussön is, as far as we know, the first one who uses it. Remarkably enough, in his lexicon, Fritzner has only under the word "Akr" the explanation of the direct meaning of it—*cultivated corn-field*; but we have noticed, when going later through the Icelandic sagas and old Norwegian documents, that when the word "Akr" is used in direct connection with the account of a church, it has a religious significance, meaning the field of the Lord, the old German "Gottes Acker"—therefore, the name for the cemetery with its graves round the church. Consequently there is no doubt that in Sverre's saga it is meant that the encounter took place in the churchyard, from where the partisans of the King and of the Jarl were driven into the river. Through this undoubtedly correct meaning of the word "Akr" we get the conclusive proof of the truth of our previous rendering of the account of the battle.

† *Konunga Sögur*, p. 414, ed. C. R. Unger.

We can conclude with certainty, therefore, that the burials took place in the rebuilt chancel of Archbishop Eystein. As the walls of its aisles are still standing and do not contain —or, on account of their lack of thickness, could not contain—any graves, it is certain that the "Steinvegginn" ("stone wall") means the screen wall round the chancel, as explained before. The chancel of Archbishop Eystein must then have been finished before his flight from the land in 1180.

We do not know whether the work was continued or if it stopped during the three years that the Archbishop was away, but there is every reason to believe that, from 1183 until his death, he made every effort to conduct the rebuilding with all possible speed—all the more so because he took no part now in worldly politics.

When we turn to the chronology of the nave we think again of the account given in the *Saga of Haakon Haakonsson,* where we read: "Thetta Sumar hafdi hann látit setja Grundvollinn til Kirkju svá langt vestr, sem nu er" * ("That summer [1248] he had set [Archbishop Sigurd] the foundations of the church as far west as they now stand"). This part of the saga has been read until now, by all writers, as a direct evidence that the rebuilding of the nave from Norman to Gothic style—that is to say, all the work west of the central tower—was started in that year. We must consider this rendering of the text as being quite arbitrary, because this is not the direct meaning of it according to the words. The translation is done, moreover, quite uncritically, without parallel studies of the existing remains of the nave, which contain, however, clear chronological signs. We refer, in this connection, to the highly characteristic fact that there is an obvious jump between the architecture of the side-walls of the aisles and that of the two flanking corner towers (see fig. 333).

On the other hand, there is an agreement just as remarkable in the architecture of the windows in the aisles of the nave and chancel; it indicates forcibly a connected period of building. It is clearly visible already, from this fact, that the side-walls of the aisles of the nave must be older than 1248. In the octagon and chancel we still find the Norman manner of building, with thick masonry and projecting supports for the wall. Lights are small, in pairs, and lancet-shaped, without divisions; they have ornamental columns, the capitals of which are still Byzantine in the octagon, but in the chancel have already a more indicated Gothic design. Arches over the windows are just as little divided as the lights, but decorated with the tooth ornament of the transition period. Over the arches is a round "eye" inserted in the wall without being joined architecturally with the lancet windows. We still find in the frieze of the cornice the ancient wave, like a Byzantine reminiscence (see fig. 296). If we examined the aisle west of the transept we would still find the twin lancet windows, but here joined together by the "eye"—therefore approaching the fully developed Gothic windows under a common arch; similarly in the elevation (fig. 333) and in the plan (fig. 275) it will be seen that the system of abutment of the Gothic construction has arrived to its full development. The ornate frieze under the cornice is seen here relieved of all antique manner. If we look now at the window of the north flanking tower we find the window formation further developed in the richer divisions of the splay as well as of the arch, its lobed opening and its decoration. It will also be seen that it is only because of its narrowness that the window is not divided with mullions and tracery. As shown on the plan, exactly the same shape of window is found on the east side of the tower.

From these observations we can draw the conclusion that the rebuilding of Archbishop Eystein had reached on the west, at the time of his death in 1188, the place where his aisle met the Norman west front of Olaf Kyrre, or—to keep to the still standing building—to the east side of the actual flanking towers, which were raised just at the time of the removal of the west

* *Konunga Sögur*, ed. C. R. Unger, p. 414.

front in 1248. This appears quite clearly in a comparative examination of the plan of Archbishop Eystein (in fig. 275) with the two plans, drawn together, of this cathedral and the cathedral of King Olaf Kyrre (fig. 253). In the first plan it can be seen that the rebuilt aisles cut into the east wall of the flanking tower, which had here to be given a chamfered plane in order not to shut out the light of the farthest west windows of the aisles. On examining Olaf Kyrre's plan it will be seen that the new wall of the aisle has stopped just against the west front of Olaf Kyrre, the flanking tower of which was going to be pulled down.

The mentioned chamfered plane in the east wall of the corner towers, which can

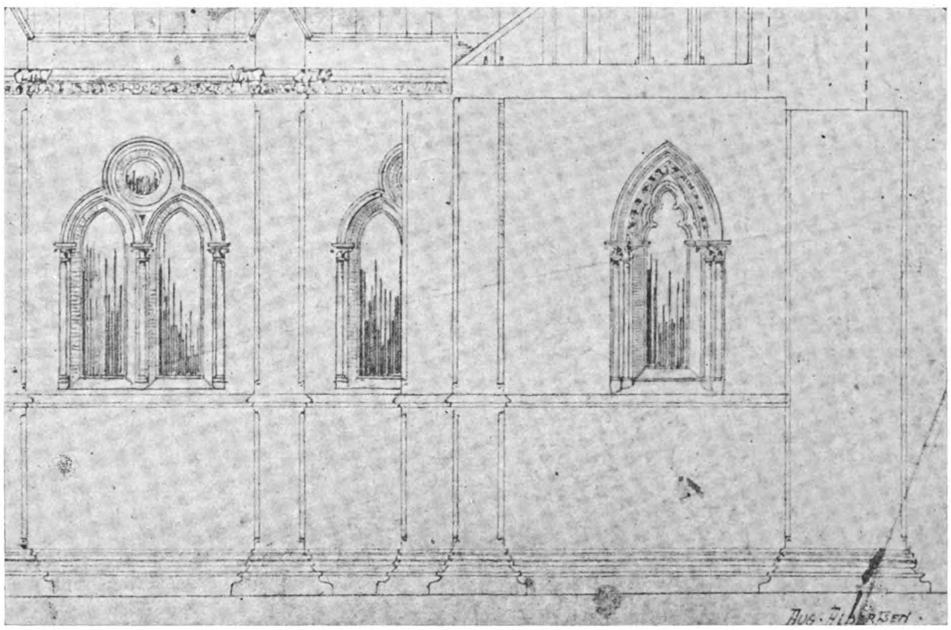


Fig. 333.—Cathedral of Nidaros.—Aisle windows compared with the window of the flanking tower.

easily be considered as something accidentally "amusing," is, however, a direct outcome of the most western bay of the nave having been made shorter because the front is planned as a screen front with the system of abutment adjusted in the wall at the back.

We have no direct written evidence of how much of the nave was rebuilt in Gothic style before 1248. But the narrative of Snorre, in all its briefness, is here a complete reliable document. He speaks of Archbishop Eystein's great minster which now stands, showing that the nave also must have been rebuilt, in all its important parts, when he visited Nidaros the first or second time.

If this had not been the case two-thirds of the church at least, the transept and the nave, would still have been the church of Olaf Kyrre, and the chancel only would have been raised by Eystein. It is obvious then that Snorre, who is always precise in his choice of words, would have considered the cathedral he saw as the work of Olaf Kyrre, and all the more so

that he had quite as many reasons to be proud of his relationship with that King, whose cathedral was in its way as wonderful as Archbishop Eystein's.

As for the work of rebuilding the nave after 1188 we find an evidence of it in Sverre's saga. The successor of Eystein, Archbishop Eirik, mixed himself up also in worldly politics, to the great annoyance of King Sverre; this led to a climax in 1190. King Sverre reproached the Archbishop with keeping a large Court and warships with a crew of 120 men, and he suggests that it would be more in accordance with his office to use the revenue of the Church in continuing its building, in the same way as it was already started.* The King does not mention anything here about the building operations having been stopped.

We have an evidence of the further continuation of the building of the cathedral in the thirteenth century, when King Haakon Haakonsson gave Archbishop Guthorm (1214–24) the right to coin money in order to increase the revenues of the Church. It is probable that the rebuilding went steadily from year to year, until it arrived at the still standing Norman front of Olaf Kyrre, at the time of the first or second visit of Snorre. Therefore he had every right to call the work of Archbishop Eystein "thetta hit mikla musteri sem nu stendr" ("this great minster which now stands").

The cathedral stood then in the following conditions: rebuilt in Gothic style according to the plan of Archbishop Eystein, but having still the Norman front of Olaf Kyrre, until it "was removed so far west as it now stands" by Archbishop Sigurd in 1248. It is psychologically impossible that the author of the saga should have noted this removal as a remarkable event, if it had happened as an obvious part of the continuous building operations. But, on the contrary, the long hoped for undertaking, whereby the Gothic work of Archbishop Eystein would get a front, the design of which had no doubt been much discussed, was an historical event which the saga had to record.

We do not know when the spire of the cathedral was raised, but, as Peder Claussön, who had his evidence from a previous member of the chapter, says that the church in the Middle Ages had sustained three fires through lightning, it is obvious that it must have had a spire, or, in other words, that the cathedral was then completely finished. As regards the covering of the roof and of the spire, Absalon Pederssön says that at one time the church was covered with copper and at another time with lead.

* * *

If we now resume our construction of the whole church (fig. 325, Plate XXVII) we find here the expression of the chronology which we explained above. We shall see the development continuing from the octagon, to the wall of the aisle of the chancel, then to the wall of the central aisle of the same, with its early Gothic masking colonnade; we see the further advanced early Gothic style in the walls of the aisles of the nave, and we notice that, according to the very nature of Gothic architecture, its constructiveness is expressed quite consciously in the system of abutment, lifting up its flying buttresses to support the fully developed Gothic wall of the clerestory of the nave.

And lastly we find on the towers the transition to the rich west front.

We have therefore proved and realised fully what we said previously, p. 276, that all the Gothic parts of the cathedral are built according to a united and settled plan made before-

* *Konunga Sögur*, p. 121, ed. Unger. The expenses of the Archbishop for the keeping and pay of this crew amounted to 128 marks of silver = 52,000 crowns, therefore, a small sum relatively to the fixed income of his church, which had, from landed property only, about three million crowns per annum (see Fr. Macody Lund *Norges ökonomiske System og Værdiforhold i Middelalderen* (*The Economical System and Values in the Middle Ages*), Christiania Videnskabsselskabs, Skrifter 1908.

hand by the Archbishop Eystein and his architect, a plan strictly carried out until the completion of the work, although some details have undergone successive development, being the natural result of the long time which it took to realise the foundation—that is to say, from the establishment of the Archbishop's see in 1152 to the full completion of the cathedral in the years following 1248. As regards the raising and completion of the front, we must, with our knowledge of mediæval building, maintain that it did not take many years.

* * *

We complete our arguments by producing the appeal with which Count Minutoli concludes the preface of his great work, *Der Dom zu Drontheim,* written by order of King Frederick William IV of Prussia, and published at his expense, in 1852:

"An Alle aber, welche noch ein besonderes Interesse an der Erhaltung des Denkmals haben, richte ich die Bitte, für dieselbe beizusteuern.

"*Es ist das erhabenste Denkmal der Geschichte der Normannen, werth nicht allein erhalten, sondern auch wieder hergestellt zu werden.*

"Alle diejenigen Nationen, welche von den Normannen abstammen und deren Geschichte mit der Normannischen verkettet ist, sollten ein lebhaftes Interesse darfür haben. An sie vorzugsweise richte ich diese Aufforderung."

("To all who have an interest in keeping up this memorial I address the appeal to contribute towards it. It is the greatest memorial in the history of Normans, not only worthy to be kept, but to be raised again. All the nations descended from Normans and whose history is tied to theirs should have a vital interest in it. It is to these especially that I address this appeal.")

CHAPTER XXI

THE ORNAMENTATION OF THE CATHEDRAL

AFTER having gone through the architecture of the cathedral, and having proved its scientific conception, it may be of interest to give a short account of its ornamental sculpture.

As the nature of our task did not require any special analysis of the ornamentation, we did not prepare any systematic collection of photographs. What is reproduced in Plate XXX*a*, figs. 1–8, is only an accidental gathering, representing capitals and voussoirs of arches from the chancel, the nave, and the west front, whereas we have no examples of the richness of the cathedral in sculptured figures.

The specimens given will be sufficient, however, to form an idea of the technical and artistic execution of its ornamentation.

We gave previously several examples of the few remains of ornamental sculpture from Olaf Kyrre's cathedral and other churches of Nidaros dating from the beginning of the eleventh century, and we indicated how this ornamentation represented a direct continuation of the ancient Norwegian art of wood carving. These examples, however, are of a simple geometrical kind, and can give only a limited knowledge when it comes to judge of the achievements of Norwegians in Christian stone ornamentation.

In the specimens given here, however, we see examples of the fully developed Gothic ornamentation of the twelfth and thirteenth centuries.

In spite of their damaged state, they show quite sufficiently that we have here a fully ripe decorative sculpture, which can compare, from every point of view, with the best of its kind in England and in France. Our opinion is more than justified when we remember that this pregnant, clear, and conscious handling belongs, as we just mentioned, to the oldest Gothic art in Europe. It is older than the excellent ornamentation of Lincoln, for which the cathedral of Nidaros has undoubtedly served as model, and from which Lincoln has probably obtained its artists.

Among the examples given, fig. 1 belongs to the arcading of the wall in the aisle of the chancel; that is, previous to 1180.

It will be seen that the capitals have an original shape quite special; from the bell of the capital, there does not grow the usual leaf ornament, but a cluster of small leaves like the shooting buds of spring.

In spite of the hesitating forms, not yet quite clear, we find already the plastic lines of Gothic art at work to assimilate the living flora used in its ornamentation.

In fig. 7 we find a related but more developed capital from the wall of the aisle of the nave—that is, from 1180–88.

Figs. 3, 5, and 6 are from the same place, and show how the style is already in full development. The double twisted capital of fig. 8 is also from the nave. We see here that the style begins to get clearer in the firmer lines of the plastic vine ornament; at the back of the deeply cut leaf there are birds pecking at bunches of grapes.

In fig. 9 we have an example of Christie's carved capital full of respect and understanding, related to the one in fig. 7. It is a pleasure for anyone who understands, to see how he has succeeded in reproducing the elegance and plastic life of medieval sculpture.

The eminent German historian of architecture, Dr. Rudolf Adamy, when mentioning old Norwegian ornamentation, writes: "The material employed—steatite—was very suited to the ornamentation of Romanesque and Gothic buildings. On account of its tenderness and texture, the carving could be extraordinarily sharp, deep, and with undercut mouldings; so much so that, according to the declaration of our witnesses, the details in many Norwegian churches surpass those of the English churches related to them. Steatite is, moreover, very durable, and in consequence the ornamentation of Romanesque churches is well preserved." ("Den Ornamenten der romanischen und gothischen Bauten kam das verwendete Material, der Speckstein, zu Gute. Dasselbe gestattete wegen seiner Weichheit und Struktur einen ausserordentlich scharfen Schnitt derselben und ein tieferes Ausarbeiten und Unterschneiden der Profile, so dass die Einzelarbeiten mancher norwegischen Kirchen nach den Versicherungen unseres Gewehrsmannes die der englischen Schwestern übertreffen. Da zudem der Speckstein sehr dauerhaft ist, haben sich auch die Ornamente der romanischen Kirchen noch wohl erhalten.")* The reference claimed by Adamy is the talented German-born architect, H. E. Schirmer, senior, the man who originated the rebuilding of the cathedral of Nidaros.

It can be seen that Christie has followed in the steps of this true appreciation reverently. His efforts during a whole generation made him successful in gathering round himself a group of clever well-trained, and thoroughly competent workers, who devoted themselves with keen interest to this beautiful and difficult art.

The successor of Christie, Mr. Nordhagen, has been anxious to show his "originality" in this branch also, by introducing new ornamentation, which, according to his teacher, the Danish architect, Mr. Nyrop, is "an attempt" to show the artistic ability of our time. On Plates XXX*a* and *b* (figs. 10, 15, 16–22) we have reproduced an accidental collection of these "attempts." They need no explanation; the reader can see for himself that they represent a retrograde step in the art which Christie had succeeded in creating again. It forms also a brutal and violent break with old Norwegian ornamental art—an art which is still fully living in the ornamental wood carving of Norwegian peasants.

It would be an insult to make a direct comparison between these productions of Mr. Nordhagen and the original sculptures, and also with Christie's reproductions. We will point out in fig. 21, however, the only example where Mr. Nordhagen has tried to reproduce the character of medieval ornamentation. We see here a compact mass of coarsely modelled leaves stuck on the clumsy bell of the capital.

If this attempted imitation of Mr. Nordhagen is unsuccessful, the original attempt at new ornamentation is still worse.

While in the previous examples of original ornamentation we have the proof of the living and joyful feelings of the Middle Ages regarding the exuberance of life, in Mr. Nordhagen's "attempt" we find only a collection of accidentally stuck-on, clumsy indications of flowers, dead and stiff, without organic connection with the capital or with the part from which they should naturally spring. Nowhere do we find an idea of ornament, the unifying line denoting the mastery of artistic development over the composition. Everything is equally unripe

* Dr. Rudolf Adamy, *Architektonik des muhamedanischen und romanishen Stils*, p. 455. Hanover, 1887.

and uncultivated, therefore it is not easy to emphasise any special example. The clumsy shape of the capitals has nothing in common with Gothic art.

The same unripeness, added to unfortunate ignorance of architecture, especially of the harmonising law of Gothic art, is crudely evident in fig. 22, which reproduces the arch over the niches, in the triforium of the west front, reconstructed by Mr. Nordhagen. In order to understand it we refer to fig. 4, which reproduces a corresponding part of the preserved first row of niches in the same front. As it will be seen from this and fig. 309 (Plate XXII) the arches on the niches of this row are supported by a cluster of columns consisting of a column in front and two at the sides, receding on another plane. It follows that the arcade is in two orders on the plan, and that the archivault is the same as shown in fig. 4.

In the triforium also Mr. Nordhagen has a cluster of columns in two orders, but it will be seen in fig. 22 that he has given the archivault no less than three orders. The dimensions of the three parts of this archivault, as of the hood-mould lying above, indicate completely, moreover, a lack of knowledge of the simplest law of harmony. He has given all parts the same thickness, in childish ignorance that medieval architects, knowing very well that harmony is obtained by the play of contrasts, always divided their archivaults according to determined unequal values, set against one another, often according to the sectio aurea (see figs. 1, 2, and 4). It will be seen also in figs. 2 and 4 how the ornamental foliage in the original arches of the first row of niches shoots out full of life from the folds of the arch and increases their action thereby. In contrast to this, in his original "attempt," fig. 22, Mr. Nordhagen has put in, between the folds, quite small, inorganic clusters of leaves, like the bow of a tie.

This ornamentation is repeatedly held out by Mr. Nordhagen himself and his admirers, as being both original and national. It is neither. Obviously it agrees little with the national ornamentation preserved in the cathedral. Nor does it make it better, that its woolliness reminds one of the wool embroideries of Hallingdal.

As far as originality is concerned, this consists in a cheerful copy of foreign models. As an explanation it is sufficient to say that Mr. Nordhagen has been educated as an architect in Denmark, so that it is easy to understand where he has found the subjects for this ornamentation, those he imitated coarsely and clumsily.

In the Middle Ages, Denmark did not have in its near proximity any stone suitable for plastic treatment. Except for a porous limestone and imported sandstone (for instance, in the cathedrals of Viborg and of Lund) Danish master-builders had nothing else than large pebbles found in the earth of the same kind of stone as the Norwegian granite. It follows that this influenced their ornamentation. The talented Danish artist, *Bindesböll*, has created in our time a renaissance in ornamentation, naturally influenced by the hardness of the granite, both as regards form and technical treatment. It is interesting just at this point and at the same time educating to see in French medieval art how the ornamentation changes technique and character according to the different kinds of stone used to build cathedrals in the different districts. In Norway, also, as everywhere else, the same remarks apply.

Mr. Nordhagen's uncritical introduction of shapes used in Danish granite ornamentation—in spite of the different character of the steatite—will be obvious. The case is the same with his technique. This is not seen so clearly in the reproductions as in the original photographs. But it is clearer in figs. 12, 13, 16, 17, 18, and 19.

Already from the name given to the stone used in the cathedral, the early Norwegians showed that they knew that each kind of stone requires its own tool and its own technique in carving. The stone used in the cathedral was called Talgustein, viz. a stone which can be carved, shaped, cut with a knife, or, in other words, with the same tool that is used in wood carving. The reproduced specimens of old sculptures show by their sharpness, their deep incision with under-cutting, and their firm shape, that this tool has been used. It will then be understood

that Mr. Nordhagen's uncritical transfer of the technique of granite with its surface visibly cut with the chisel is an offence against the history of art, and, to use Minutoli's words, against "the greatest memorial of Normans."

We have criticised this time after time, during many years, but with no result. Shielded by the Supervising Committee, incompetent in all branches, Mr. Nordhagen has been able to continue unhampered his "attempts" of dilettante.

We find good reason to repeat here our criticism and our warning.

CHAPTER XXII

THE ICONOGRAPHY OF THE SCREEN FRONT

OUR previous statement, p. 71, that Goethe was the first who roused interest in the beauty of Gothic architecture, needs a modification. This interest became general and vital only after the work of Chateaubriand, *Le Génie du Christianisme,* had influenced the French and brought them back from their exile in the desert of sterile rationalism, and from Rousseau's wilderness of misunderstandings regarding history.

As concerns the eighteenth-century historians and their doctrines of spiritual parvenus, one can apply to them, more or less, what Diderot said of Voltaire, that, like the monks in the Middle Ages, each wrote for his monastery.

With their demand that everything had to be judged apart from its environment and its given conditions, with their burning enthusiasm for the beauty of Christian religion, its devoted martyrs, its civilisation, its art, its cathedrals, the writings of Chateaubriand were like a flaming apology of the past and the right of mankind to be connected with it.

Chateaubriand obtained the acquittal of the past. What the revolution had discarded or doomed to disappear—institutions and thoughts of the past, the old France—he gave back to Frenchmen; the light of a new day shone on his country.

Although not a methodical historian, he was the founder, or, at any rate, the guide, in the modern writing of history.

He kindled a new light in the mind of his contemporaries and those who came after. Augustin Thierry, who has been called the Homer of history, bears witness to this when he says that reading *Les Martyrs* brought out the historian in him; it roused his understanding of local colour, of given conditions, of everything showing line and perspective. Historians, poets, artists, appeared as in a new spring-time: Guizot, Victor Hugo, Didron, Viollet-le-Duc, two generations, owe their inspiration to Chateaubriand. At the instigation of his older friend, Victor Hugo, Didron gave up his legal career and threw himself into the study of Christian iconography. Similar to Viollet-le-Duc as regards architecture, we owe to Didron the knowledge and the understanding of medieval religious figure-work in art, its iconography, which is connected in the closest manner with medieval architecture and supplements it. Interest in the magic beauty of medieval cathedrals became general, after the appearance of Victor Hugo's novel on Notre-Dame, built upon exhaustive studies of history and on Viollet-le-Duc's and Didron's scientific researches.

Before Didron's time, people had a very childish idea of the figure decoration of a cathedral, which was thought to be an arbitrary placing of statues of angels and saints. In *Notre-Dame de Paris,* which came out, as well known, in 1831, Victor Hugo saw in what is called

" la Galerie des Rois " on the west front of the cathedral, a representation of the Kings of France, and the architects had even cut inscriptions on the old statues, by which a King David was made into a Hugues Capet, a Solomon into a Robert, etc.

Didron sought in Greece, in the Byzantine Church and its fixed traditional iconography, the solution of the problem, which the iconography of the Latin Church had been, to all who had tried to find its meaning. He was accompanied by a whole group of interested people, among whom the artist, Paul Durand, who, like Didron, was well acquainted with the Greek language. In Athens they found a student to act as their interpreter in modern Greek. Didron and his friends visited a number of churches. He was struck to find here the same representations of figures, the same identifying attributes, inscriptions, and quotations from the Scriptures, in paintings of the seventeenth and eighteenth centuries, which he had noticed in mosaics of the tenth century in the cathedral of St. Mark in Venice. The draperies of the figures, not only their shape, but the drawing, the colour, and the number of folds, were everywhere uniform.

This applied also to the great biblical scenes, wherewith often whole walls in Greek churches are "storied." The disposition of the painting, the grouping of figures, and the placing of each figure in the group, seemed to be determined according to an iconographic system. Each picture was everywhere treated and grouped in the same manner, when it represented the same scene. The saints had scrolls with inscriptions written on, taken from their writings or the story of their lives. Biblical scenes had everywhere the same inscriptions, taken from their respective places in the Bible.

In one of the monasteries of Mount Athos, Didron had the good luck to see a monk decorating a church with frescoes. His pupils, who were all younger " brothers," were helping him. One applied the plaster to the wall, the master drew the picture, while another pupil traced the outlines and painted the under-surfaces. A younger one gilded the halos, painted the inscriptions, another painted the ornaments, while the youngest of all the brothers crushed the colours and mixed them.

In the meantime the master continued to draw his pictures—as it were from memory or inspiration. In one hour he composed on the walls a picture representing Christ ordering His disciples to teach and baptize. The monk drew from memory, without cartoon, without sketch, without model, just as he dictated, without book or notes, the inscriptions and sentences fixed for each picture and each of the figures. During that time Didron and his companions, who had examined other finished paintings, asked if these also were done in the same manner; they were answered in the affirmative, while the master added that he seldom took out the line which he had made. Concerning the rich pictorial decorations of the church of Salamis, with its nearly four thousand figures, Didron remarked that in France, where the great painters of his day had learned their art on general lines, if a painter was given the task of representing the ordinary history of religion, such as it is known from its usual facts and its characters, it is doubtful if he could carry out such a far-reaching composition without long and deep studies. He even insisted that in the whole of France not a single master could be found capable of carrying out such a work, not one learned and strong enough to bear such a burden. But in 1835 a master with three pupils painted figures and represented scenes, distributed inscriptions and epigraphs, named individuals and explained historical scenes, taking these epigraphs from the Holy Scriptures or from a whole lot of religious books of the Church Fathers, the lives of the saints, etc.

The monk of Mount Athos had got his paintings on the walls in the space of a few weeks, as if by magic. Inscriptions were the same which Didron had noted in Attica, in Peloponnesus, and in Salamis. When he and his companions expressed their astonishment, the monk was the most astonished—that they could really believe that these scenes and pictures of holy men were

the invention of his own humble person! It would not have been, then, a sacred art, but only a private work, maybe only to be understood by himself. No, what he painted was the art of the Church, as ancient as itself! And he showed them an old manuscript on parchment with beautiful initials in gold, blue, red and green, consisting of four parts. The first contained only quite technical indications how to get good brushes and colours, a good plaster, etc.; the second gave a practical and precise explanation of the symbolism and history which the picture had to represent; the third part determined carefully how the picture had to be placed, also the order in which the single figures had to stand, whether in a church, a hall, or a refectory. Finally, it gave an indication of the character which the paintings of Christ and of the Holy Virgin had to possess. The title of the book is *ἑρμηνεία τῆς ζωγραφικῆς*.

The writing was hardly three hundred years old, and in that time the various possessors had entered into it remarks to be added to the text in the next copy. It was explained that painting studios in other monasteries had also their copy. Didron succeeded later, in another monastery, in ordering a copy of a somewhat older sample to be written on parchment. As far as he could judge, after the lapse of a year, when the copy arrived in Paris, the writing of the beautiful initials was reproduced with faithful exactness. That beautiful book was then published at the expense of the French State, with notes of Didron, and dedicated to Victor Hugo.

The book which was thought by the monk of Athos to date from the tenth century was written in the fifteenth or in the beginning of the sixteenth by a monk whose name was Dionysios, and who was a painter in the monastery of Fourna, near Agrapha. By comparing the rules of the book with paintings from the ninth century, it seems, however, that the contents are very old, as it only refers to an old iconographic tradition, kept either orally or in writing, from the oldest time of the Church. This is shown, as explained by Didron, by a decision of the second Council of Nicæa, in 787, mostly brought about no doubt by Oriental, Jewish, and Mohammedan influence, and their dislike of figures in churches—known as iconoclasm. The decision is as follows: "The composition of the pictures is not invented by the painter, but it is from the prescription and narratives of the Catholic Church. What is delivered unto us from the past is venerable, says St. Basilius. This is shown by the remote age of the case in question, and the inherited learning of our forefathers, who were born of the Holy Ghost. When they saw pictures in holy temples they also made temples with their humble minds, and thereby offered their thanksgivings and their bloodless sacrifice to the Lord Almighty. The representation and inheritance of the painter are not his own (the execution only is his); these things are arranged and established by our forefathers who have built."

We find in this the same view which the monk of Mount Athos had explained to Didron.

As already remarked, the Bible was the main source of religious iconography. Rules for the pictorial representation of patriarchs and judges, of Kings and prophets, were taken from the Old Testament's descriptions of the various characters' appearance, position, etc. Apostles, martyrs, confessors, and other saints are pictured according to the New Testament, the Saints' Legends, and other sources. Guidance for the representation of the heavenly life was found in the Apocalypse, and, not least, in the very important medieval treatise of theology, *The Heavenly Hierarchy*, which the tradition of the Church ascribes to Dionysios Areopagita. This work had all the more importance because Dionysios, the first Bishop of Athens, was baptized by the Apostle Paul, and he had been a friend of the Apostle John, the writer of the Apocalypse. In addition to pictorial art, medieval poetry went also to John and Dionysios for its inspiration; the beautiful hymn of St. Ambrose, the "Te Deum," and Dante's "Inferno" have taken some of their subjects from these.

But, apart from this, iconography has been guided by the writings of the Church Fathers and by the teachers of the Church, the histories of the saints, and not least by the Church dogma and by medieval theology.

Regarding Didron we have added a number of explanatory remarks, for which we are responsible, but which those interested in the subject will recognise.

When Didron came to compare the iconography of the Roman Catholic churches he discovered that it was developed, in its main parts, quite as systematically as in the Greek churches. The disposition, the grouping of figures, and the placing within this, were the same; when searching carefully, he found here and there inscriptions and sentences which were not quite obliterated—the same sentences on scrolls as in the corresponding figures in Greece. He discovered that statues and stained glass figures had their systematic and determined places in the different parts of the church. "Clovis and Charles le Gros" and other French Kings had again to make room for the Kings of Judah, dukes and duchesses for holy men and women. It gave a new life to the population of the cathedral. Instead of empty arbitrariness there was meaning and connection—with the Church and its teaching.

Just as it was with the proportioning of the temple, so it was with its iconography—included within a system according to liturgic rules, a symbolical illustration of the history of religion and of the Church, and its teaching of the heavenly life. Throughout the whole pictorial representation of the Church, there was, in the Middle Ages, a method requiring understanding and an absolute identification with the way of thinking which is the basis of the system of iconography and its symbolism. It is always dramatic, and it gets its grouping, like a well-constructed drama, round a fixed idea. As long as medieval theology was living, a uniform but not monotonous use of this iconographic system can be found in the figure ornamentation of medieval churches, whether Greek or Roman. It is everywhere developed according to the same law. With the decline of theology in the fourteenth and fifteenth centuries, this spiritual art shared the fate of religious architecture: it lost its vital principle, the understanding of its deep symbolism and of its dramatic side.

The hatred of reformation for symbols, always too high to be understood by the vulgarity and banality of the crowd, caused the ancient masterpieces of the Church to be the object of a wild destruction, because, in their scanty imagination, people thought they represented popish idolatry. So, by degrees, the substance of this art was forgotten and with it also the power to understand it.

Not only Frenchmen, but all civilised people are indebted to these two spirited investigators, Didron and Viollet-le-Duc, with their vocation inspired by Chateaubriand and Victor Hugo, because, in the wide field of historical science, they have enlarged the understanding of the past and of its greatest monuments, the cathedrals, with their multitude of great figures belonging to the history of Christian religion.* It is obvious that Archbishop Eystein and his architect in Nidaros would not have built the front of the cathedral with its system of abutment hidden by a flat *screen* wall, ornamented with a rich architecture of arcades and niches, as shown by the two lower arcades of the front, if it had not been their inten-

* In 1831 Didron was in Greece, and in 1845 he published the book of Dionysios. During that time he had published several books on medieval iconography. In 1845, what we have referred to here, was known in France; but in this forsaken corner of the world, Christiania, history is written solely to flatter the people's vanity, while studying valueless branches of it, and the intendants of our museums give their unreserved support to Mr. Nordhagen and praise his spiritedness, when he proposes, for the decoration of the front, to put alternately a bishop and a king, a king and a bishop. This idea, which cannot be called a thought, was received rapturously, and called a rhythmic thought. After exhibiting our scheme of reconstruction in 1916, and explaining the big lines of the iconographic programme, the requirements of a medieval church regarding decoration were not understood any better, and the church historian officially appointed to the cathedral, Mr. O. Kolsrud, succeeded by request in issuing the following interpretation of Mr. Nordhagen's design of the front, of 1914 and 1917: the front is to symbolise the history of the Norwegian Church; the remaining rich medieval part will be the symbol of the Catholic Church, while Mr. Nordhagen's simple, rough addition over it will symbolise the Protestant Church! This home-made "iconographic" treatment was naturally read with great assurance by Mr. Nordhagen, in his report to the members of the Norwegian Storthing in 1918.

tion to continue in height. We have also convinced ourselves that the elevation reconstructed by us is included in its entirety and in its divisions of arcades as a completely reckoned part in a previously planned harmony.

After the explanation just given, of medieval iconography being an art of literary composition, it will be apparent that the Archbishop Eystein and his architect must have had their iconographic problem quite clear when they designed the architectonic distribution and the decoration of the west front.

In consequence we accompany our explanation of it by a representation of the iconographic drama, which is apparent in the engraving of Maschius, when this is considered together with the fragments found and the distribution of the front as previously proved historically.

We see that, over the main porch, Maschius has a relief of the Crucifixion as the central subject of the first row of figures. Anyone will be able to discover, in the other niches of the first row of figures in this front, a series of figures of apostles, distributed on each side of this relief.

The sixth figure from the relief towards south is a figure of a *bishop* (it does not come out clearly in the reproduction). Excepting St. Peter, none of the apostles were bishops, and the engraving of Maschius can thus suggest that the row of apostles has not been complete. But it contains a satisfactory explanation of this circumstance, as Maschius explains, in a note, that he has only drawn four niches in the farthest groups on each side, instead of the five existing, in reality.* Of the statues drawn on the engraving, the sixth and the seventh towards south from the relief, a bishop and a St. Dionysios, are still existing, not as No. 6 and 7, however, but as Nos. 7 and 8; the niche left out by Maschius has been the sixth from the relief, the one which has contained the twelfth apostle. The row of apostles has, then, been complete, with six apostles on each side of the relief. Outside this row there have been three statues on each side. It will be important, for our further explanation, to settle which persons could be represented, apart from the apostles. One of these statues, still wholly preserved, is a St. Dionysios.

St. Dionysios Areopagita suffered martyrdom towards the year 95, but he was mistaken, by Hilduin (814 to 824), Abbot of the monastery of St. Denis, near Paris, for Dionysios, Bishop of Paris, the patron saint of Frenchmen, who suffered martyrdom in about 286.

According to the narrative written by Hilduin, which was believed to be historical up to the eighteenth century, Dionysios Areopagita, after a visit to Rome, was sent by Clement "Romanus" as missionary to France, where he suffered martyrdom on Montmartre, then outside Paris. It is through a mistake that Dionysios Areopagita stands on the west front of Nidaros cathedral as the apostle of western Europe.

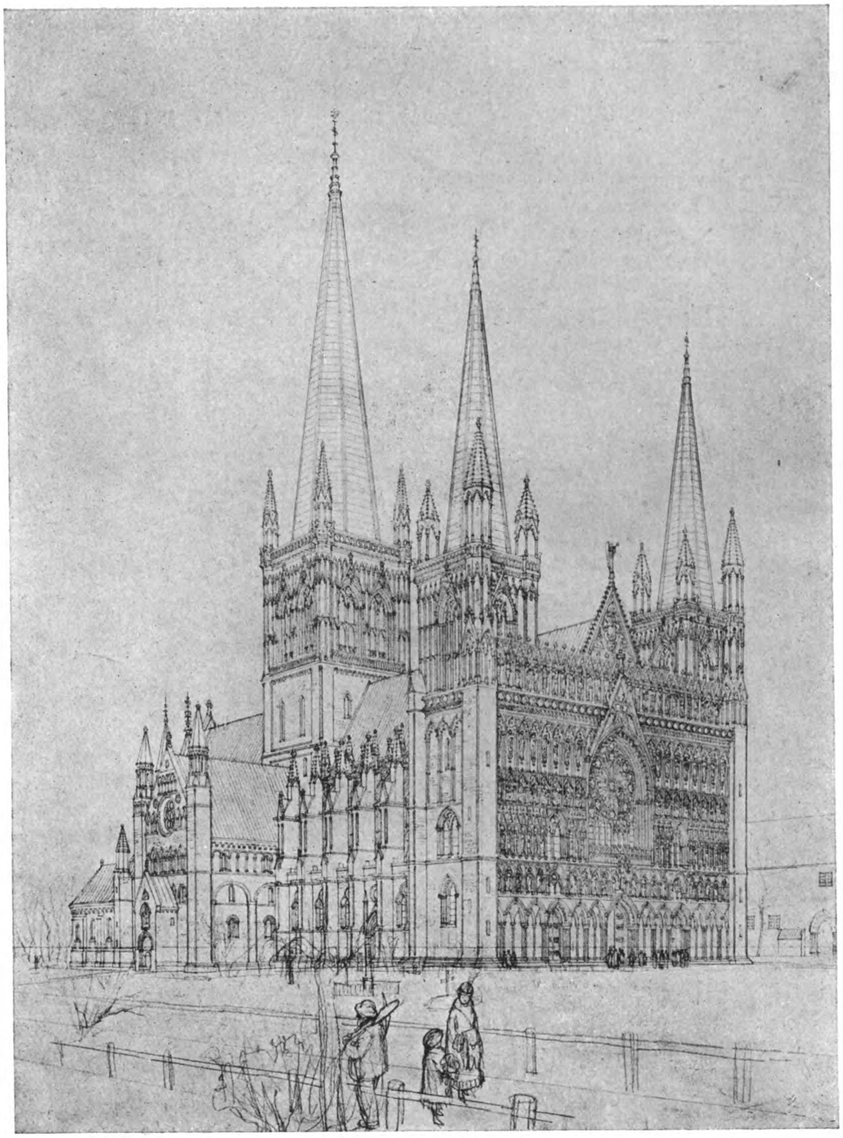
The bishop's statue on his right, that is, between him and the apostles, cannot be anyone else than he who sent him, Clement,† who, according to the medieval tradition, was baptized by the apostle Peter, to whom he succeeded as Bishop of Rome.

These two bishops, Dionysios and Clement, were included among the holy bishops because, as we said, they had been baptized by the apostles and continued their mission directly. Clement is included also, for the same reason, among the apostolic fathers.

It appears, from this, that the composer of the decoration had intended to represent the spreading of Christian teaching as part of a fixed programme. This is confirmed by the figure farthest

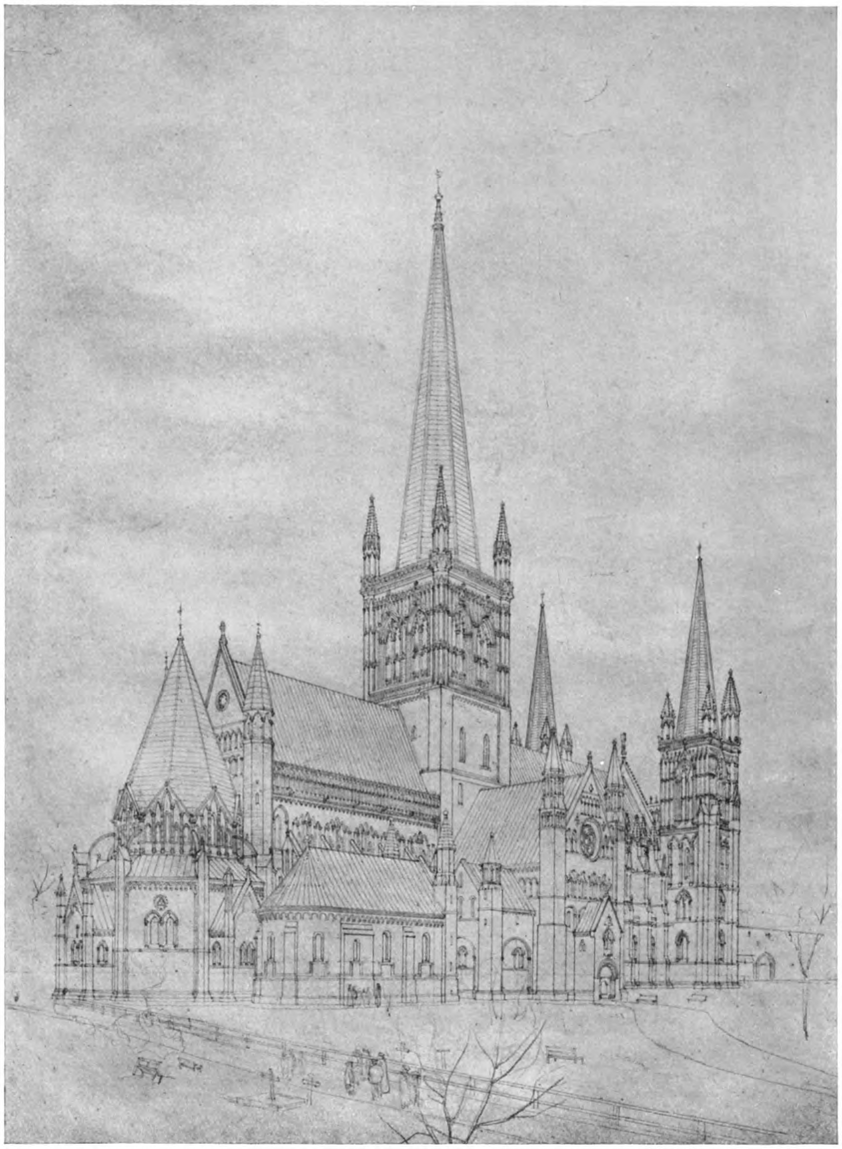
* At the foot of the engraving Maschius has written: "Ob graviores ruinas pro 5,4 modo extant vestigia," which can only be translated as follows: On account of the condition of the ruins, there remain only four instead of five (statues). He tries, therefore, to excuse his incorrectness of drawing by pretending that he has left out one of the niches in the group, because there remained only four statues.

† The first church built in Nidaros was also dedicated to Clement.

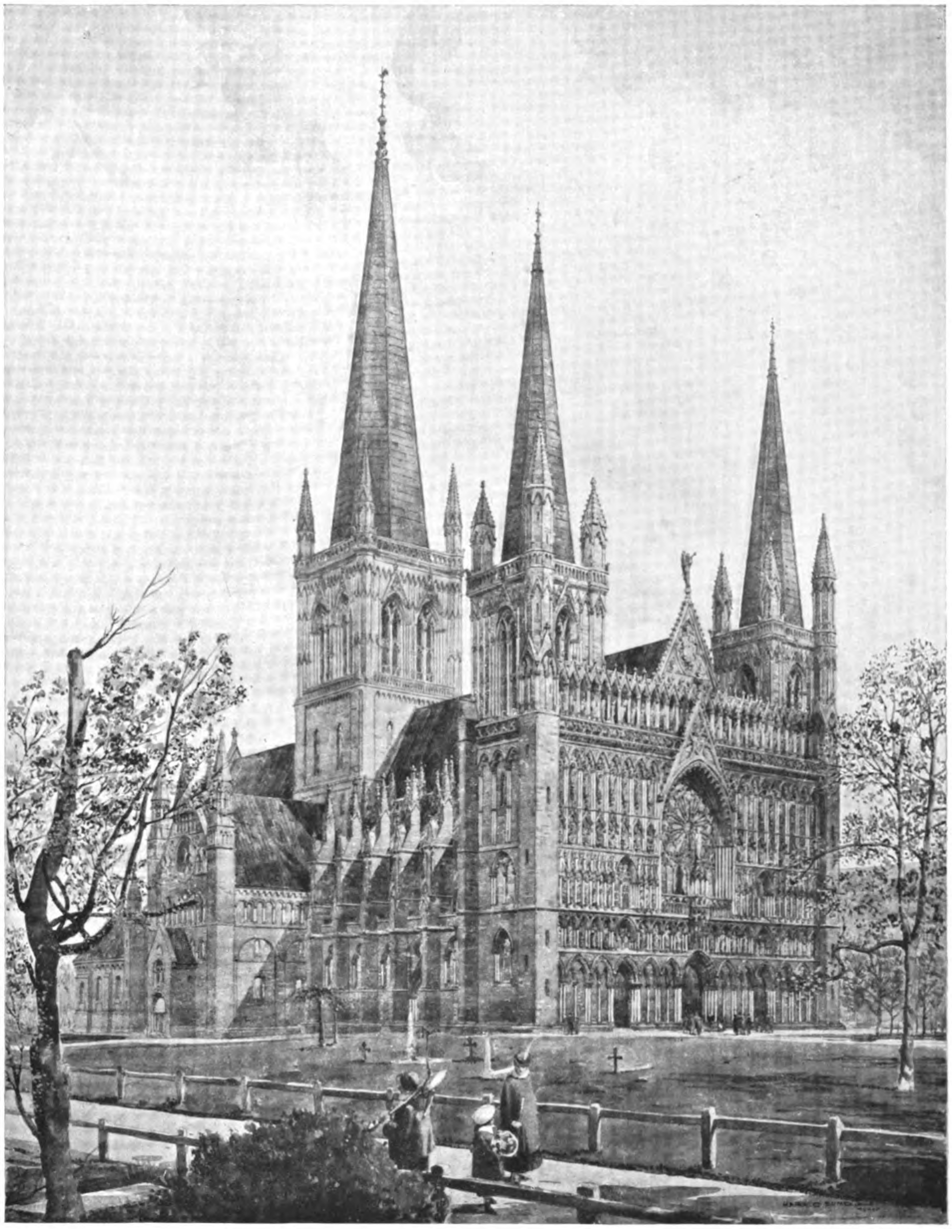


Cathedral of Nidaros. Perspective drawing from north-west, by Architect Harald Sund, after Macody Lund's reconstruction.

Plate XXXII.

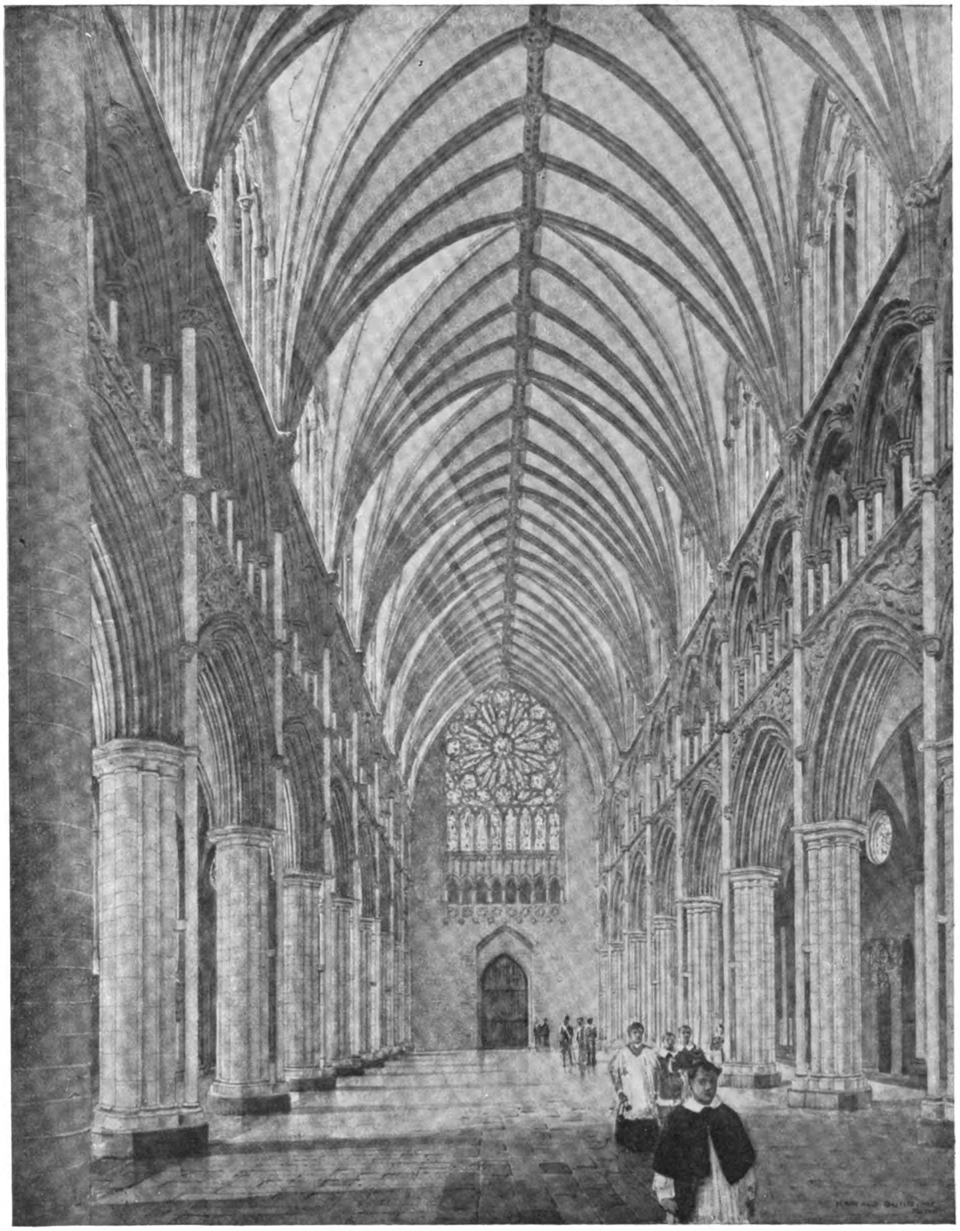


Cathedral of Nidaros. Perspective drawing from north-east, by Architect Harald Sund, after Macody Lund's reconstruction.

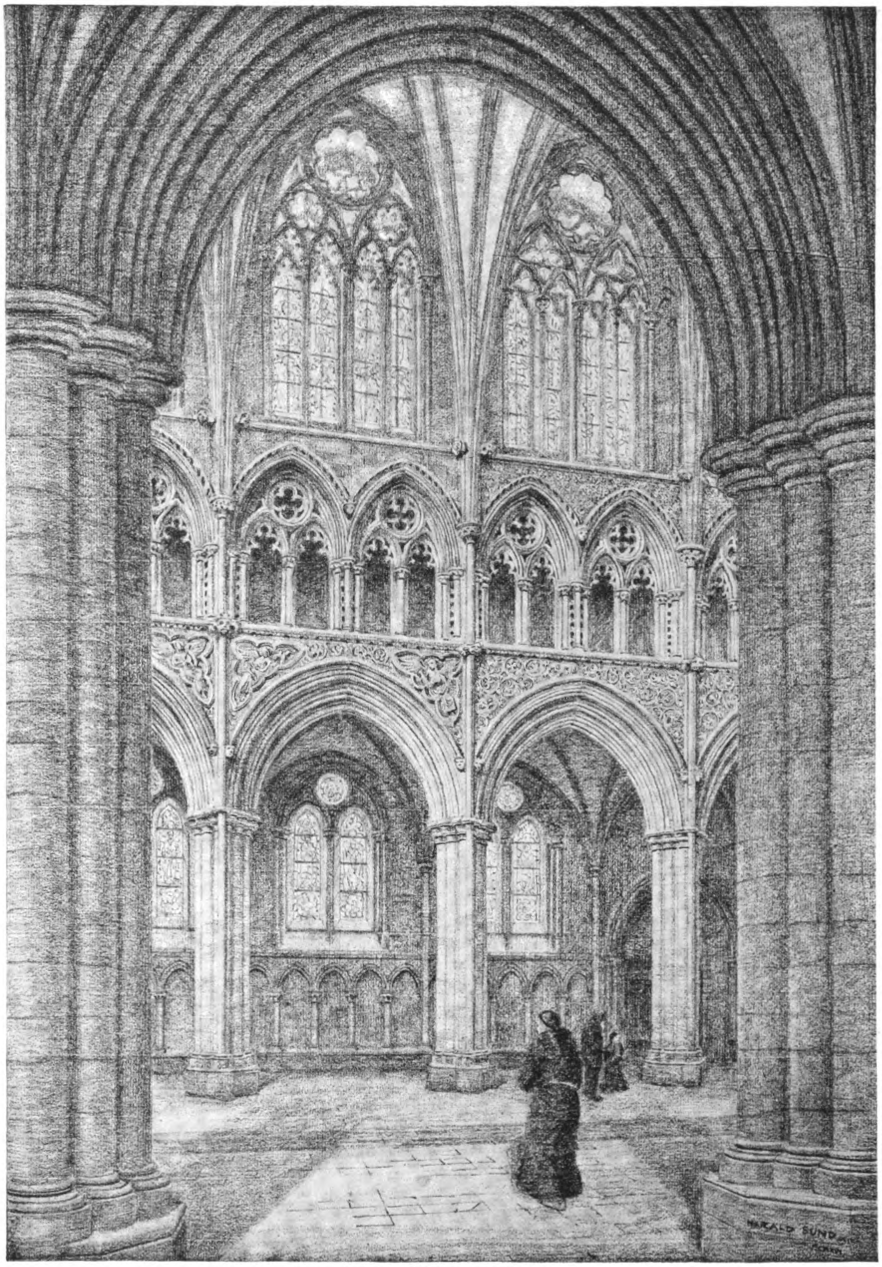


Cathedral of Nidaros from north-west, after a water-colour by Architect Harald Sund, after Macody Lund's reconstruction.

Cathedral of Nidaros from north-east, after water-colour by Architect Harald Sund.



Cathedral of Nidaros. The nave towards west, from water-colour by Architect Harald Sund, after Macody Lund's reconstruction.



Cathedral of Nidaros. Perspective drawing in charcoal towards north, after Macody Lund's reconstruction.

north on the engraving of Maschius, from its dress and attributes: he holds a stone in his hand, and represents St. Stephanus, the first martyr.

It is well known that St. Stephanus was one of the seven almoners of the apostles, but, having the gift of eloquence, he obtained an apostle's mission.

Taken broadly, we have in these three figures, apostles who all suffered martyrdom for their faith.

Regarding the second row of figures, our only guide is the engraving of Maschius. In the northern pier-stump, with the three remaining niches, two figures are preserved, male and female. Not only do these figures stand on a central part of the front, but, what is of great importance in church symbolism, they stand on the right-hand side. Therefore they cannot represent ordinary saints. As a female saint, according to the law of iconography, can least of all stand in such a central position, there is no other possible solution except that these two, male and female, saints represent persons who have stood in near relation to the central figure in the iconographic story of the front—that is to say, Christ. The two statues can only represent members of the "Holy Family," the nearest earthly relatives of Jesus. We must deduct, from this, that the other members of the Holy Family have been placed on the corresponding pier-stump.

In the two elucidated rows of figures we have thus found already a fixed guide to help us understand the meaning of the decoration, and we find it supplemented, or directly fixed, by the previously mentioned preserved relief of a seated figure, reproduced here from a photograph in fig. 334, and which is from the collection of stones belonging to the cathedral.

It will be remembered that, when explaining our reconstruction of the clerestory of the front, it was stated that this relief takes automatically its place in the front.

Some people have seen, in this, a statue of St. Olaf, others of Christ; it has been set up as a figure of Christ on the top part of the gable, and as St. Olaf in the canopy over the central window. Both positions are wrong iconographically. A figure of St. Olaf—that is to say, of a saint—is never put in a central part, because this would be pure paganism, a worship of saints, and absolutely against the teaching of the Catholic Church. As for the incorrectness of placing the relief in the gable as a figure of Christ, it is sufficient to attract attention to the fact that it is much too small and worked out in details, showing that it has been meant for a place lower down on the front. To this is added the decisive point, that the figure is seated on an elephant throne. In medieval heraldry and symbolism, the elephant was the sign of a high and ancient descent. Such a symbol applied to the representation of Christ as the *Son of God* would be a blasphemy. Christ, as such, is God and exists from all eternity. He does not need any evidence of His descent. When Christ is represented as God in medieval sculpture, He is always seated on a usual throne. The figure found is therefore undoubtedly that of a King, and the throne, this symbol of ancient descent, as well as the position of the arms, show decidedly that it represents King David playing the harp, the earthly forefather of Jesus, who can trace His ancestors, in the Bible, to our first parents.

We have demonstrated previously how the background of the relief bears marks of a capital, and that, judging from its size, the relief agrees, within an inch, with the lower part of the panels of the clerestory, divided according to the system.

This figure of King David indicates that, in the sixteen niches of this row, Christ's royal forefathers were placed, and in the centre of the row was Christ Himself.

We see, then, that as regards iconography also, there are important, not to say conclusive, indications in the remaining fragments; they give us both axes in the development of the drama: in the vertical axis the glorification of Christ as God, and in the horizontal transverse axis the representation of his royal forefathers, the symbol of His earthly descent.

In other words, the front has represented the Christian drama of the world: the Saviour,

His work, and His heavenly life. The artist has no doubt obtained his inspiration from *The Heavenly Hierarchy* of St. Dionysios.

The drama continues with a representation of apostles and martyrs and holy men and women, having in the centre Christ, to whose faith they had sacrificed their life, and they are united with God's angels in the heavenly paradise.

We have thus a subject which is not rare on French cathedrals, as Chartres, Amiens, Reims, and others, but which has nowhere been so perfectly and so methodically carried out as in Nidaros, because the whole front is used for it, while in France its development has been limited to porches, where such a systematic and extensive representation could not be realised, on account of the limited space.

From the indications found in the remains of the front, and in the engraving of Maschius, we have continued logically and achieved our iconographic reconstruction.

Over the main porch, of which fragments of relief are found, we have placed the Crucifixion as a symbol that those who enter the church do so with the Christian thought of sacrifice: the death and crucifixion of Christ for the salvation of mankind, the centre of Christian teaching. On each side of this scene are the apostles as the disseminators of this teaching, each according to his relation to Christ and his importance in the spread of the Gospel. On the right of the Crucifixion—that is, on the left of the spectator—there is the apostle Peter, on the left the apostle Paul, and so on, alternately. In accordance with the liberty allowed in the Middle Ages, we have thought of replacing two of the apostles by the Evangelists St. Mark and St. Luke. The remaining six niches are filled with figures of direct followers of the apostles and with church Fathers.

We have given the scene of the Crucifixion the royal decoration of a baldaquin. This is built from a gallery which we have introduced at the bottom of the large central window. Over this baldaquin we have placed a pulpit in connection with the mentioned gallery. Such a pulpit is justified in a church of pilgrims. In the Middle Ages, when open-air preaching was in favour, sermons were delivered from here on great festival days. It was on the gallery that the drama of the Passion took place during Easter week.

On the pulpit a rich baldaquin is raised, on which stands Christ, risen, victorious, and blessing, as the central figure of the whole front. At the back of Him is the "rose," famous in written tradition. According to this same tradition we have imagined it gilt and developed in rays, like an enormous halo.

We have thought that John the Baptist could be placed on the south pier-stump, among the figures of the Holy Family in the second row of figures. The sixteen other spaces in this row are to be filled with the most deserving martyrs and confessors of the Christian faith. In the most northern panels—that is, on the right side of the church—St. Olaf is placed, the patron saint of the Norwegian people, and on his left, St. Halvard. The two farthest panels towards south contain St. Magnus of the Orkneys and St. Thorlak of Iceland (accidentally the architect has drawn the figure of a bishop instead of St. Magnus). In the third row, the row of kings, the genealogy of Christ is represented. In this case King David, being the first, according to the law of symbolism, finds his place to the right on the wall of the church. The farthest on the left—that is, on the south side—is his son, Solomon; near David is Solomon's son, Roboam. Afterwards the kings follow alternately on both sides, until the royal forefathers Joachim and Jekonia, on each side of the figure of Christ, in front of the "rose;" while we have thought of placing the prophets, who had announced the coming of Christ, in the splays of the windows.

According to this arrangement we get on both sides of the Saviour His royal forefathers, and just underneath His nearest earthly relatives. In the row above the kings the female saints are placed, according to the rules of medieval iconography.

Highest in the gable of the nave, Christ as Judge is seated on His throne, supported by adoring angels. Down at the feet of the Saviour in the open colonnade on each side of Him are placed six seraphim with six wings; after these on each side are three cherubim with four wings, while the farther openings contain playing and singing angels with two wings.

And on the very summit of the gable the Archangel Michael stands in gilt bronze, calling with his trumpet to "the last day of judgment."

If we take the ascending axis of the drama we have Christ crucified, risen again, and

Fig. 334.—Cathedral of Nidaros. Statue of King David.

judging, and in the horizontal axis we have His royal forefathers, the symbol of His manhood.

In the canopy above the central window, geometrical tracery is introduced, the usual medieval symbol of the Trinity, to whom the cathedral was consecrated.

At the back of the perforated balustrade, and in front of the detached colonnade of white marble columns, with the singing angels, the so-called "Gloria in Excelsis Gallery" is placed. This gallery rests on the main cornice which forms a finish to the rich screen front with its powerful projection. Such a gallery, like the one of the central window, had its liturgic

functions in the fully developed medieval cathedrals. Singers stood here on great festival days and sang "Gloria in Excelsis," while priests in the gallery underneath the rose gave their benediction to the kneeling faithful.

In the Middle Ages, when pilgrims on their journey to Nidaros came up Stenbergene, and got their first glimpse of the cathedral, they fell on their knees and thanked God that they had reached "Stadarins hins helga Olafs Konungs" ("the town of the holy King Olaf"). The hill coming down from Stenbergene is called Feginsbrekka (the hill of joy), like the corresponding place on the way to Jerusalem, Mons Gaudii, from which the first glimpse of the Holy City was obtained and which had a similar name.

A stream of pilgrims was not only a crowd of cripples, but it consisted also of people going to a fair, and of persons who had arranged to meet at this great festival. They came from all countries, even from Spain. They all had expectations, not only in the benefaction of the saint-King, but no doubt they were also full of expectancy concerning his far-famed church. They found one which in its splendour illustrated the venerable hymn of praise of St. Ambrose which they all knew.

Te Deum laudamus,
te Dominum confitemur,
te æternum Patrem
omnis terra veneratur.

Tibi omnes Angeli,
tibi cœli et universæ Potestates,
tibi Cherubim et Seraphim
incessabili voce proclamant:

Sanctus, sanctus, sanctus
Dominus Deus Sabaoth!
Pleni sunt cœli et terra
majestatis gloriæ tuæ.

Te gloriosus Apostolorum chorus,
te Prophetarum laudabilis numerus,
te Martyrum candidatus laudat exercitus.

The pilgrims had never seen such a complete and systematic representation of the heavenly host in any other church, and they went back spreading the fame of the cathedral of St. Olaf as the "most beautiful church in Christendom."

The coinciding evidence of all written sources is no exaggeration. Anyone acquainted with medieval architecture must admit, that another church with such richness and splendour, and such emotion under the control of one harmony, is not to be found in Europe.

To this church only there can belong a front like the one we have produced from the church itself.

In spite of this richness and this variety, there is quietness and rest in this front, as if it were a part of nature. This is due to the raising of the church *ad quadratum*, according to the ancient classic law of harmony.

The honour is not ours. We say with the singing Angels of the front:

Gloria in Excelsis Deo.

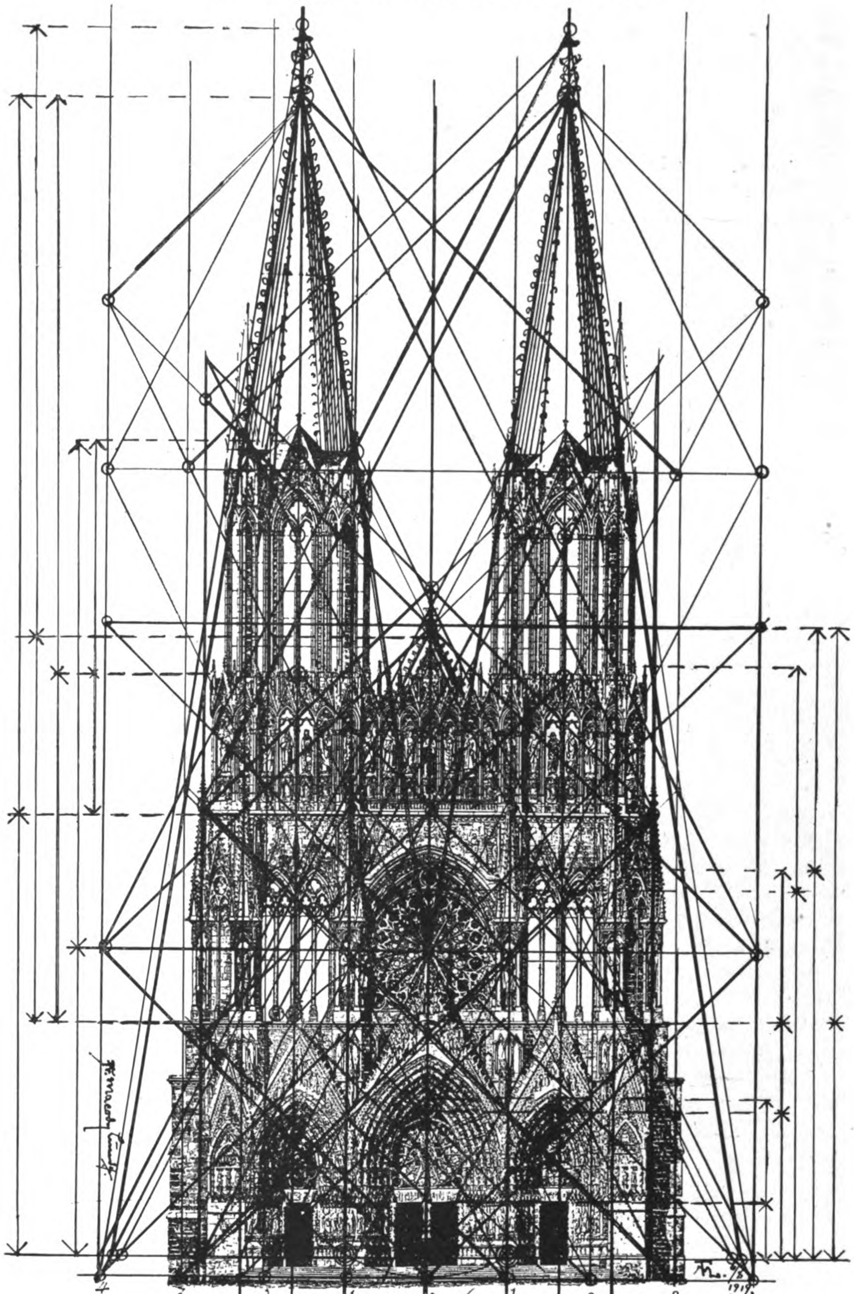


Fig. 335.—Cathedral of Reims. West front with the spires added, analysis.

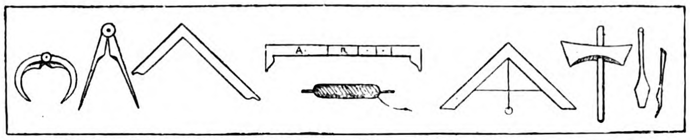


Fig. 336.—Measuring Rod or Embates with the scale of the sectio aurea, sculptured on the sarcophagus of a Roman architect.

APPENDIX

WE remarked previously that after the greater part of the work was printed, we came accidentally across an orthogonal drawing of the west front of Reims cathedral. We reproduce it here in fig. 335 with our analysis drawn in, showing that this cathedral also has been strictly designed *ad quadratum*. It will be seen that the spire drawn according to our method finishes harmoniously and solves the elevation of the front. The drawing does not need any comment. To use the words of the compiler, Mr. Hans Dedekam, it appears like an excellent " symptom of the naïveté and ignorance of a certain group of people " in the Middle Ages.

As the next proof concerning this, we give a drawing (fig. 336) reproducing the sculpture on the sarcophagus of a Roman architect from the first or second century of the Church, taken from Didron's *Iconographie chrétienne*, Paris, p. 364, 1848. Among the tools of the architect reproduced here, we notice an *Embates*, or measuring-rod with a scale according to the *sectio aurea*. This drawing, which also we noticed too late, gives a tangible proof of the truth of our representation of the principle of classic architecture.

Finally, we give the reproduction (fig. 337) of a relief from a church of Spalato, Dalmatia, which we have come across recently in the publication given out for the 250 years jubilee of the University of Lund, in 1918, where the relief is proved to have been done about A.D. 500—that is to say, during the lifetime of the philosopher and mathematician Boëtius. We see that it represents a regular pentagram, constructed nearly 800 years before the anonymous French *Traité de Géométrie*, to which the compiler of Trondhjem (see previously, pp. 29 and 141) attributes the honour of containing the oldest examples of this geometrical figure.

Fig. 337.—Pentagram as ornament on a relief in a church of Spalato, Dalmatia.

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